Övningsuppgifter

1,1 $\lambda = 200\text{nm}$

$f = \frac{1}{\lambda} = \frac{1}{200}$

$E = \frac{hc}{\lambda} = 6.2\,eV$ – kommer

1,2 $m_e c^2 = \frac{hc}{\lambda} \Rightarrow 2{,}42\,pm$ $\quad p = \frac{h}{\lambda} \Rightarrow p = 2{,}73 \cdot 10^{-22}\,Js/m$

1,3 $I = 1\,kW/m^2$

$\lambda = 600\,nm$

a) $W_{foton} = \frac{hc}{\lambda}$ – gör om till watt $\quad P = E/t$

$\frac{1000}{W_{foton}}$ = elektroner som infaller per sekund $= n$ $\Rightarrow 3{,}02 \cdot 10^{21}$ fotoner

b) 

$p = \frac{F}{A}$ $\quad p = \frac{h}{\lambda} \approx 1{,}1 \cdot 10^{-27}$

$p \cdot 2$ = trycket en foton ger

$p \cdot 2 \cdot n = 6{,}6 \cdot 10^{-6}\,Pa$ tryck som alla fotoner trycker med.

c) liten betydelse, långsam!

1,4 $\quad$ 10eV / 1KeV

$\lambda = \frac{h}{p}$

$\boxed{\lambda = \frac{h}{2Em}}$

$E = \frac{mv^2}{2} = \frac{p^2}{2m}$

	10 eV	1 keV
e^-	0,39 nm	0,039 nm
p	9, pm	0,9 pm

(1,5) $\lambda = 0{,}15\,nm$

$$E = q \cdot U = \frac{h^2}{2m\lambda^2}$$

$W_{foton} = \frac{hc}{\lambda}$

U = spänningen

(1,6)

a) $n \cdot \lambda = a \sin\Theta$ energi ↑ ⇔ våglängd ↓

b) $1 \cdot \frac{hc}{E} = \ell \sin 30°$

$E = \frac{hc}{\lambda}$

(1,8)

a) $\Psi(x,0) = \begin{cases} Nxe^{-x/a}, & x > 0 \\ 0, & x < 0 \end{cases}$ $\left(Nxe^{-x/a}\right)^2 = N^2 x^2 e^{-\frac{x \cdot 2}{a}}$

$$\int_{-\infty}^{+\infty} \Psi^2(x,0)\,dx = 1 \quad N^2\int_0^{\infty} x^2 e^{-\frac{2x}{a}}$$

$$= N^2\left(\left(\frac{-ae^{-\frac{2x}{a}}}{a} \cdot x^2\right) + \int_0^{\infty} 2x\right) = N^2\left(\left[\frac{-ae^{-\frac{2x}{a}}}{2} \cdot x^2\right]_0^{\infty} - \left(\left[-ae^{-\frac{2x}{a}} \cdot x\right] - \int 2e^{-\frac{2}{a}}\right.\right.$$

Osv. Osv $= 1$

Bryt ut N!

$|N|^2 = \frac{4}{a^3}$

b) $x < 0$ Sannolikhet $= 0$

$0 < x < a$ $1 - 5e^{-2}$

$a < x$ $5e^{-2}$

(1,9) $\Psi(x,0) = Ne^{-K|x|}$

a) $$\int_{-\infty}^{+\infty} Ne^{-K|x|}\,dx = 1 \quad \int_0^{\infty}\left(Ne^{-Kx}\right)^2 dx = N^2\left[\frac{e^{-2kx}}{-2k}\right]_0^{\infty}$$

$$1 = N^2\left(\frac{1}{2k} + \frac{1}{2k}\right) \quad N\int_{-\infty}^{0} e^{2kx}\,dx = N^2\left[\frac{e^{2kx}}{2k}\right]_{-\infty}^{0}$$

