$\boxed{6.3}$

$$r(t) = (\cos t, 2\sin t, t) \quad , t > 0$$

a) $r'(t) = (-\sin t, 2\cos t, 1) \qquad t = \pi$

$$r'(\pi) = (-\sin\pi, 2\cos\pi, 1) = \boxed{(0, 2, 1)}$$

b) $|(0,2,1)| = \sqrt{4+1} = \boxed{\sqrt{5}}$

c) $r''(t) = (-\cos t, -2\sin t, 0)$

$$\boxed{r''(\pi) = (1, 0, 0)}$$

$\boxed{6.5}$

$$r = \begin{cases} x = (2-\cos t)\cos s \\ y = (2-\cos t)\sin s \\ z = \sin t \end{cases} \quad , \begin{matrix} 0 \leq s \leq \pi \\ -\pi \leq t \leq \pi \end{matrix}$$

$r_t' = (\sin t \cos s, \sin t \sin s, \cos t) \qquad P: (1, \sqrt{3}, 1)$

$r_s' = ((2-\cos t)(-\sin s), (2-\cos t)\cos s, 0) \quad \rightarrow \boxed{t = \frac{\pi}{2}} \Rightarrow \cos s = \frac{1}{2}$

$$\boxed{s = \frac{\pi}{3}}$$

$r_t'(\frac{\pi}{2}, \frac{\pi}{3}) = (\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$, $r_s'(\frac{\pi}{2}, \frac{\pi}{3}) = (-\sqrt{3}, 1, 0)$

$$r'_t\left(\frac{\pi}{2}, \frac{\pi}{3}\right) \times r'_s\left(\frac{\pi}{2}, \frac{\pi}{3}\right) =$$

$$= \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right) \times \left(-\sqrt{3}, 1, 0\right) =$$

$$\frac{1}{2}\begin{vmatrix} e_x & e_y & e_z \\ 1 & \sqrt{3} & 0 \\ -\sqrt{3} & 1 & 0 \end{vmatrix} = \frac{1}{2} e_z \begin{vmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{vmatrix} = \frac{1}{2} e_z (1+3) =$$

$$= 2e_z = \boxed{(0,0,2)} = \boxed{(0,0,1)}$$

$\boxed{6.5}$

$$\gamma = \begin{cases} x = \cos t \\ y = \sin t \\ z = \cos 2t \end{cases}, \quad 0 \le t \le 2\pi$$

Yta: $z = x^2 - y^2$

$\cos^2 t - \sin^2 t = \cos 2t$ #

b) $\gamma'_t = (-\sin t, \cos t, -2\sin 2t)$

$$v = (\underbrace{\sin^2 t + \cos^2 t}_{1} - 4\sin^2 2t)^{1/2} = (1 - 4\sin^2 2t)^{1/2} \le \boxed{1}$$

$\sin^2 2t = 0 \Rightarrow 2t = n \cdot \pi \Rightarrow \boxed{t = \frac{n \cdot \pi}{2}}$

6.9

a) $\begin{cases} y_1 = x_1^2 + 2x_2 \\ y_2 = x_1 + x_2 \end{cases}$

$$f' = \begin{pmatrix} \frac{\partial y_1}{\partial x} & \frac{\partial y_1}{\partial x_2} \\ \frac{\partial y_2}{\partial x} & \frac{\partial y_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 2x_1 & 2 \\ 1 & 1 \end{pmatrix}$$

b) $\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$ c) $\begin{pmatrix} 2\cos\varphi & -2r\sin\varphi \\ 3\sin\varphi & 3r\cos\varphi \end{pmatrix}$

d) $\begin{pmatrix} 2x_1 & -2x_2 \\ 2x_2 & 2x_1 \end{pmatrix} = \boxed{2\begin{pmatrix} x_1 & -x_2 \\ x_2 & x_1 \end{pmatrix}}$

6.11 $\bar{y} = \overline{f(x)} = (x_1^2 + x_2^2, x_1)$

$\bar{x} = \overline{g(t)} = (t_1 + 4t_2, -2t_1 + 2t_2)$

$$f \circ g = f(g(t)) = \begin{pmatrix} 2x_1 & 2x_2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ -2 & 2 \end{pmatrix} = \begin{pmatrix} 2x_1 - 4x_2 & 8x_1 + 4x_2 \\ -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2t_1 \cancel{+8t_2} \cancel{+8t_1} \cancel{-8t_2} & 8t_1 + 32t_2 \cancel{-8t_1} \cancel{+8t_2} \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 10t_1 & 40t_2 \\ +1 & 4 \end{pmatrix}$$

