$\boxed{6.1}$

$$f(z) = \frac{2z^2 + 3z + 1}{z^3 - z^2 + z - 1} = 0$$

$$z^2 + \frac{3}{2} z + 1 = 0$$

$$z = -\frac{3}{4} \pm \sqrt{\frac{9}{16} - \frac{8}{16}} = -\frac{3}{4} \pm \frac{1}{4}$$

$$\boxed{z_1 = -1,\ z_2 = -\frac{1}{2}}$$ nollställen

$$z^3 - z^2 + z - 1 = 0$$

$$z_1 = 1$$

$$(z_1 - 1)(z - z_2)(z - z_3) = 0$$

$$\begin{array}{r|l} & z^2 + 1 \\ z - 1 & z^3 - z^2 + z - 1 \\ & -(z^3 - z^2) \\ \hline & z - 1 \\ & -(z - 1) \\ \hline & 0 \end{array}$$

$$\Rightarrow (z - 1)(z^2 + 1) = 0$$

$$\Rightarrow \boxed{\begin{array}{l} z_1 = 1 \\ z_2 = i \\ z_3 = -i \end{array}}$$ poler

faktoriserat $$\boxed{f = \frac{(z+1)(z+\frac{1}{2})}{(z-1)(z^2+1)}}$$

$\boxed{6.2}$ Ange på formen $a+bi$

a) $\sin(i)$

$$\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\sin(i) = \frac{e^{i2} - e^{-i2}}{2i} = \frac{e^{-1} - e^{1}}{2i} = \frac{\frac{1}{e} + \frac{e^2}{e}}{2i} =$$

$$= \frac{\frac{1+e^2}{e}}{2i} = \frac{1+e^2}{e} \cdot 2i$$

b) $$\cos(2-i) = \frac{e^{(2-i)i} + e^{-(2-i)i}}{2} =$$

$$= \frac{e^{2i+1} + e^{-2i-1}}{2} = \frac{e^{2i+1}(1+e^{-1})}{2}$$

$\boxed{6.4}$

$$|\sin(x+iy)|^2 = \sin^2 x + \sinh^2 y \quad , x,y \in \mathbb{R}$$

$$|\sin(x+iy)|^2 = |\cos(x)\sin(iy) + \sin(x)\cos(iy)|^2 =$$

$$\sin^2(z) = \frac{1}{(2i)^2}\left(e^{iz} - e^{-iz}\right)^2 = \frac{1}{-4}\left(\qquad \right)^2 =$$

$$= \frac{1}{-4}\left(e^{2iz} - 2e^{iz-iz} + e^{-2iz}\right) = -\frac{1}{4}\left(e^{2iz} + e^{-2iz} - 2\right) =$$

$$=$$

$\boxed{6.5}$

$\tan(ix) \stackrel{?}{=} \tanh(x)$

$\boxed{6.7}$

$$4\cos(z) - 3 = i$$

$$\boxed{\cos(z) = \frac{3+i}{4}} \quad *$$

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2} = \frac{3+i}{4}$$

$$\Leftrightarrow \quad e^{iz} + e^{-iz} = \frac{3+i}{2}$$

$$\boxed{w = e^{iz}}$$

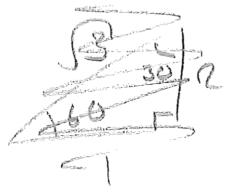
$$\Rightarrow \quad w + \frac{1}{w} = \frac{3+i}{2}$$

$$\Leftrightarrow \quad w^2 + 1 = \frac{3+i}{2} w$$

$$w^2 - \frac{3+i}{2} w + 1 = 0$$

$$w = \frac{3+i}{4} \pm \sqrt{\frac{(3+i)^2}{16} - \frac{16}{16}} =$$

$\boxed{6.10}$



a) $\log(z) = \ln|z| + i \cdot \arg(z)$

$\log(2) = \ln(2) + i \cdot \arg(2) = \ln 2 + 0 = \boxed{\ln 2}$

b) $\log(i) = \ln(i) + i \arg(i) = \boxed{\ln(i)}$

c) $\log(-1) = \boxed{\ln(-1)}$ ej def!

d) $\log(1 + i\sqrt{3}) = \ln|1 + i\sqrt{3}| + i \cdot \frac{\pi}{3} = \boxed{\ln(2) + \frac{i\pi}{3}}$

e) $\log(-1-i) = \ln(\sqrt{2}) - i\frac{\pi 3}{4} = \ln(2^{1/2}) - i\frac{\pi 3}{4} =$

$= \boxed{\frac{1}{2}\ln 2 - i\frac{3}{4}\pi}$

$\boxed{6.12}$

* $\log(1-i)$ och $\log(1-i)^5$

* $\log(1-i) = \ln|1-i| + i\arg(1-i) =$

$= \frac{1}{2}\ln(2) + i\left(-\frac{\pi}{4}\right) = \frac{1}{2}\ln 2 - \frac{i\pi}{4}$

$\log(1-i)^5 = 5 \cdot \frac{1}{2}\ln 2 - 5\frac{i\pi}{4} = \boxed{\frac{5}{2}\ln 2 - \frac{i\pi}{4}}$

$\boxed{6.15}$ $w = z^i$

a) $z = 2 \Rightarrow w = 2^i = e^{i\log 2} = e^{i\ln 2} = \cos(\ln 2) + i\sin(\ln 2)$

b) $z = -2 \Rightarrow w = e^{i\log(-2)}$

c) $z = -1$

d) $z = 1+i$

e) $z = i$