$$k = N^2 \Rightarrow |N| = \sqrt{k}$$

(1,11) $\Psi_1(x,0) = Ae^{ikx}$ $\Psi_2(x,0) = Be^{i(kx+\varphi)}$ ♡

$(\Psi_1^* \Psi_2 + \Psi_1 \Psi_2^*)$ = sannolikstätheten

$$Ae^{-ikx} \cdot Be^{i(kx+\varphi)} + Ae^{ikx} \cdot Be^{-i(kx+\varphi)}$$

$$ABe^{-ikx+i(kx+\varphi)} + ABe^{ikx-i(kx+\varphi)}$$

$$ABe^{-i\varphi} + ABe^{i\varphi}$$ $-ikx+i(kx+\varphi)$

$$AB(e^{-i\varphi} + e^{i\varphi})$$ $i\varphi$

$$AB(\cos\varphi - i\sin\varphi + \cos\varphi + i\sin\varphi)$$ $ikx-ikx+i\varphi$

$$A^2 + 2AB\cos\varphi + B^2$$

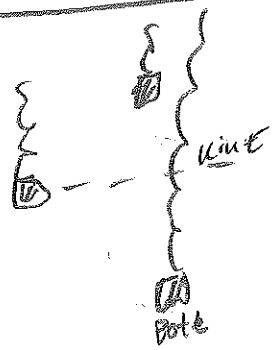
(2,1)

a) Kraften är återförande mot jämviktsläget

b) $\bar{F} = -\nabla V$ $F_x = -cx^3$

$$\boxed{F = -\frac{cx^4}{4} + D}$$

c) $$-\frac{cx^4}{4} = \frac{mv^2}{2}$$



$$x^4 = -\frac{mv^2 \cdot 2}{c}$$

$$x = \pm\left(\frac{-mv^2 \cdot 2}{c}\right)^{1/4}$$

(2.2)

$$V(x) = \begin{cases} 0 & x < 0 \\ \frac{kx^2}{2} & x > 0 \end{cases}$$

$$x_{\text{vänd}} = V_0 \sqrt{m/k}$$

(2.3)

$$\Psi(x,t) = Ae^{(ikx - i\omega t)}$$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\Psi''(x) + V(x)\Psi \qquad \omega = \omega(k)$$

$$\Psi'_t = -Ai\omega e^{ikx - i\omega t}$$

$$\Psi''_{xx} = -Ak^2 e^{ikx - i\omega t}$$

$$\Psi'_x = Aik e^{ikx - i\omega t}$$

$$-i\hbar Ai\omega e^{ikx - i\omega t} = +\frac{\hbar^2}{2m} Ak^2 e^{ikx - i\omega t} + V(x) A e^{(ikx - i\omega t)}$$

$$\hbar\omega = \frac{\hbar^2}{2m} k^2 + V(x)$$

$$\omega = \frac{\hbar k^2}{2m} + \frac{V(x)}{\hbar}$$ konstant i 1:din

$(V(x) = 0)$

$$\omega = \frac{\hbar k^2}{2m}$$

(2.4) $$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \psi + V(x)\psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \psi + \frac{c}{4} x^4 \psi$$

(3.1) $E = 0{,}1$ eV   500 partiklar/s

strömmen

(3.2) a) $R + T = I = 1$

$$\overbrace{\left(\frac{K-k}{K+k}\right)^2}^{R} + \overbrace{\frac{4kK}{(K+k)^2}}^{T}$$

