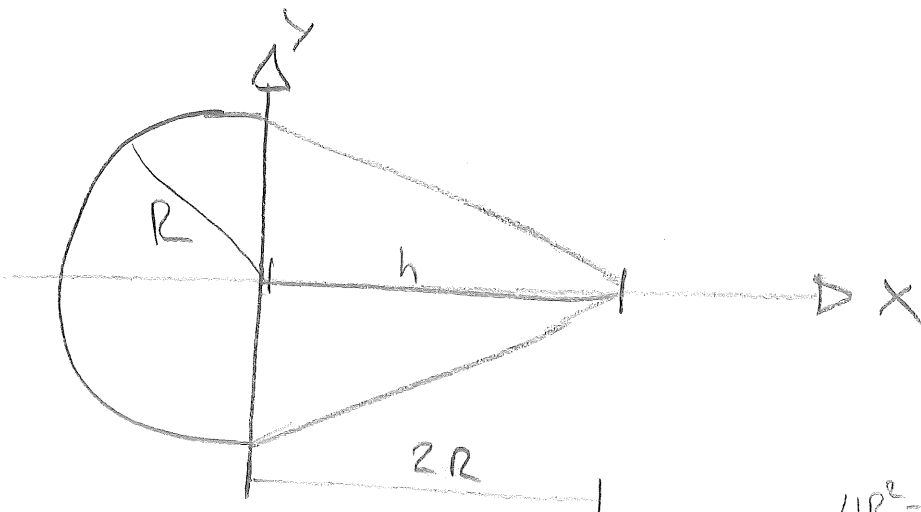
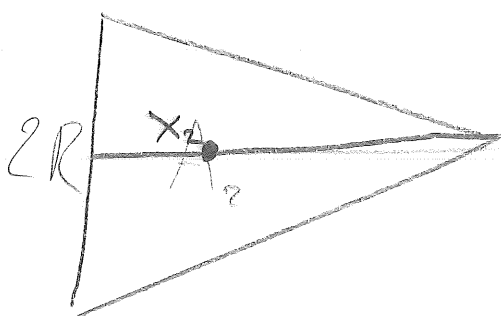
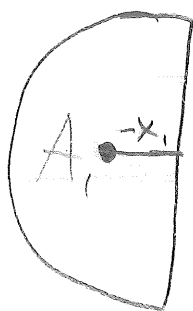


