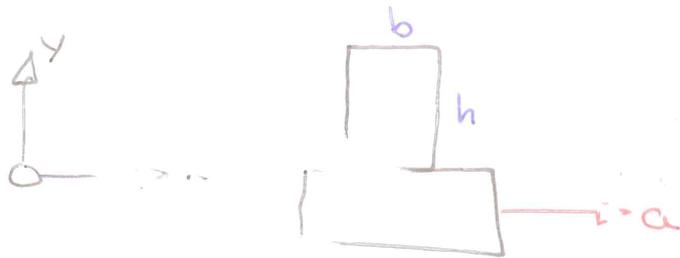


Stela kroppars plana kinetik

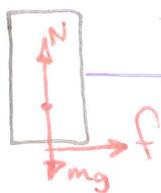
KAPITEL 4

4.3

Glidning eller stjälpning?



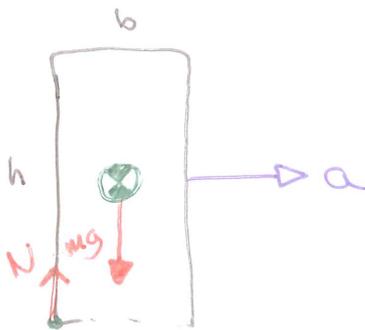
Glidning



$$\Rightarrow ma = -f = -mg\mu \text{ (vid glidn.)}$$

$$\Rightarrow a = -\mu g$$

Stjälpning



$$mg \cdot \frac{b}{2} + ma \cdot \frac{h}{2} = 0$$

$$a = \frac{-gb}{h}$$

Detta kommer från gränsvill för balans

Glidning sker om $\frac{b}{h} > \mu$ och

stjälpning sker om $\frac{b}{h} < \mu$.

4.4



Kraftequationen

$$(P \cos \theta, P \sin \theta + N - mg, 0) = (ma, 0, 0)$$

$$\begin{cases} P \cos \theta = ma \\ P \sin \theta + N - mg = 0 \end{cases}$$

Moment

$$M_B = \vec{r}_{BG} \times m\vec{g} + \vec{r}_{BA} \times \vec{P} =$$

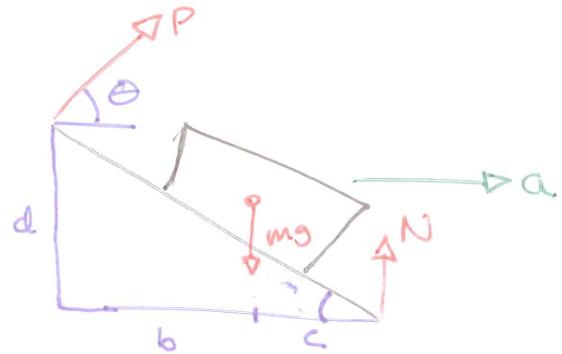
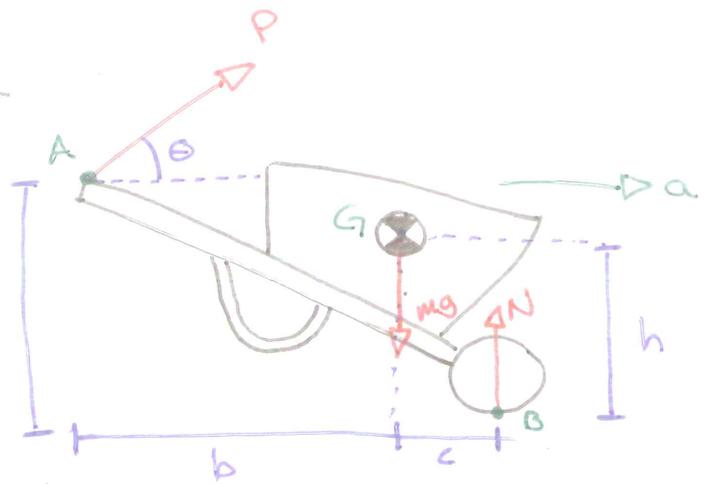
$$= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -c & h & 0 \\ 0 & -mg & 0 \end{vmatrix} + \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -(b+c) & d & 0 \\ P \cos \theta & P \sin \theta & 0 \end{vmatrix} =$$