$$\frac{(K-k)^2 + 4kK}{(K+k)^2}$$

$$= \frac{K^2 - 2kK + k^2 + 4kK}{(K+k)^2}$$

$$= \frac{K^2 + 2kK + k^2}{(K+k)^2}$$

$$= \frac{(K+k)^2}{(K+k)^2} = 1$$

om $E \to \infty$

b) $R \to 0$

$T \to 1$ Mer energi $\to$ större chans att den transmitteras

Om k går mot ∞

K antingen är potentialen 0 eller V_0

$4kK \approx 4k^2 \quad \frac{4k^2}{(2k)^2} \to 1$

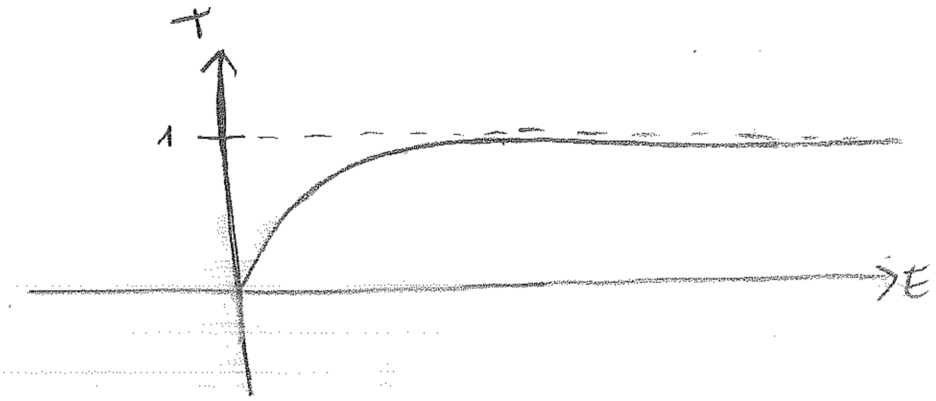
c) Om $V_0 \to \infty$

$K \gg k$

$$T = \frac{4kK}{(k+K)^2} = \frac{k}{K^2} \to 0$$

$$R \to 1$$

d)



3) a) $-\frac{\hbar^2}{2m}\phi''(x) + V\phi(x) = E\phi(x)$ $V=0$

$$-\frac{\hbar^2}{2m}\phi''(x) = E\phi(x)$$

$$\phi''(x) = \underbrace{\frac{-E2m}{\hbar^2}}_{k_1^2}\phi(x)$$

$$\phi''(x) + k_1^2\phi(x) = 0$$

$$\phi(x) = Ae^{ikx} + Be^{-ikx}$$

$$-\frac{\hbar^2}{2m}\phi''(x) + V_0\phi(x) = E\phi(x)$$ $V_0 \quad x>0$

$$\phi''(x) = \underbrace{\frac{2m}{\hbar^2}(E - V_0)}_{k_2^2}\phi(x)$$ $E > V_0$

$$\phi''(x) + k_2^2\phi(x) = 0$$

$\alpha = \pm ik_2$ $\phi = Ce^{ik_2x} + \underbrace{De^{-ik_2x}}_{0 \text{ inget ?}}$

i) $A + B = C$

ii) $ik_1(A-B) = ik_2C$

$\boxed{X=0}$

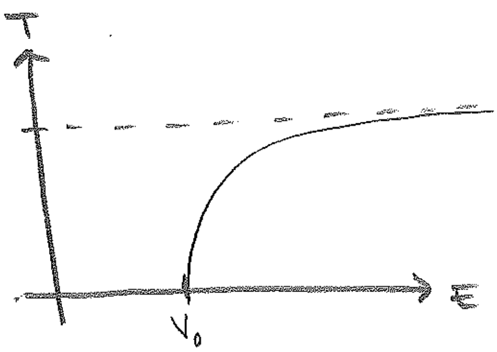
b) Fortsätt med passning.

Beskriv A, B och C

Få ut T och R beroende på A, B, C och sedan sätt in energi sedan

Plats

c)



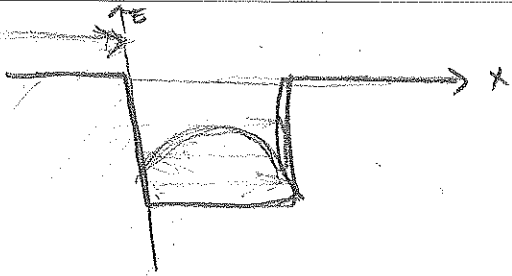
T är en monotont växande funktion för $E > V_0$

d) $R + T = 1$

$$\frac{\left(\sqrt{E}-\sqrt{E-V_0}\right)^2}{\left(\sqrt{E}+\sqrt{E-V_0}\right)^2} + 4\sqrt{E(E-V_0)} = \frac{(\sqrt{E})^2 - 2\sqrt{E-V_0}\cdot\sqrt{E} + 4\sqrt{E(E-V_0)} + \left(\sqrt{E-V_0}\right)^2}{\left(\sqrt{E}+\sqrt{E-V_0}\right)^2}$$

$$= \frac{(\sqrt{E})^2 + 2\sqrt{E(E-V_0)} + \left(\sqrt{E-V_0}\right)^2}{\sqrt{E}+\sqrt{E-V_0}} = \frac{\left(\sqrt{E}+\sqrt{E-V_0}\right)^2}{\left(\sqrt{E}+\sqrt{E-V_0}\right)^2} = 1$$

