5.1

b) $f(x,y) = (1+x+2y)^{-1}$

$$f_x' = -(1-x+2y)^{-2} = -\frac{1}{(1-x+2y)^2}$$

$$f_y' = -\frac{2}{(1-x+2y)^2}$$

$$\begin{cases} P_1 = f(1,1) + f_x'(1,1)\cdot h + f_y'(1,1)\,k \\ h = (x-1) \quad , k = (y-1) \end{cases}$$

$$\boxed{P_1 = \frac{1}{4} - \frac{1}{16}(x-1) - \frac{1}{8}(y-1)}$$

$$f_{xx}'' = 2(1-x+2y)^{-3}$$

$$f_{xy}'' = 4(1-x+2y)^{-3}$$

$$f_{yy}'' = 8(1-x+2y)^{-3}$$

$$P_2 = P_1 + \frac{1}{2}\left(\frac{2}{4^3}h^2 + 2\cdot\frac{4}{4^3}hk + \frac{8}{4^3}k^2\right) = \boxed{P_1 + \frac{h^2}{64} + \frac{hk}{16} + \frac{k^2}{16}}$$

5.4

a) $Q(h,k) = h^2 + 6k^2 + 4hk + 4k^2 - 4k^2$

$(h+2k)^2 + 2k^2 > 0$

pozitiv DEFINIT!

b) $Q(h,k,l) = h^2 + 2k^2 + 8l^2 + 2hk + 2hl$

$= h^2 + 2k^2 + 8l^2 + 2h(k+l) =$

$= h^2 + 2 \cdot h \cdot (k+l) + (k+l)^2 - (k+l)^2 + 2k^2 + 8l^2 =$

$= (h+k+l)^2 - (k+l)^2 + 2k^2 + 8l^2 =$

$= (h+k+l)^2 - k^2 - l^2 - 2kl + 2k^2 + 8l^2 =$

$= (\quad)^2 + k^2 + 7l^2 - 2kl =$

$= (\quad)^2 + (k-l)^2 + 6l^2 \geq 0$

Pozitiv definit.

5.5

a) h^2+k^2 är $\boxed{+\text{DEFINIT}}$

b) h^2-k^2 är $\boxed{\text{Indefinit}}$

c) hk är $\boxed{\text{Indefinit}}$

d) $h^2+k^2+hk = \dfrac{2h^2+2k^2+hk\cdot 2}{2} = \boxed{\dfrac{h^2+k^2+(k+h)^2}{2}}$

$\boxed{+\text{DEFINIT}}$

SEMI → $\boxed{\text{OM } h=-k\neq 0}$

e) $h^2+k^2+2hk=(h+k)^2 \geq 0$ $\boxed{+\text{DEFINIT}}$

f) $h^2+k^2+4hk=(h+k)^2+2hk$

$\boxed{\text{Indefinit}}$

5.7 $f(x,y)=3x^2+3xy+y^2+y^3$

$f_x' = 6x+3y = 0 \iff 2x+y=0 \iff \boxed{y=-2x}$

$x=-\frac{y}{2}$

$f_y' = 3x+2y+3y^2=0$

$(x,y)=(0,0)$

$3x^2-4x+12x^2 = \boxed{12x^2-x=0} \Rightarrow \boxed{x=\frac{1}{12} \Rightarrow y=-\frac{1}{6}}$

5.8 $f(x,y) = x^3y^2 + 27xy + 27y$

$$\begin{cases} f'_x = 3x^2y^2 + 27y = 0 \\ f'_x = 2x^3y + 27x + 27 = 0 \end{cases}$$