$$= (0, 0, cmg) + (0, 0, -(b+c)P \sin \theta - dP \cos \theta) =$$

$$= (0, 0, cmg - P((b+c) \sin \theta + d \cos \theta)) \stackrel{!}{=} \dot{H}_G + \vec{r}_{BG} \times m\vec{a}_G$$

$$= 0 + m \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -c & h & 0 \\ a & 0 & 0 \end{vmatrix} = (0, 0, -mah)$$

$$\begin{cases} P \cos \theta = ma \\ cmg - P((b+c) \sin \theta + d \cos \theta) = -mah \end{cases}$$



$$\Rightarrow cmg - \frac{ma}{\cos\theta} \left((b+c)\sin\theta + d\cos\theta \right) = -mah$$

$$\Leftrightarrow c\cancel{mg} - \cancel{ma}(b+c)\tan\theta - d\cancel{ma} = -\cancel{ma}h$$

$$\Leftrightarrow cg - a(b+c)\tan\theta - da = -ah$$

$$\Rightarrow \tan\theta(b+c)(-a) = -ah + da - cg$$

$$\Leftrightarrow \tan\theta = \frac{a(h-d) + gc}{(b+c)a}$$

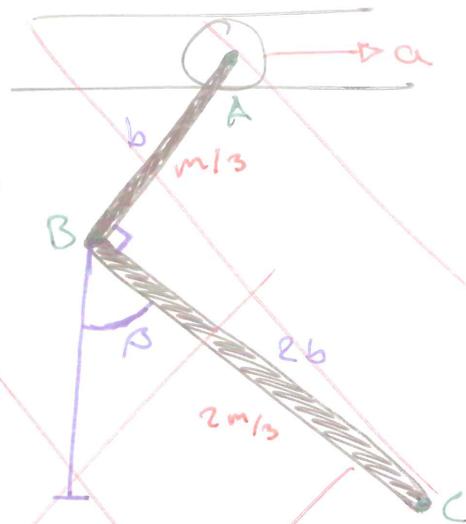
Lösa ut N:

$$\frac{ma}{\cos\theta} \sin\theta + N - mg = 0$$

$$\Leftrightarrow N = mg - ma \tan\theta = m \left(g - \frac{a(h-d) + gc}{b+c} \right)$$

4.7 Bestäm accelerationen a

(stängeln har massan m .)



$$r_{\perp} = \sqrt{b^2 + x^2}$$

$$dm = \frac{m}{3b} \cdot dx$$

$$\Rightarrow I_{zz} = \frac{mb^2}{9} + \int_0^{2b} \frac{2m}{3} \cdot (b^2 + x^2) \frac{m}{3b} dx =$$

$$= \frac{mb^2}{9} + \frac{m}{3b} \int_0^{2b} (b^2 + x^2) dx =$$

$$= \frac{mb^2}{9} + \frac{m}{3b} \left[b^2x + \frac{x^3}{3} \right]_0^{2b} = \frac{mb^2}{9} + \frac{2mb^3}{3b} + \frac{8b^3m}{9} =$$

$$= \frac{15}{9} mb^2 = \boxed{\frac{5}{3} mb^2}$$

4.7

$$\bar{M} = \dot{H}$$

$$\Sigma \bar{F} = m \bar{a}$$

Kraftequationen

v-led: $N = mg$

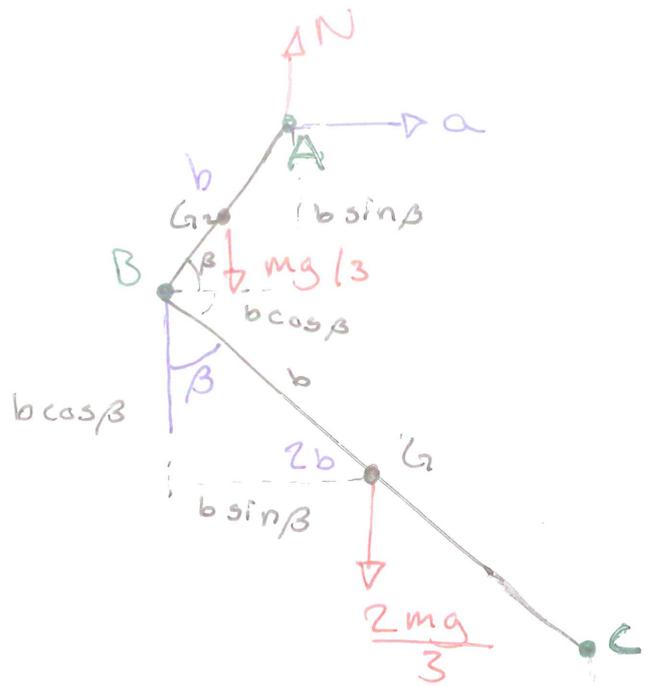
Momentengleichungen ($\bar{M} = \dot{H} = 0$)