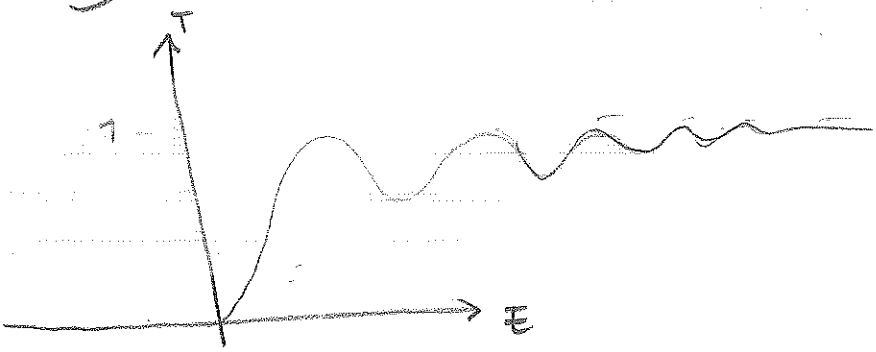
3.4. a)



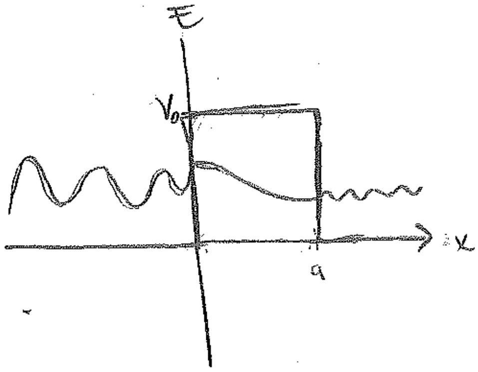
Interferens
reflekteras fram och tillbaka
Stående våg

– interferens i tunna skikt

b)



3.5



$V_0 = 2eV$

$a = 2nm$

$E < V_0$

$$T = e^{-2a\sqrt{2m(V_0-E)/\hbar^2}}$$

$$T = e^{-2\cdot 2\cdot 10^{-9}\sqrt{2m_e(2-E)/\hbar^2}}$$

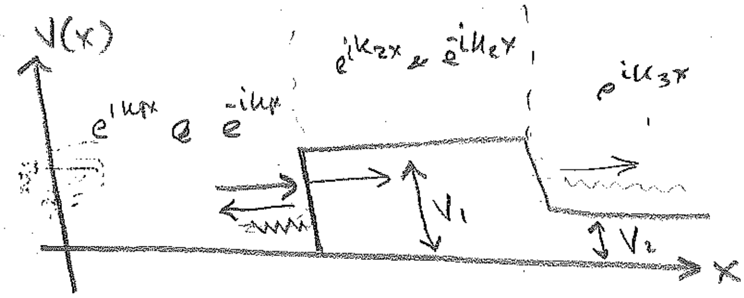
$$0,01 = e^{-2\cdot 2\cdot 10^{-9}\sqrt{2m_e(2-E)/\hbar^2}}$$

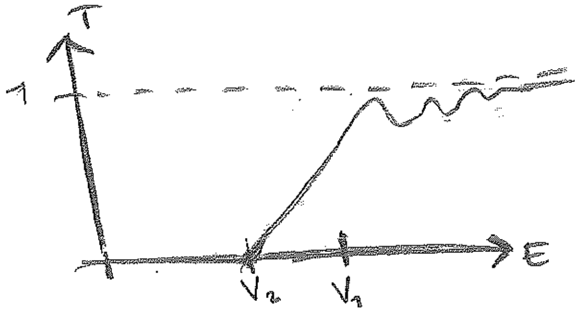
$$\ln 0.01 = -2\cdot 2\cdot 10^{-9}\sqrt{2m_e(2-E/\hbar^2)}$$

$$\frac{\left(\frac{\ln 0,01}{-2\cdot 2\cdot 10^{-9}}\right)^2}{2m_e}\cdot\hbar^2 = 2-E$$

$E = 1,95eV$

3.7.