$y = 0 \Rightarrow x = -1$ **Lösn: $(x,y) = (-1, 0)$**

$$\begin{cases} 3y(x^2 \cdot y + 9) = 0 \Rightarrow x = -\frac{9}{y}, \; y = -\frac{9}{x^2} \\ 2x^3y + 27x + 27 = 0 \end{cases}$$

$$2x^3 \cdot \left(-\frac{9}{x^2}\right) + 27x + 27 = 0$$

$$\Leftrightarrow -18x + 27x + 27 = 0$$

$$9x + 27 = 0$$

$$x + 3 = 0$$

$$x = -3 \Rightarrow y = -\frac{9}{9} = -1$$

Lösn: $(x,y) = (-3, -1)$

5.10

a) $f(x,y) = (1+\sin(x+y))\ln(1+2x+y) - 2x - y$

$f(0,0) = (1+0)(\ln 1) - 0 - 0 = 0$

$f'_x = (1+\cos(x+y))\ln(1+2x+y) + \frac{1+\sin(x+y)}{1+2x+y}\cdot 2 - 2 = 0$

$f'_y = 1+\cos(x$

b) $f(x,y) = 4x^2 + 12xy + 9y^2 + x^4$

$f'_x = 8x + 12y + 4x^3 = 0$

$f'_y = 12x + 18y = 0 \Rightarrow y = \frac{2}{3}x$

$8x + 8x + 4x^3 = 0$

$4x + x^3 = 0$

$x(4+x^2) = 0$

$x = 0$, ~~$x = 2i$~~

$\Rightarrow y = 0$

JA

5.10

a) $f(x,y) = (1+\sin(x+y))\ln(1+2x+y) - 2x - y$

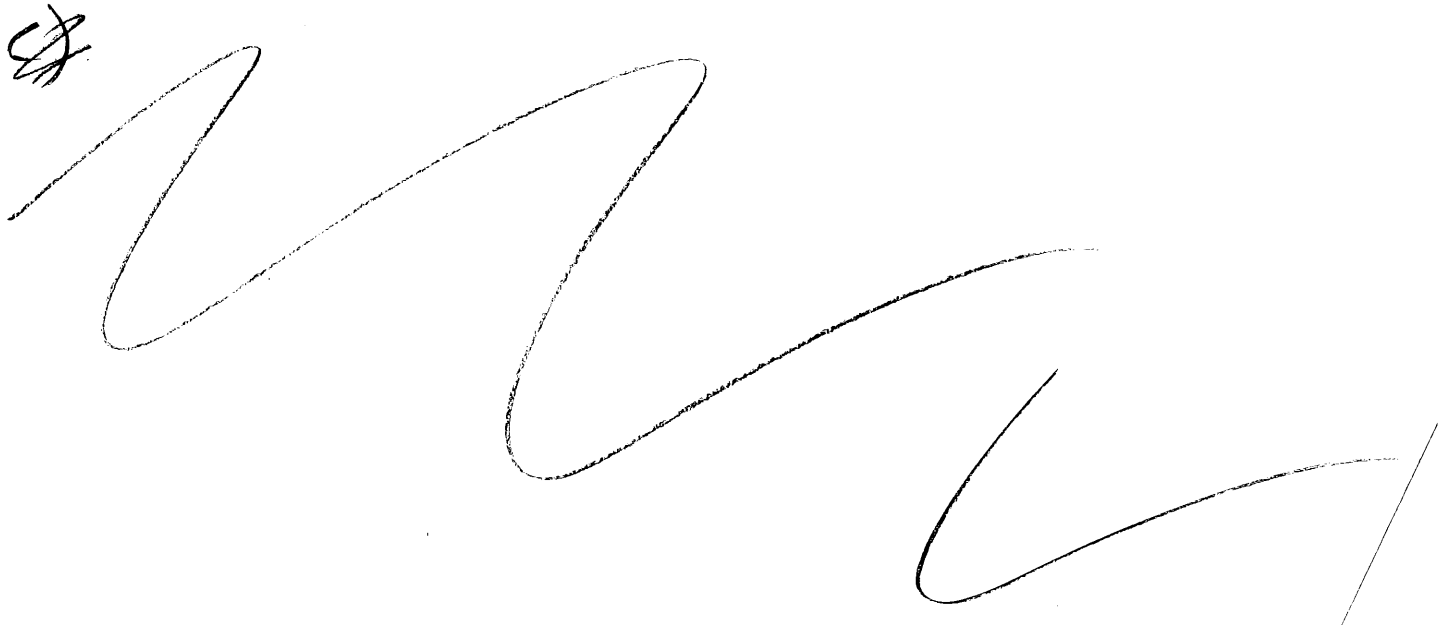
$$f'_x = (1+\cos(x+y))\ln(1+2x+y) + \frac{(1+\sin(x+y))}{1+2x+y}\cdot 2 - 2 = 0$$