$$\Sigma \bar{M}_B = b \sin \beta \cdot \frac{2mg}{3} + \frac{b}{2} \cos \beta \cdot \frac{mg}{3} - b \cos \beta N + m a b \sin \beta =$$

$$= b mg \left(\frac{2}{3} \sin \beta + \frac{1}{6} \cos \beta - \cos \beta \right) + m a b \sin \beta =$$

$$= b mg \left(\frac{2}{3} \sin \beta - \frac{5}{6} \cos \beta \right) + m a b \sin \beta = 0$$

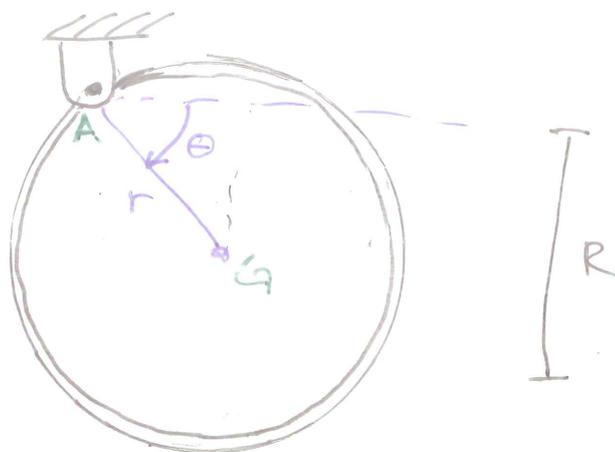
$$\Rightarrow a = \frac{-g \left(\frac{2}{3} \sin \beta - \frac{5}{6} \cos \beta \right)}{\sin \beta} = g \frac{5 \cos \beta - 4 \sin \beta}{6 \sin \beta}$$



4.13

$$I_{zz}^G = \int r^2 dm = \int_0^{2\pi} r^2 \cdot \frac{m}{2\pi r} r d\theta =$$

$$= \frac{mr^2}{2\pi} \int_0^{2\pi} d\theta = \boxed{mr^2}$$



Steiners Formel

$$I_{zz}^A = I_{zz}^G + mr^2 = \boxed{2mr^2}$$

Energiegleichungen

$$T_1 = \frac{1}{2} I_{zz}^A \omega^2 = mr^2 \omega^2, \quad T_0 = 0$$

$$V_1 = mgr - mgr \sin \theta =$$

$$= mgr(1 - \sin \theta)$$

$$V_0 = mgh = mgr$$

$$T_1 - T_0 + V_1 - V_0 = 0$$

$$\Leftrightarrow mr^2 \omega^2 - 0 + mgr(1 - \sin \theta) - mgr = 0$$

$$\Leftrightarrow mr^2 \omega^2 + mgr(\cancel{1 - \sin \theta} - 1) = 0$$

$$\Leftrightarrow mr^2 \omega^2 - mgr \sin \theta = 0$$

$$\Leftrightarrow \boxed{r \dot{\theta}^2 = g \sin \theta} \Rightarrow \cancel{2r \ddot{\theta}} = \cancel{\dot{\theta}} g \cos \theta$$