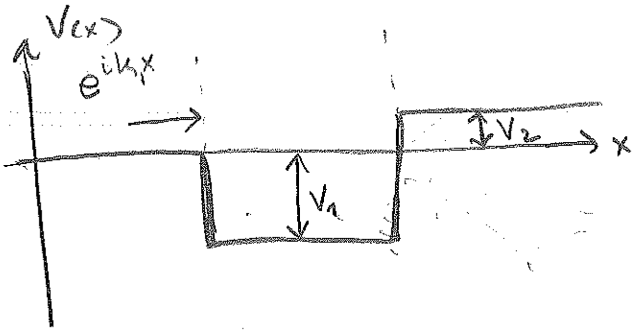


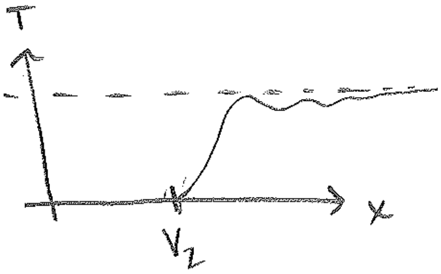


om $E < V_2$
$T = 0$

om $V_2 < E < V_1$
kan du tunnla

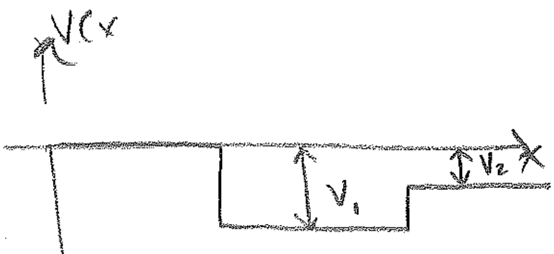
om $E > V_1$
kan reflekteras,
resonanser
förekommer.



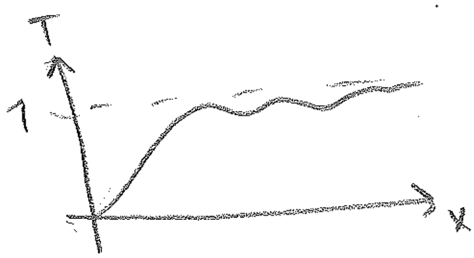


$E < V_2$
$T = 0$

$E > V_2$
förekommer
resonanser



$T > 0$ för alla $E > 0$ Resonanser finns

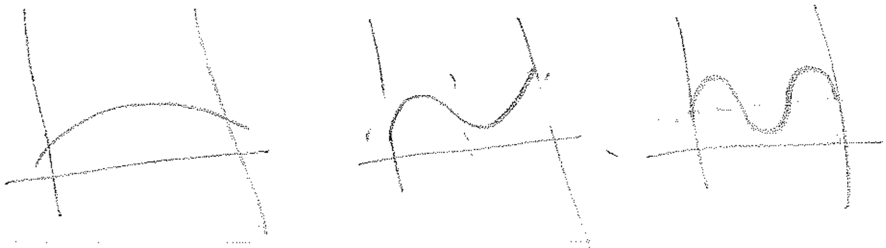


4.1

a) ~~?~~ nm

b) 0,3 eV

4.2



~~för tillstånd~~ med udda n.

4.4

$$\tan\left(\sqrt{2mb^2(E+V_0)/\hbar^2}\right) = -\sqrt{(E+V_0)/(-E)}$$

$$\phi = \begin{cases} a_2 e^{Kx} & x<-b \\ C_2 \sin(kx) & \text{udda} \\ -a_2 e^{-Kx} & \end{cases}$$

kontinuerlig i $-b$

$$C_2 \sin(kb) = a_2 e^{-Kb} \quad (1)$$

ϕ' kontinuerlig i $-b$

$$kC_2 \cos(kx) = K a_2 e^{-Kb} \quad (2)$$

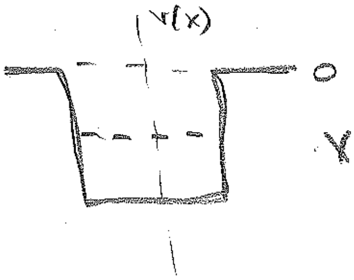
$$\frac{(1)}{(2)} \quad \frac{\tan(-kb)}{k} = \frac{1}{K}$$