$\boxed{6.15}$

$$\det(f) = \begin{vmatrix} 10t_1 & 40t_2 \\ 1 & 4 \end{vmatrix} = \boxed{40t_1 - 40t_2} =$$

$$= \boxed{40(t_1 - t_2)}$$

$\boxed{6.17}$
$$\begin{cases} u = x^2 + y^2 \\ v = \sin(x^2+y^2) \end{cases}$$

$$\begin{vmatrix} 2x & 2y \\ 2x\cos & 2y\cos \end{vmatrix} = 4xy\cos() - 4xy\cos() = \boxed{0}$$

$\boxed{6.21}$ a) $\begin{cases} u = x + 2y \\ v = 3x + 4y \end{cases}$ bijektiv $\mathbb{R}^2 \rightarrow \mathbb{R}^2$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = \boxed{-2 \neq 0}$$ Invers finns!

b) $\frac{\partial u}{\partial x} = \boxed{1}$ c) $\boxed{v - 2u = x \Rightarrow \frac{3u - v}{2} = y}$

d) $\frac{\partial x}{\partial u} = \boxed{-2}$

$\boxed{6.22}$

$~~f(x,y)=~~ x^3y + 2y^3x = 3$

$f'_y = x^3 + 6y^2x \quad , f'_x = 3x^2y + 2y^3$

$f'_y(1,1) = 1 + 6 = \boxed{7 \neq 0} \quad , f(1,1) = 3+2 = \boxed{5}$

$f(x,y)=3$ nära $(1,1)$ kommer att definiera y som en funktion av x.

$$\Longrightarrow f(x, y(x)) = 3$$

$$f'(x, y(x)) = f'_x(x, y(x)) \cdot 1 + f'_y(x, y(x)) \cdot y'(x) = 0$$

$$y'(1) = -\frac{f'_x(x, y(x))}{f'_y(x, y(x))} = \boxed{-\frac{5}{7}}$$

$\boxed{6.23}$

$f(x,y) = x^y + \sin y$, $x^y + \sin y = 1$

omgivning: $(1,0)$

Beräkna: $y'(x)$

$f(x,y) = e^{y \cdot \ln x} + \sin y$

$f'_x = y \cdot x^{y-1} \Rightarrow f'_x(1,0) = \boxed{0}$

$f'_y = \ln x \cdot x^y + \cos y \Rightarrow f(1,0) = \boxed{1}$

$f(x, y(x)) = f'_x \cdot 1 + f'_y \cdot y'(x)$

$$\boxed{y'(x) = -\frac{f'_x}{f'_y} = \frac{y x^{y-1}}{\ln x \cdot x^y + \cos x}}$$

$\boxed{6.29}$

$$f = e^x - x \cdot \cos y - 1$$

$$f'_x = -\cos y \Rightarrow f'_x(0,0) = \boxed{-1}$$

$$f'_y = e^x + x\sin y \Rightarrow f'_y(0,0) = 1 + 0 = \boxed{1}$$

$$f = e^{y(x)} - x \cdot \cos(y(x)) - 1$$

$$f'_x = y'(x) \cdot e^{y(x)} + \cos(y(x)) + x \cdot y'(x) \sin y(x)$$

$$y'(x) = \frac{\cos y(x)}{e^{y(x)} + x \sin y(x)} \qquad y(0) = 0$$

$$y'(0) = \frac{\cos y(0)}{e^{y(0)} + 0} = \frac{1}{1+0} = \boxed{1}$$

$$y(x) = x + m$$

$$0 = 0 + m \Rightarrow m = 0$$

svar:

$$\boxed{y = x}$$

[redacted]

$\boxed{6.29}$

b) $y(x) = a_0 + a_1 x + a_2 x^2 + x^3 B(x)$

$a_0 = y(0) = 0$

$a_1 = y'(0) = 1$

$a_2 = \frac{y''(0)}{2} = ???$

$\boxed{\text{Derivera } \frac{\cos y(x)}{e^{y(x)} + x \cdot \sin y(x)}}$