$$2\ln 1 + \frac{2\cdot 1}{4}\cdot 2 - 2 = 0$$

$$f'_y = (1+\cos(x+y))\ln(1+2x+y) + \frac{1+\sin(x+y)}{1+2x+y} - 1 = 0$$

$(0,0)$ ger $f'_x \neq 0$

Svar: NEJ

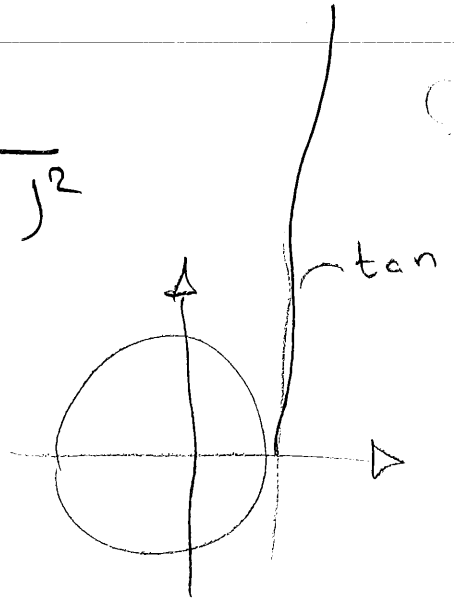


c) $f(x,y,z) = e^{xyz}(1-\arctan(x^2+y^2+2z^2))$

$$f_x' = yz\,e^{xyz}(1-\arctan(\,)) + e^{xyz}\cdot\frac{2x}{1+(x+y^2+2z^2)^2}$$

$$f_x' = xz\,e^{xyz}(1-\arctan(\,)) + e^{xyz}\,\frac{2x}{1+(\quad)^2}$$

$$f_z' = xy\,e^{xyz}(1-\text{art}\,) + e^{xyz}\cdot\frac{4z}{1+(\quad)^2}$$



$f_x'(0,0,0) = 0+0 = 0$

$f_y'(0,0,0) = 0$

$f_z'(0,0,0) = 0$

JA

f är ~~indefinit~~ => Sadelpunkt

Tydligen Negdef... VF?

5.10

b) $f(x,y) = 4x^2 + 12xy + 9y^2 + x^4$

$f'_x = 8x + 12y + 4x^3 = 0$

$f'_y = 12x + 18y = 0 \Rightarrow \boxed{x = \frac{3}{2}y}$

$8x + 12 \cdot \frac{2x}{3} + 4x^3 = 0$

$8x + 8x + 4x^3 = 0$

$4x(4 + x^2) = 0 \Rightarrow \boxed{x = 0 \Rightarrow y = 0}$

JA

$f(x,y) = (2x + 3y)^2 + x^4 > 0$

Pozitiv definit

MIN

5.12 $f: \mathbb{R}^3 \to \mathbb{R}$

$x + y(x+z)$

$$f(x,y,z) = (x + xy + yz) \cdot e^x$$

$$f'_x = (1+y)e^x + e^x(x+xy+yz) = e^x(1+x+y+xy+zy) = 0$$

$$f'_y = (x+z)e^x = 0$$

$$f'_z = y \cdot e^x = 0 \Rightarrow \boxed{y = 0}$$

$$1 + x = 0 \Rightarrow x = -1$$

$$-1 + z = 0 \Rightarrow \boxed{z = 1}$$

$\boxed{(x,y,z) = (-1,0,1)}$ – ej extrempunkt, (* indefinit)

5.10

d) $f(x,y,z) = 1 + x^2 + 2y^2 + 4z^2 - 2xy + 6yz - 2xz$

$f'_x = 2x - 2y - 2z$

$f'_y = 4y - 2x + 6z$

$f'_z = 8z + 6y - 2x$

$(0,0,0)$ är en extrempunkt, ~~JA~~

Nej för f är indefinit (går ej att kvadratkomplettera fullständigt.