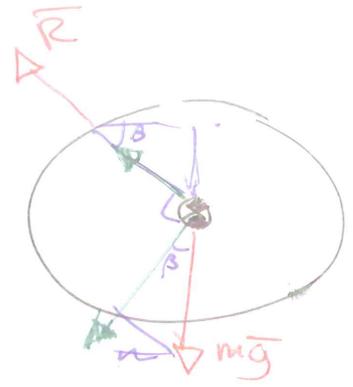
Cylinderkoordinaten $\Rightarrow r \ddot{\theta} = \frac{1}{2} g \cos \theta$

$$\vec{a} = (\cancel{\ddot{r}} - r \dot{\theta}^2, r \ddot{\theta} + \cancel{2\dot{r}\dot{\theta}}, \ddot{z}) = \boxed{(-r \dot{\theta}^2, r \ddot{\theta}, 0)}$$

$$\vec{a} = (-g \sin \theta, \frac{1}{2} g \cos \theta, 0)$$

Naturligt koordinatsystem

$$\vec{a} = (\frac{1}{2} g \cos \theta, g \sin \theta, 0)$$



Kraftekvationen

$$(R_t, R_n, 0) + (mg \cos \theta, -mg \sin \theta, 0) = m \vec{a}$$

$$\begin{cases} R_t + mg \cos \theta = \frac{m}{2} g \cos \theta \\ R_n - mg \sin \theta = mg \sin \theta \end{cases}$$

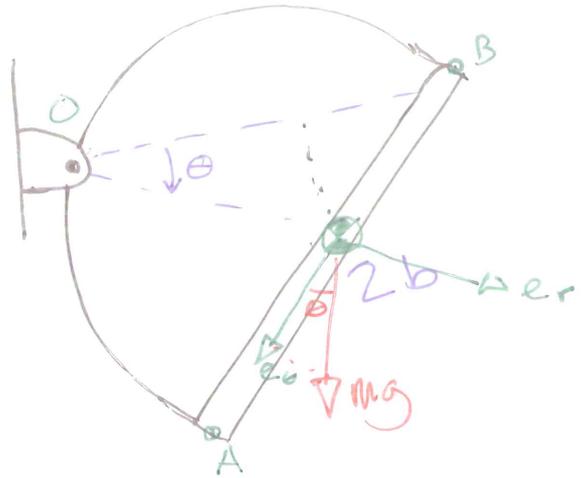
$$\Rightarrow \begin{cases} R_t = -\frac{1}{2} mg \cos \theta \\ R_n = 2mg \sin \theta \end{cases}$$

4.17

$$\underline{I_{zz}^G} = \frac{m(2b)^2}{12} = \boxed{\frac{mb^2}{3}}$$

Steiners Formel

$$\underline{I_{zz}^O} = I_{zz}^G + m r^2 = \frac{mb^2}{3} + \frac{3mb^2}{3} = \boxed{\frac{4}{3} mb^2}$$



Energiegleichungen

$$T_0 = 0, \quad T_1 = \frac{1}{2} I_{zz}^A \dot{\theta}^2 = \underline{\underline{\frac{2}{3} mb^2 \dot{\theta}^2}}$$

$$V_0 = \underline{mgb}, \quad V_1 = mgb - mgb \sin \theta = \underline{mgb(1 - \sin \theta)}$$

$$T_1 - T_0 + V_1 - V_0 = 0$$

$$\Leftrightarrow \frac{2}{3} mb^2 \dot{\theta}^2 + mgb(1 - \sin \theta) - mgb = 0$$

$$\Leftrightarrow \frac{2}{3} mb^2 \dot{\theta}^2 = mgb \sin \theta$$

$$\Leftrightarrow \boxed{b \dot{\theta}^2 = \frac{3}{2} g \sin \theta} \Rightarrow 2b \dot{\theta} \ddot{\theta} = \frac{3}{2} g \cos \theta \cdot \dot{\theta}$$

$$\Leftrightarrow \boxed{b \ddot{\theta} = \frac{3}{4} g \cos \theta}$$

Cylin. koordinaten

$$\underline{\underline{\vec{a}}} = \underline{\underline{(-b \ddot{\theta}^2, b \ddot{\theta}, 0)}} = \underline{\underline{(-\frac{3}{2} g \sin \theta, \frac{3}{4} g \cos \theta, 0)}}$$

$$\boxed{\vec{R} + m \vec{g} = m \vec{a}} \quad \vec{R} = (R_r, R_\theta, 0)$$

$$m \vec{g} = (mg \sin \theta, mg \cos \theta, 0)$$

$$\begin{cases} R_r + mg \sin \theta = -m \frac{3}{2} g \sin \theta \\ R_\theta + mg \cos \theta = m \frac{3}{4} g \cos \theta \end{cases}$$