$$\tan kb = \frac{k}{K} = \frac{\sqrt{\frac{-2m}{\hbar^2}(V_0+E)}}{\sqrt{\frac{-2mE}{\hbar}}} = \sqrt{\frac{\frac{-2m}{\hbar}(V_0+E)}{\frac{-2mE}{\hbar}}}$$

$$\tan(-kb) = \sqrt{\frac{V_0+E}{E}}$$

$$\tan(kb) = -\sqrt{\frac{V_0+E}{E}}$$

4.5



$$V(x) = \begin{cases} -V_0 & |x| \leq b \\ 0 & |x| > 0 \end{cases}$$

$$E_1 = \frac{-V_0}{2}$$

SE utanför:

$$-\frac{\hbar^2}{2m}\phi'' + V\phi = E\phi \qquad V_0 = 0$$

$$-\frac{\hbar^2}{2m}\phi'' = -\frac{V_0}{2}\phi$$

$$\phi'' = \underbrace{\frac{-2mE}{\hbar^2}}_{K^2}\phi$$

$K > 0$ då $E < 0$

$$\underline{\phi = K^2\phi}$$

$$\phi = Ae^{\alpha x}$$

$$\alpha^2\phi = k^2\phi$$

$$\Longrightarrow \alpha = \pm K$$

$$\phi(x) = Ce^{-Kx} + De^{Kx}$$

måste begränsas för $x \to \pm\infty$

$$\phi = \begin{cases} Ce^{-Kx} & x > b \\ De^{Kx} & x < b \end{cases}$$

exponentiellt avklingande

SE innanför:

$$-\frac{\hbar^2}{2m}\phi'' - V_0\phi = -\frac{V_0}{2}\phi$$

$$\phi'' = -\underbrace{\frac{2m}{\hbar^2}(E + V_0)}_{k^2}\phi \qquad k > 0$$

$$\phi'' = -k^2\phi$$

$$\phi = Ae^{\alpha x}$$

$$\boxed{\phi = Ae^{ikx} + Be^{-ikx}}$$

$$\begin{cases} Ce^{-Kx} \\ Ae^{ikx} + Be^{-ikx} \\ De^{Kx} \end{cases}$$

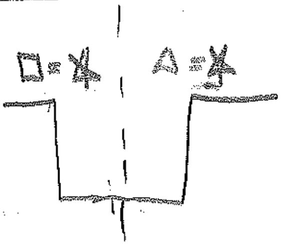
$V(x) = V(-x)$

udda $\phi(x) = -\phi(-x)$

jämna $\phi(x) = \phi(-x)$

4.5

b) $X = b$



$$\phi = \begin{cases} Ce^{-Kx} & x > b & \triangle \\ Ae^{ikx} + Be^{-ikx} & |x| < b & \cancel{\ast} \\ De^{Kx} & x < -b & \square \end{cases}$$

$$Ae^{ikx} + Be^{-ikx} = Ce^{-Kx} \qquad \text{i } b$$

$$(Ae^{ikx} + Be^{-ikx} = De^{Kx} \qquad \text{i } -b)$$

$$ikAe^{ikx} - ikBe^{-ikx} = -KCe^{-Kx} \qquad \text{i } b$$

$$(ikAe^{ikx} - ikBe^{-ikx} = KDe^{Kx} \qquad \text{i } -b)$$

Inkenför

Jämna

$$Ae^{ikx} + Be^{-ikx} = Ae^{-ikx} + Be^{ikx} \qquad \phi(x) = \phi(-x)$$

$$A = B$$

$$A(e^{ikx} + e^{-ikx}) = \underbrace{2A}_{C_1} \underbrace{\tfrac{1}{2}(e^{ikx} + e^{-ikx})}_{\cos(kx)}$$