~~$f = (x-y)^2 + 1 + (2y+3z)$~~

5.14

$f(x,y) = e^y(y+1-(y-1)\sin x)$

$f'_x = -e^y \cos x$

$f'_y = e^y(\quad) + e^x(1-\sin x) = 0$

$x = \frac{3\pi}{2} + n \cdot 2\pi$

5.15

a) $D = \{(x,y); |x| \leq 1, |y| \leq 1\}$

Nej

b) Ja

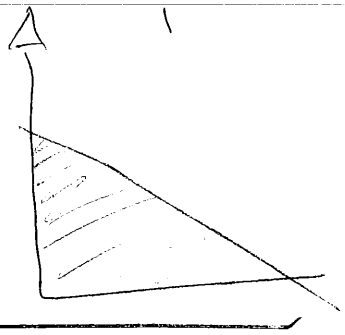
$x + y \leq 1$

c) Nej

$y \leq 1 - x$

d) Nej

e) Ja



5.17 $f(x,y) = xy + x^2y^2$, $|x| \leq 1, |y| \leq 1$

$$f_x' = y + 2xy^2 = y(1+2xy) = 0$$

$$f_y' = x + 2x^2y = x(1+2xy) = 0$$

$f(0,0) = \boxed{0}$ ①

$f(1, -\frac{1}{2}) = -\frac{1}{2} + \frac{1}{4} = \boxed{-\frac{1}{4}}$

$1 + 2xy = 0$

$y = -\frac{1}{2x}$

$\boxed{x = 1 \Rightarrow y = -\frac{1}{2}}$

$f(x,-1) = -x + x^2 = \boxed{\frac{3}{4}}$

$f_x' = -1 - 2x \Rightarrow x = -\frac{1}{2}$

$f(x,1) = x + x^2 \Rightarrow 1 + 2x \quad x = -\frac{1}{2}$

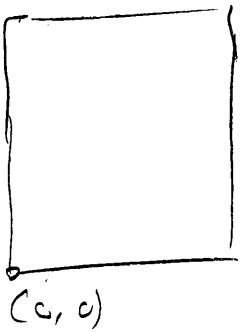
$f(-1,y) = -y - y^2 - 1 - 2y$

$f(1,1) = \boxed{2}$

$\boxed{2 \text{ och } -\frac{1}{4}}$

5.16 $f(x,y) = y \cdot e^x - xy^2$

$0 \le x \le 1 \;,\; 0 \le y \le 2$



$f_x' = y \cdot e^x - y^2 = y(e^x - y) = 0$

$f_x' = e^x - 2xy = 0$

$y \neq 0 \quad e^x = y \Rightarrow e^0 = 1$

$f(0,1) = 1 - 0 = \boxed{1}$

Horn

$f(0,0) = \boxed{0}$, $f(0,2) = \boxed{2}$

$f(1,0) = 0$, $f(1,2) = \boxed{2e - 4}$

Rand

$f(x,0) = \boxed{0}$

$f(x,2) = 2e^x - 4x$

$f'(x,2) = 2e^x - 4 = 0 \Rightarrow x = \ln 2$

$f(\ln 2, 2) = \boxed{4 - 4\ln 2}$

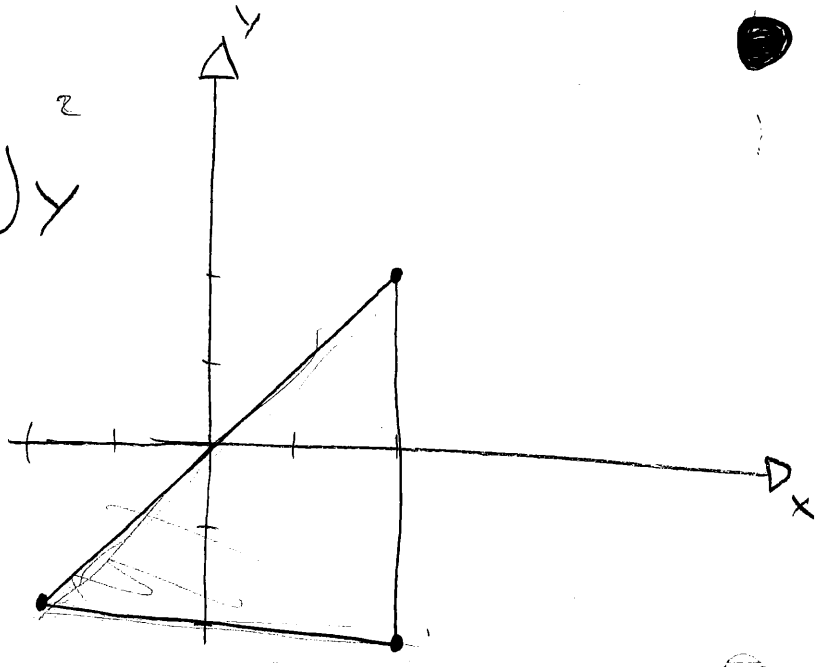
$f(0,y) = y \Rightarrow f' = 1$

$f(1,y) = y \cdot e - y^2 \Rightarrow f' = e - 2y \Rightarrow \boxed{y = \frac{e}{2}}$