4.2



$$4R^2 - R^2 = 3R^2$$



$$x_1 = \frac{4R}{3\pi}$$

$$x_2 = \frac{2R}{3}$$

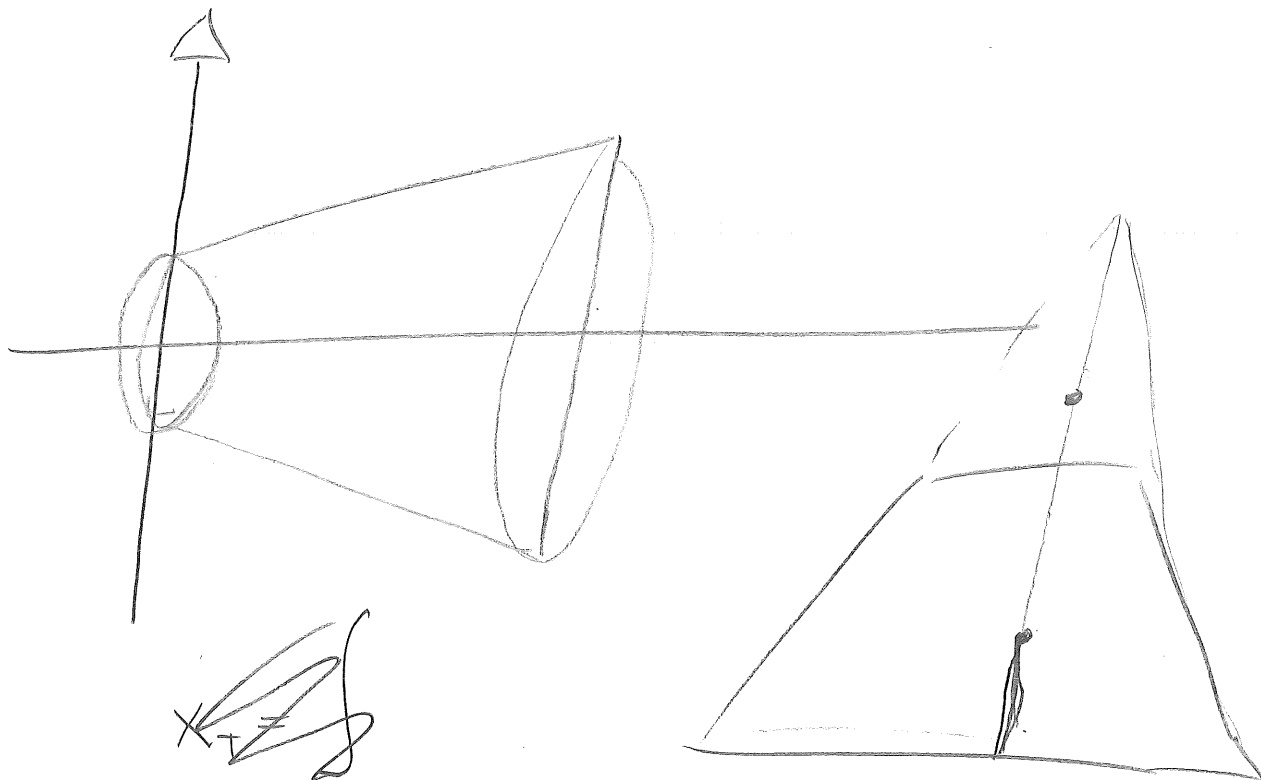
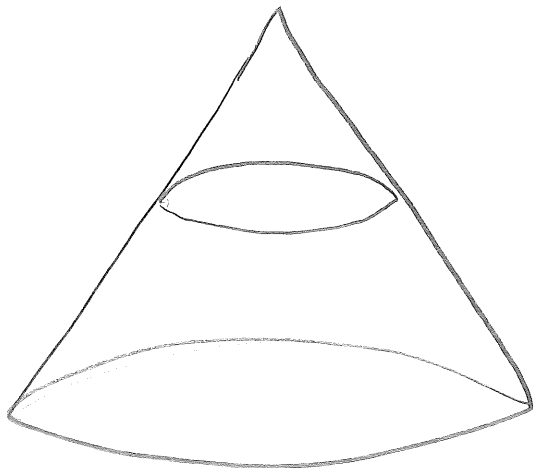
$$A_1 = \frac{5\pi R^2}{2}$$

$$A_2 = \frac{4R^2}{2} = 2R^2$$

$$\frac{x_1 A_1 + x_2 A_2}{A_1 + A_2} = \frac{\frac{4R}{3\pi} \cdot \frac{5\pi R^2}{2} + \frac{2R}{3} \cdot 2R^2}{\frac{5\pi R^2}{2} + \frac{4R^2}{2}} =$$

$$= \frac{\frac{2R^3}{3} + \frac{4R^3}{3}}{\frac{R^2}{2}(\pi + 4)} = \frac{2R^3}{\frac{R^2}{2}(\pi + 4)} = \frac{2 \cdot R \cdot 2}{3(\pi + 4)} = \boxed{\frac{4R}{3\pi + 4}}$$

4.5



$$X_{G_1} = \frac{4H}{3}$$

$$m_1 = \frac{\pi R^2 H}{3}$$

$$X_{G_2} = \frac{4h}{3} + (H-h)$$

$$m_2 = \frac{\pi r^2 h}{3}$$

$$\frac{4H}{3} \cdot \frac{\pi R^2 H}{3} - \frac{4h}{3} \cdot \frac{\pi r^2 h}{3}$$