2A Si

Udda

$$Ae^{ikx} + Be^{-ikx} = -Ae^{-ikx} - Be^{ikx} \qquad \phi(x) = -\phi(-x)$$

$$A = -B$$

$$B = -A$$

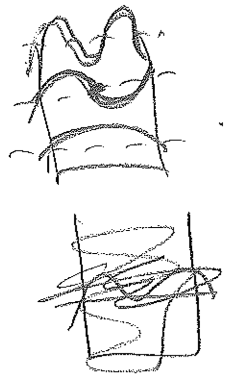
$$\underbrace{2iA}_{C_2} \underbrace{\tfrac{1}{2i}(e^{ikx} - e^{-ikx})}_{\sin(xk)}$$

$$\phi = \begin{cases} C_1 \cos(xk) & \text{Jämn} \\ C_2 \sin(kx) & \text{udda} \end{cases}$$

$$Ae^{ikb} + Be^{-ikb} = Ce^{-Kb}$$
$$ikAe^{ikb} - ikBe^{-ikb} = -KCe^{-Kb}$$

Passningsvillkoret

c) antar att grundtillståndet i en potentialbrunn är alltid jämnt



Kontroll!

$$C_1\cos(kb) = Ce^{-Kb} \quad (1)$$
$$-C_1 k\sin(kb) = -KCe^{-Kb} \quad (2)$$

$$\frac{(2)}{(1)} = -k\tan(kb) = -K$$

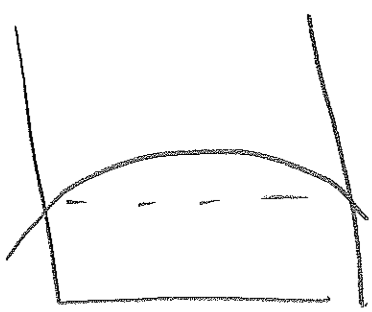
$$\tan(kb) = \frac{K}{k} = \frac{\sqrt{\frac{-2m\left(-\frac{V_0}{2}\right)}{\hbar^2}}}{\sqrt{\frac{2m}{\hbar^2}\left(-\frac{V_0}{2}+V_0\right)}}$$

$$\tan\left(\left(\frac{2m}{\hbar^2}\left(\frac{V_0}{2}\right)\right)b\right) = \sqrt{1} = 1$$

$$\tan(kb) = 1$$

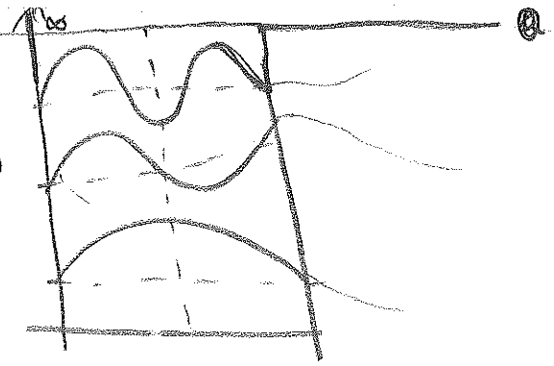
$$\boxed{kb = \frac{\pi}{4}} \quad \ldots \quad V_0 b^2 = \frac{\pi^2\hbar^2}{16m}$$

d)



8.

a)



b) $$\boxed{E_n = \frac{\hbar^2\pi^2}{2ma^2}(n^2)}$$

9.

a) $$E_1 - E_2 = \frac{\hbar^2\pi^2}{2ma^2}\left(n_1^2 - n_2^2\right)$$

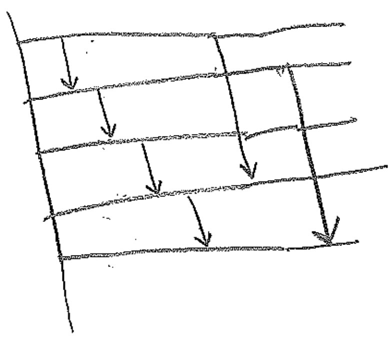
$$2{,}5 = \frac{\hbar^2\pi^2}{2ma^2}\left(n_1^2 - n_2^2\right)$$

$$a = \frac{\hbar^2\pi^2}{2m\,2{,}5\,(1-4)} \qquad a = 0{,}67\ \mathrm{nm}$$

b) $$E = \hbar\omega = 2\pi f \cdot \hbar = \frac{hc}{\lambda}$$

$\underline{\underline{496\mathrm{nm}}}$

11. s. 80 kolla möjligheterna



$$E = \frac{\hbar^2\pi^2}{2ma^2}(n_4^2$$

$$E = \frac{hc}{\lambda}$$