$f(1, \frac{e}{2}) = \boxed{\frac{e^2}{4}}$

5.21

$$f(x,y)=y^4+(x^2-1)y^2$$



$$\begin{cases} f'_x = 2xy \\ f'_y = 2y+(x^2-1) \end{cases}$$

$$f'_y = 0 \Rightarrow 2y+x^2-1=0$$

$\boxed{x=0 \Rightarrow y=\frac{1}{2}}$ ⟶ finns ej med

$\boxed{y=0} \Rightarrow x^2-1=0 \Rightarrow \boxed{x=\pm 1}$ — finns med!

$$f(-1,0)=0-(1-1)0=\boxed{0}$$

$$f(x,-2)=4-2(x^2-1)$$

$$f'_x=-4x=0 \Rightarrow x=0, \quad f(0,-2)=4+2=\boxed{6}$$

$$f(2,y)=y^2+3y, \quad f'_y=2y+3=0 \Rightarrow y=-\frac{3}{2}$$

$$f\left(2,-\frac{3}{2}\right)=\frac{9}{4}-\frac{18}{4}=\boxed{-\frac{9}{4}}$$

$$f(x,x) = x^2 + (x^2-1)x = x^2 + x^3 - x$$

$$f'_x = 2x + 3x^2 - 1 = 0$$

$$x + \frac{2}{3}x - \frac{1}{3} = 0 \Rightarrow x = -\frac{2}{6}x \pm \sqrt{\frac{4}{36} + \frac{12}{36}} = -\frac{2}{6} \pm \frac{4}{6}$$

$$\boxed{x_1 = \frac{1}{3}, \; x_2 = -1}$$

$$f\left(\frac{1}{3}, \frac{1}{3}\right) = \frac{1}{9} + \left(\frac{1}{9} - \frac{9}{9}\right)\frac{1}{9} = \frac{1}{9}\left(1 - \frac{8}{9}\right) = \boxed{\frac{1}{81}}$$

$$f(-1,-1) = 1 + (1-1)(-1) = \boxed{1}$$

$$f(-2,-2) = \boxed{-2}$$

$$f(2,-2) = \boxed{-2}$$

$$f(2,2) = \boxed{10}$$

$$\boxed{\text{MAX: } 10, \; \text{MIN: } -\frac{9}{4}}$$

5.25

$$f(x,y) = x^2 + x(y^2-1)$$

$$f'_x = 2x + y^2 - 1$$

$$f'_y = 2xy$$

$x = 0 \Rightarrow y = \pm 1$

$f(0,\pm 1) = \boxed{0}$

RAND

$f = x^2 + y^2 = 1$

$(x,y) = (\cos t, \sin t)$

$0 \le t \le 2\pi$

Randen

$y = 0 \Rightarrow x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}, 0\right) =$