$$\frac{4\pi}{3} (R^2 H^2 - r^2 h^2)$$

$$\frac{\pi R^2 H}{3} - \frac{\pi r^2 h}{3}$$

$$\frac{\pi}{3} (R^2 H - r^2 h)$$

4.5

$$X_{G_1} = \frac{3h}{4}$$

$$m_1 = \frac{\pi r^2 h}{3}$$

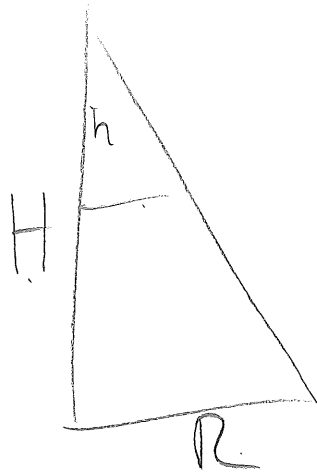
$$X_{G_2} = \frac{3H}{4}$$

$$m_2 = \frac{\pi R^2 H}{3}$$

$$\frac{3\pi R^2 H^2}{12 \cdot 4} - \frac{3\pi r^2 h^2}{12 \cdot 4} = \frac{\pi}{4} (R^2 H^2 - r^2 h^2)$$

$$\frac{\pi}{3} (R^2 H - r^2 h)$$

$$\frac{3 (R^2 H^2 - r^2 h^2)}{4 (R^2 H - r^2 h)}$$



$$3 \left( R^2 H^2 - \frac{R^2 h^2}{H^2} \cdot h^2 \right)$$

$$\frac{3 \left( R^2 H^2 - \frac{R^2 h^2}{H^2} \cdot h^2 \right)}{4 \left( R^2 H - \frac{R^2 h^2}{H^2} h \right)} =$$

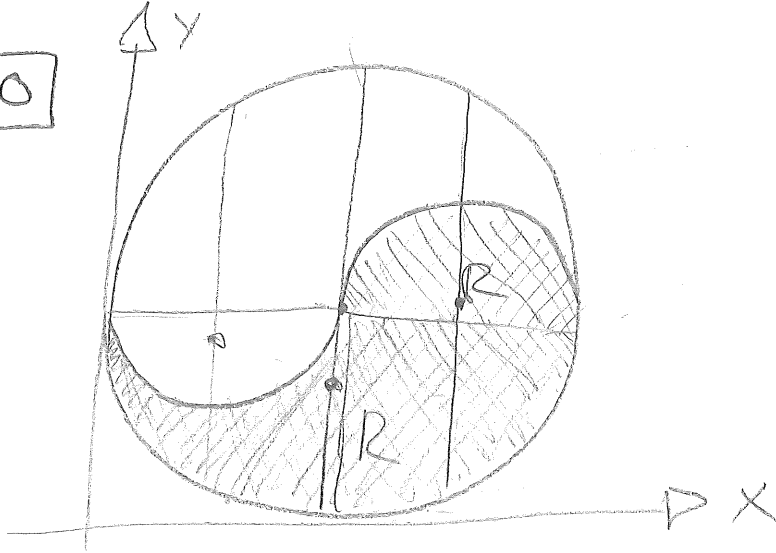
$$\frac{R}{H} = \frac{r}{h}$$

$$r = \frac{R h}{H}$$

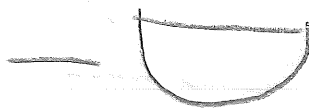
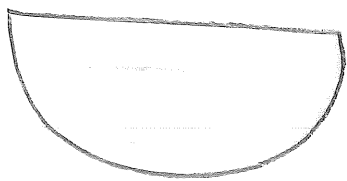
$$\frac{3 (H^4 - h^4)}{4 (H^3 - h^3)}$$

$$\frac{3 (H^4 - h^4)}{4 (H^3 - h^3)}$$

4.10



$$r - \frac{4r}{3\pi} = R \left(1 - \frac{4}{3\pi}\right) = R \left(\frac{3\pi - 4}{3\pi}\right)$$