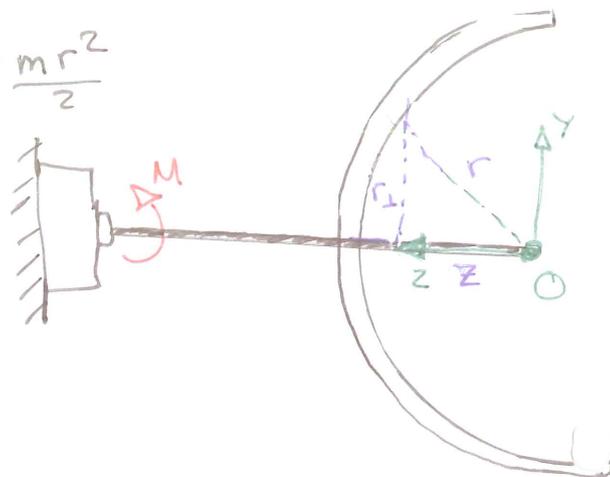
$$\Rightarrow R_r = -\frac{5}{2} mg \sin \theta$$

$$R_\theta = -\frac{1}{4} mg \cos \theta$$

4.25

$$a) I_{zz} = I_{yy} = \frac{I_x}{2} = \frac{1}{2} \int_0^{2\pi} r^2 \cdot \frac{m}{2\pi r} r d\theta = \frac{mr^2}{2}$$

$$I_{zz} = \frac{mr^2}{2}$$



$$b) H_z = I_{zz} \dot{\theta}$$

$$\Rightarrow M = I_{zz} \ddot{\theta} \Leftrightarrow \ddot{\theta} = \frac{M}{I_{zz}} = \frac{2M}{mr^2}$$

$$c) \dot{\theta} = \int \ddot{\theta} dt = \int \frac{2M}{mr^2} dt = \frac{2M}{mr^2} t$$

$$d) I_{xz} = \int xz dm = \int 0 \cdot z dm = 0$$

$$I_{yz} = \int yz dm = 0 \quad (y \text{ är udda})$$

$$e) \quad \vec{H}_0 = (-I_{xz} \dot{\theta}, -I_{yz} \dot{\theta}, I_{zz} \dot{\theta}) = \left(0, 0, \frac{mr^2}{2} \cdot \frac{2Mt}{mr^2}\right) =$$

$$= \boxed{(0, 0, Mt)}$$

$$f) \quad I_{xz} = \int 0 \cdot z \, dm = \underline{0}$$

$$I_{yz} = I_{yz}^{\text{gammal}} + I_{yz}^{\text{partikel}} = \int r^2 \sin \theta \cos \theta \, dm = \boxed{\frac{r^2}{2} m}$$

$$I_{zz} = I_{zz}^{\text{gammal}} + I_{zz}^{\text{partikel}} = \frac{mr^2}{2} \int \sin^2 \theta \, dm = \boxed{r^2 m}$$

$$\Rightarrow \vec{H}_0 = \left(0, \cancel{\frac{r^2}{2} m} \cdot \frac{2Mt}{\cancel{mr^2}}, r^2 m \cdot \frac{2Mt}{mr^2}\right) = \boxed{(0, -Mt, 2Mt)}$$