$\frac{1}{2^2} + \frac{1}{2}(-1) = \boxed{\frac{1}{4}}$

$f(x,y) = f(2\cos t, 2\sin t) = \cos^2 t + \cos t(\sin^2 t - 1)$

$= \cos^2 t - \cos^3 t = \cos^2 t(1 - \cos t)$

$f'_t = -2\cos^2 t \sin t - \sin t(\sin^2 t - 1) + \cos t(2\sin t \cos t)$

$= -\sin 2t - \sin t(\sin^2 t - 1) + \cos t \sin 2t$

$= -\sin 2t + \sin t \cos^2 t + \cos t \sin 2t$

$= -\sin 2t + \frac{1}{2}\sin 2t \cos t + \cos t \sin 2t$

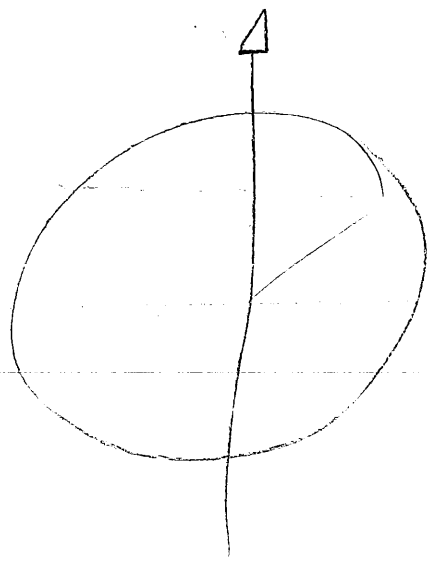
$= -\sin 2t + \frac{3}{2}\sin 2t \cos t = \sin 2t\left(\frac{3}{2}\cos t - 1\right) = 0$

5.25

$\sin 2t = 0$

$2t = 0 + n \cdot 2\pi$
$2t = \pi + n \cdot 2\pi$
$\Big\} 2t = \pi + n \cdot \pi$

$$t = \frac{\pi}{2} + \frac{n \cdot \pi}{2}$$



$t = 0 \Rightarrow f = 1 - 1 = \boxed{0}$

$t = \frac{\pi}{2} \Rightarrow f = \boxed{0}$

$t = \pi \Rightarrow f = 1 - (-1)^3 = \boxed{2}$ — MAX

$t = \frac{3\pi}{2} \Rightarrow f = \boxed{0}$

$\cos t = \frac{2}{3}$

$f = \frac{4}{9} - \frac{8}{27} = \frac{12}{27} - \frac{8}{27} = \boxed{\frac{4}{27}}$

SVAR

MAX: 2

MIN: $-\frac{1}{4}$

5.32

$$f(x,y) = x^2 \cdot y \cdot e^{-x^2-2y^2} \qquad (x,y) \in \mathbb{R}^2$$

$$f'_x = 2xy e^{-x^2-2y^2} - x^2 y e^{-x^2-2y^2} \cdot 2x$$

$$= 2xy e^{-x^2-2y^2}(1-x^2) = 0$$

$$f'_y = x^2 \cdot e^{-x^2-2y^2} - 4y \cdot x^2 \cdot y \cdot e^{-x^2-2y^2}$$

$$= x^2 \cdot e^{-x^2-2y^2}(1-4y^2) = 0$$

$(0, y)$

$f(0,y) = 0$

$f(0,0) = 0$

$x = \pm 1 \quad , \quad y = \pm \frac{1}{2}$

$$f\left(\pm 1, -\frac{1}{\sqrt{2}}\right) = -\frac{1}{2} e^{-3/2}$$

MIN

MAX

$$f\left(\pm 1, \frac{1}{\sqrt{2}}\right) = \frac{1}{2} e^{-1-\frac{1}{2}} = \frac{1}{2} e^{-\frac{3}{2}}$$