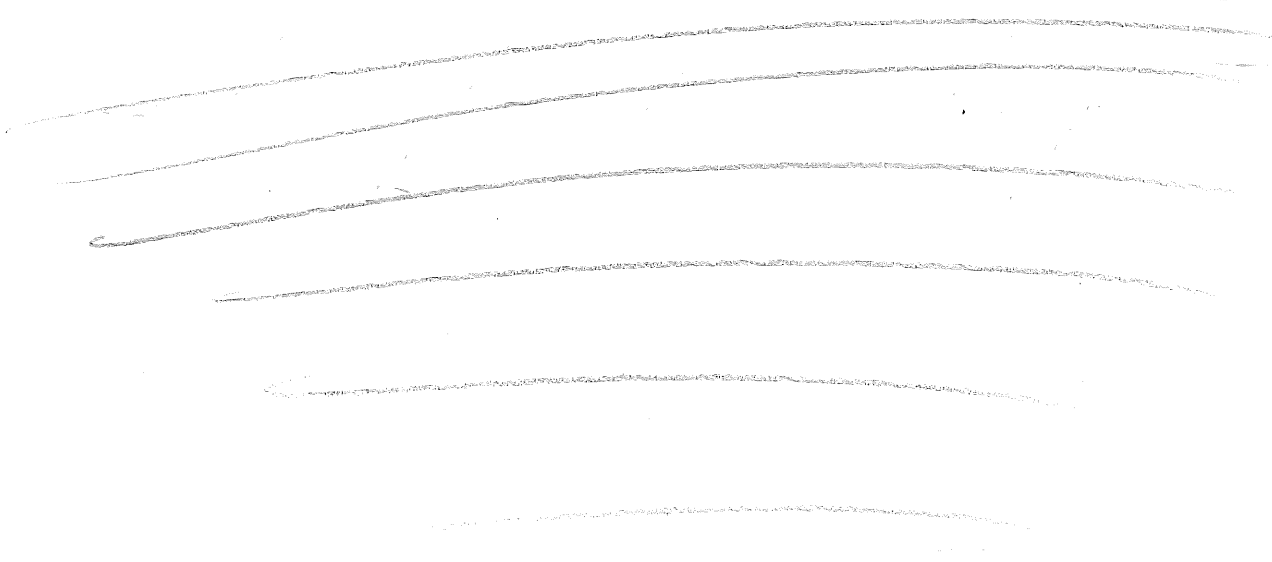
+



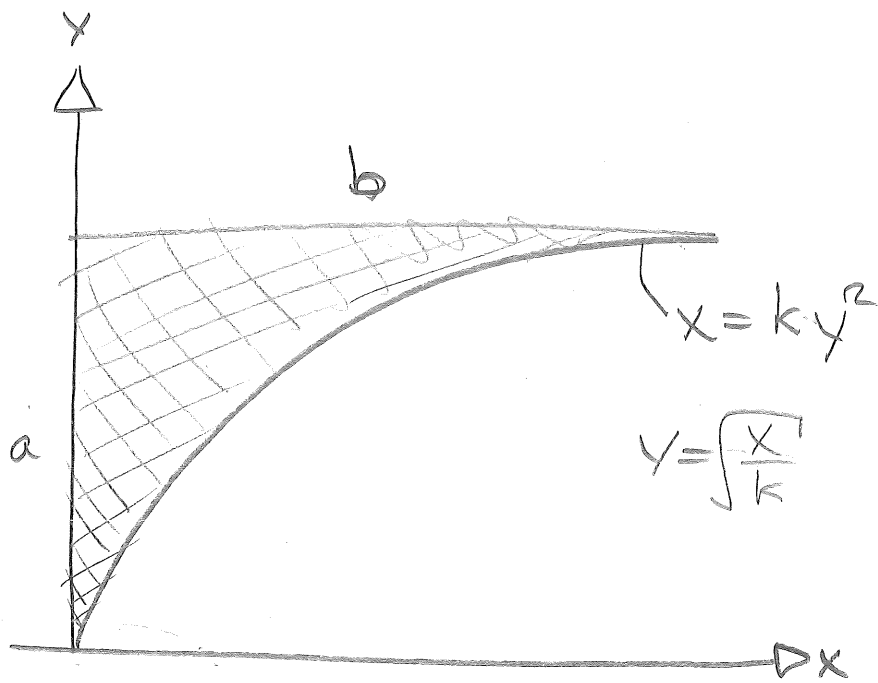
$$(x, y) = (R/2, \dots)$$

$$(x, y) = \left(R, R \left(\frac{3\pi - 4}{3\pi}\right)\right)$$

Rout + +



4.12



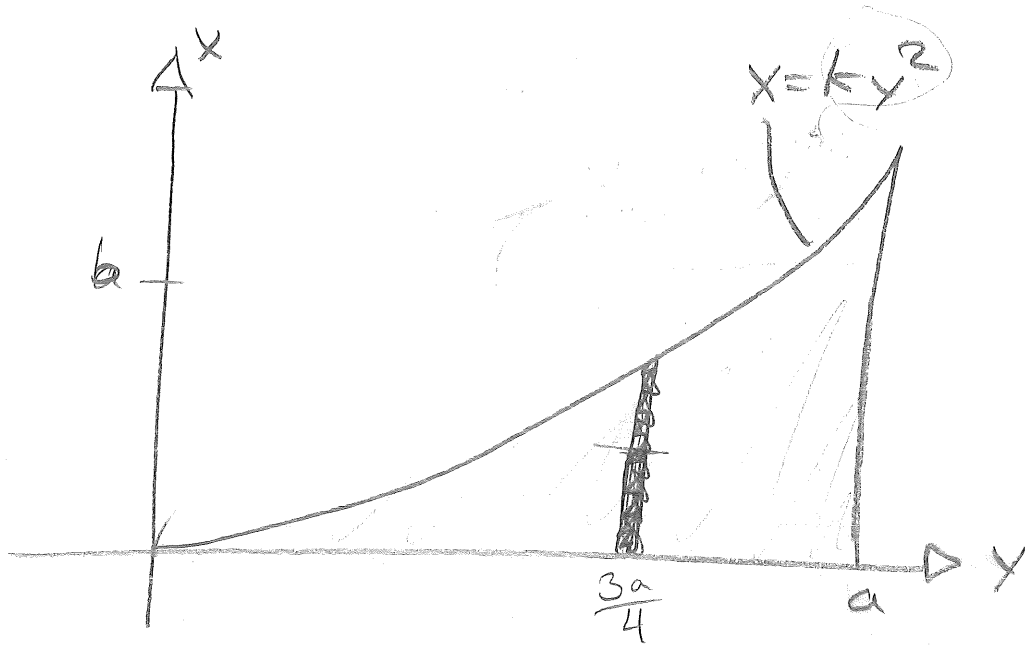
$$\frac{1}{m} \int x dm, \quad m = \int dx$$

~~$$\frac{1}{\sqrt{k}} \int_0^b \sqrt{x} dx = \frac{1}{\sqrt{k}} \left[ \frac{2}{3} x^{3/2} \right]_0^b = \frac{2b^{3/2}}{3\sqrt{k}}$$~~

$$ab - \frac{2b^{3/2}}{3\sqrt{k}} = m$$

~~$$\frac{1}{m\sqrt{k}} \int_0^b x(a - \sqrt{x}) dx = \frac{1}{m\sqrt{k}} \left[ \frac{ax^2}{2} - \frac{2}{5} x^{5/2} \right]_0^b$$~~

4.12

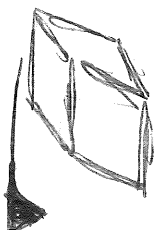


$$m = \int_0^a ky^2 dy = k \left[ \frac{y^3}{3} \right]_0^a = \frac{a^3}{3} k$$

$$\bar{y} = \frac{1}{m} \int ky^3 dy = \frac{k}{m} \left[ \frac{y^4}{4} \right]_0^a = \frac{ka^4}{4m} = \frac{ka^4 \cdot 3}{4a^3 \cdot k} = \frac{3a}{4}$$

$$X_T = \frac{1}{m} \int \left( \frac{ky^2}{2} \right) \cdot ky^2 dy = \frac{k^2}{m^2} \left[ \frac{y^5}{5} \right]_0^a = \frac{k^2 a^5}{10m} =$$

$$= \frac{k^2 a^5 \cdot 3}{10 \cdot a^3 k} = \frac{3ka^2}{10} = \frac{3b}{10} \quad b = ka^2$$



$$X_T = \frac{1}{m} \int_0^a x dm$$