$$\boxed{429} \quad d = \sqrt{\frac{I}{m}} \Leftrightarrow \boxed{I_0 = md^2}$$

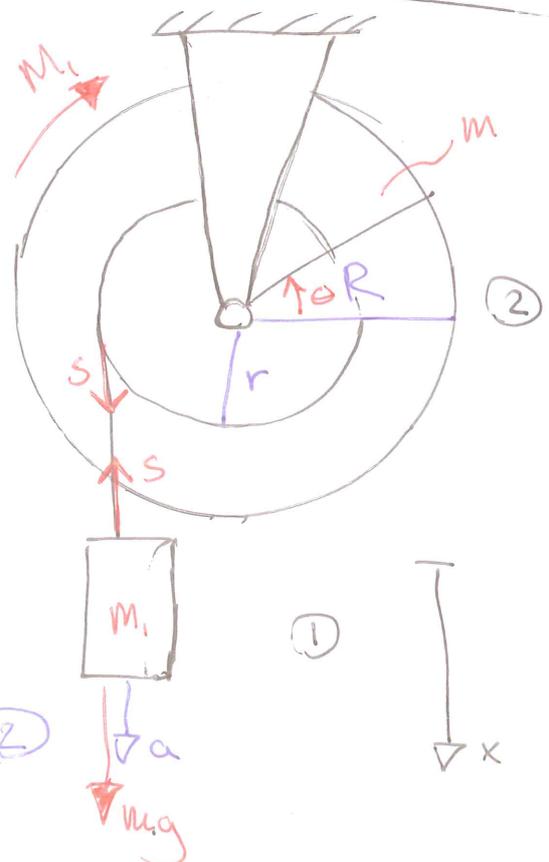
$$\textcircled{1} \quad m_1 g - S = m_1 \ddot{x} \Rightarrow \boxed{S = m_1 (g - \ddot{x})} \quad \textcircled{1}$$

$$\textcircled{2} \quad \vec{M}_G = M_i - S r$$

$$\vec{H} = \frac{d}{dt} (I_0 (-\dot{\theta})) = md^2 (-\ddot{\theta})$$

$$\vec{M} = \vec{H} \Rightarrow \boxed{M_i - S r = md^2 (-\ddot{\theta})}$$

$$\begin{cases} x = r\theta \\ \dot{x} = r\dot{\theta} \\ \ddot{x} = r\ddot{\theta} \end{cases} \Rightarrow \boxed{S = \frac{M_i}{r} + md^2 \cdot \frac{\ddot{x}}{r^2}} \quad \textcircled{2}$$



① och ② ger:

$$m_1 g - m_1 \ddot{x} = \frac{M_1}{r} + m d^2 \frac{\ddot{x}}{r^2}$$

$$\Leftrightarrow m_1 \ddot{x} + m d^2 \frac{\ddot{x}}{r^2} = m_1 g - \frac{M_1}{r}$$

$$\Leftrightarrow \ddot{x} \left(m_1 + \frac{m d^2}{r^2} \right) = m_1 g - \frac{M_1}{r}$$

$$\Leftrightarrow \ddot{x} = \frac{m_1 g - \frac{M_1}{r}}{m_1 + \frac{m d^2}{r^2}} = \frac{m_1 g r^2 - M_1 r}{m_1 r^2 + m d^2}$$



4.33 Bestäm a då glidning sker

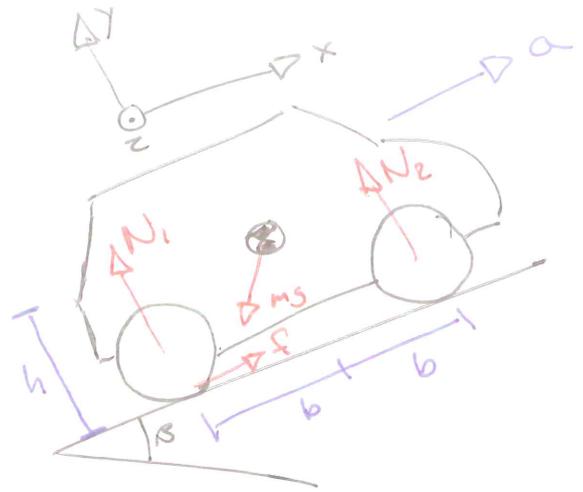
Kraftekvationen

$$\vec{F} = m\vec{a} = m(a, 0, 0)$$

$$\vec{F} = (f - mg\sin\beta, N_1 + N_2 - mg\cos\beta, 0)$$

$$\Rightarrow \begin{cases} f - mg\sin\beta = ma \\ N_1 + N_2 - mg\cos\beta = 0 \end{cases}$$