5.35

a) $f(x,y) = x^2y(30 - x^2 - 3y^2)$, $x \geq 0, y \geq 0$

$-\infty$ då $x^2 - 3x^2 \to \infty$

$f'_x = 2xy(\quad) + x^2y(-2x) =$

$= 2xy(30 - x^2 - 3y^2) - 2x^3y$

$= 2xy(30 - x^2 - 3y^2 - x^2) =$

$= 2xy(30 - 2x^2 - 3y) = 0$

$f'_x = x^2(30 - x^2 - 3y^2) + x^2y(-6y) =$

$= x^2(30 - x^2 - 3y^2 - 6y^2) =$

$= x^2(30 - x^2 - 9y^2) = 0$

$(x,y) = (0,y) \Rightarrow f(0,y) = \boxed{0}$ — MAX

$\begin{cases} 30 - 2x^2 - 3x^2 = 0 \\ 30 - x^2 - 6y^2 = 0 \end{cases} \Rightarrow$ Ingen lösning!

5.37

$$f(x,y)=(2x+3y+1)^2 \quad , x^2+y^2=1$$

$$f(\cos t,\sin t)=(2\cos t+3\sin t+1)^2$$

$$f_t'=2(2\cos t+3\sin t+1)(-2\sin t+3\cos t)$$

$$=-8\cos t\sin t+12\cos^2 t-12\sin^2 t+18\sin t\cos t-2\sin t+3\cos t$$

$$=-4\sin 2t+9\sin 2t+12\cos 2t-2\sin t+3\cos t$$

$$=5\sin 2t+12\cos 2t-2\sin t+3\cos t=0$$

NEJ

NEJ

NEJ!

5.37

$$f(x,y) = (2x+3y+1)^2 \quad , \quad x^2+y^2 = 1$$

$$f'_x = 4(2x+3y+1)$$

$$f'_y = 6(2x+3y+1)$$

$$\begin{vmatrix} 2x & 2y \\ 4(2x+3y+1) & 6(2x+3y+1) \end{vmatrix} = 12x(2x+3y+1) - 8y(2x+3y+1) =$$

$$= 4(2x+3y+1)(3x-2y) = 0$$

$$\Leftrightarrow (2x+3y+1)(3x-2y) = 0$$

$$(2x+3y+1) = 0 = \boxed{f = 0}$$

$$x = \frac{2y}{3} \quad , \quad y^2 + \left(\frac{2y}{3}\right)^2 = 1 \Leftrightarrow 9y^2 + 4y^2 = 9$$

$$\Leftrightarrow 13y^2 = 9 \Rightarrow \boxed{x = \frac{2}{\sqrt{13}} \Rightarrow y = \frac{3}{\sqrt{13}}}$$

$$f\left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right) = \left(\frac{14}{\sqrt{13}} + \frac{9}{\sqrt{13}} + 1\right)^2 = \boxed{\left(\sqrt{13}+1\right)^2}$$

5.40

$$f(x,y) = x^2 y \quad , \quad x + y = 1$$

$$f_x' = 2xy \quad , \quad f_x' = x^2$$

$$\begin{vmatrix} 2xy & x^2 \\ 1 & 1 \end{vmatrix} = 2xy - x^2 = x(2y - x) = 0$$

$$x = 0 \Rightarrow \boxed{f = 0}$$

$$x = 2y \Rightarrow 2y + y = 1 \Rightarrow y = \frac{1}{3} \Rightarrow x = \frac{2}{3}$$

$$f\left(\frac{2}{3}, \frac{1}{3}\right) = \frac{4}{9} \cdot \frac{1}{3} = \boxed{\frac{4}{27}}$$

MEN

$$f = x^2(1 - x) \longrightarrow \pm\infty \text{ då } x \longrightarrow \mp\infty$$

5.42

$$f(x,y,z) = xy\sqrt{z}, \quad \boxed{x+y+z=1 \qquad x = 1-z-y}$$

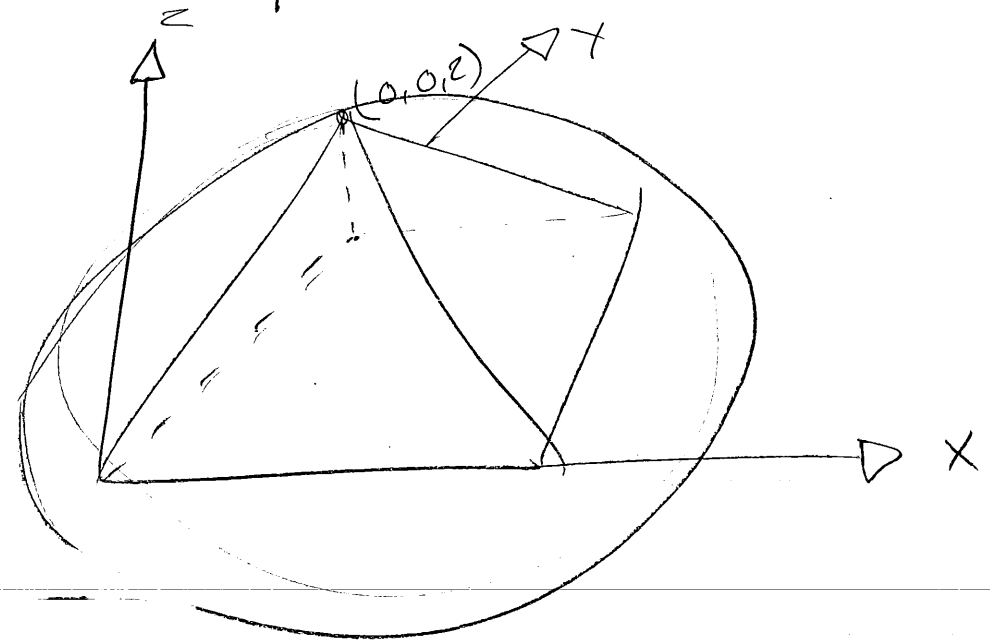
$$(y\sqrt{z}, x\sqrt{z}, xy) = \lambda(1,1,1)$$

$$\Rightarrow \begin{cases} y\sqrt{z} = \lambda \\ x\sqrt{z} = \lambda \\ \frac{xy}{2\sqrt{z}} = \lambda \end{cases} \Leftrightarrow \begin{cases} x = y \\ \frac{xy}{2\sqrt{z}} = x\sqrt{z} \end{cases} \Leftrightarrow \boxed{\begin{cases} x = y \\ y = 2z \end{cases}}$$

$$x + x + \frac{1}{2}x = 2{,}5x = 1 \Rightarrow x = \frac{1}{2{,}5} = \boxed{\frac{2}{5}}$$

$$f\left(\frac{2}{5}, \frac{2}{5}, \frac{2}{10}\right) = \frac{4}{25} \cdot \frac{\sqrt{2}}{\sqrt{10}} = \frac{4}{25} \cdot \frac{1}{\sqrt{5}} = \boxed{\frac{4\sqrt{5}}{125}}$$

5.48 $f = x^2 + \frac{y^2}{4} + (z-1)^2 = 1$



$x \geq 0, \quad y \geq 0, \quad z \geq 0$

$$V = \frac{2x \cdot 2y \cdot (2-z)}{3} = \boxed{\frac{4xy(2-z)}{3}}$$

$$\nabla V = \left(\frac{4y(2-z)}{3}, \frac{4x(2-z)}{3}, \frac{-4xy}{3} \right)$$

$$\nabla f = \left(2x, \frac{y}{2}, 2(z-1) \right)$$

5.51

$$f(x,y)=xy \quad , x^2+4y^2 \leq 8$$

$$\nabla f=(y,x)=0 \Rightarrow (x,y)=(0,0)$$

$$f(0,0)=0$$

$$\begin{vmatrix} y & x \\ 2x & 8x \end{vmatrix} = 8y^2-2x^2=0$$

$$\Leftrightarrow 4y^2-x^2=0 \Rightarrow x^2=4y^2$$

$$4y^2+4y^2 \leq 8$$

$$y^2=1 \Rightarrow y=\pm 1 \Rightarrow x=\pm 2$$

$$\boxed{f(2,1)=2,\ f(-2,1)=-2}$$