4 okända, 2 ekvationer



$f \leq \mu N_1$ ($= \mu N_1$ vid glidning) (\Rightarrow 3 okända) \odot

Momentekvationen ($\vec{M}_G = \dot{\vec{H}}_G$)

$$\vec{M}_G = \sum \vec{r}_i \times \vec{F}_i = (-b, -h, 0) \times (f, N_1, 0) + (b, -h, 0) \times (0, N_2, 0) =$$

$$= (, ,) + (, ,) = \boxed{(0, 0, -bN_1 + hf + bN_2)} = 0 \quad *$$

$$\dot{\vec{H}} = I_{zz} \cdot \ddot{\Theta} = 0 \quad (\text{bilen roterar ej})$$

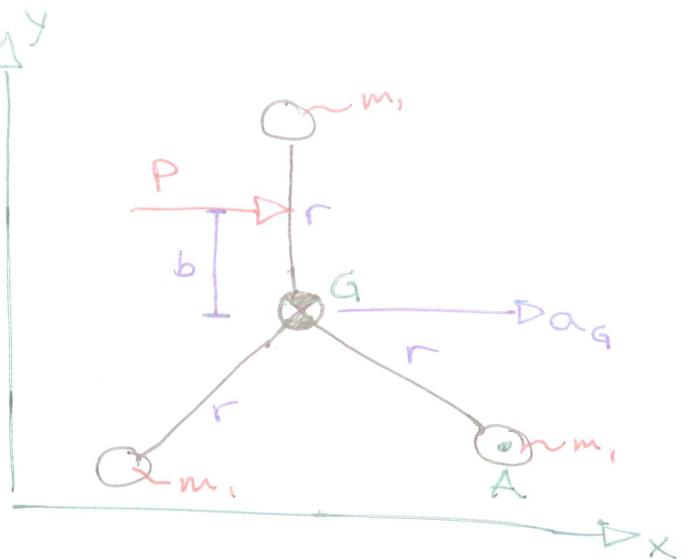
#, \odot och * ger:

$$\begin{cases} \mu N_1 - mg\sin\beta = ma \\ N_1 + N_2 - mg\cos\beta = 0 \\ -bN_1 + bN_2 + hf = 0 \end{cases} \Rightarrow a = \left(\frac{b}{h} \cos\beta - \sin\beta \right) g$$

4.37

Kroppen ligger i vila på ett glatt horisontalplan.

Bestäm accelerationen för kulan i A i det första ögonblicket.



Kraftekvationen

$$\boxed{\vec{F} = m \vec{a}_G} \Rightarrow \# \left\{ \begin{array}{l} \boxed{P = 3m_1 \ddot{x}} \\ 0 = 3m_1 \ddot{y} \Rightarrow \underline{\ddot{y} = 0} \end{array} \right.$$

Momentekvationen

$$\boxed{\vec{M}_G = \vec{H}_G} \quad * \left\{ \begin{array}{l} \boxed{M_G = Pb} \\ \vec{H}_G = \frac{d}{dt} (I_{zz} \cdot \dot{\theta}) = \boxed{3r^2 m_1 \ddot{\theta}} \end{array} \right.$$

$$* \Rightarrow \boxed{\ddot{\theta} = \frac{Pb}{3r^2 m_1}}$$

$$* + \# \Rightarrow \ddot{\theta} = \frac{3m_1 \ddot{x} b}{3r^2 m_1} = \boxed{\frac{\ddot{x} b}{r^2}}$$

$$(\vec{a}_A)_{rel_G} = r \ddot{\theta} \vec{e}_G = r \ddot{\theta} \cdot \frac{1}{2} \vec{e}_x$$

4.44 Bestäm friktionskraften!

Tröghetsradie = d

$$md^2 = I_{zz}$$

Kraftekvationer

$$\vec{F} = m\vec{a}_G:$$

$$\rightarrow: P + f = m\ddot{x}_G$$

$$\uparrow: N - mg = 0 \Leftrightarrow N = mg$$

Momentekvationen

$$\vec{M}_G = \dot{\vec{H}}_G:$$

$$\uparrow \vec{M} = f \cdot R - Pr = -I_{zz}\ddot{\theta} = -md^2\ddot{\theta} = \dot{\vec{H}} \quad (2)$$

$$(1) \Rightarrow P + f = m\ddot{x}_G = m\ddot{\theta}R$$

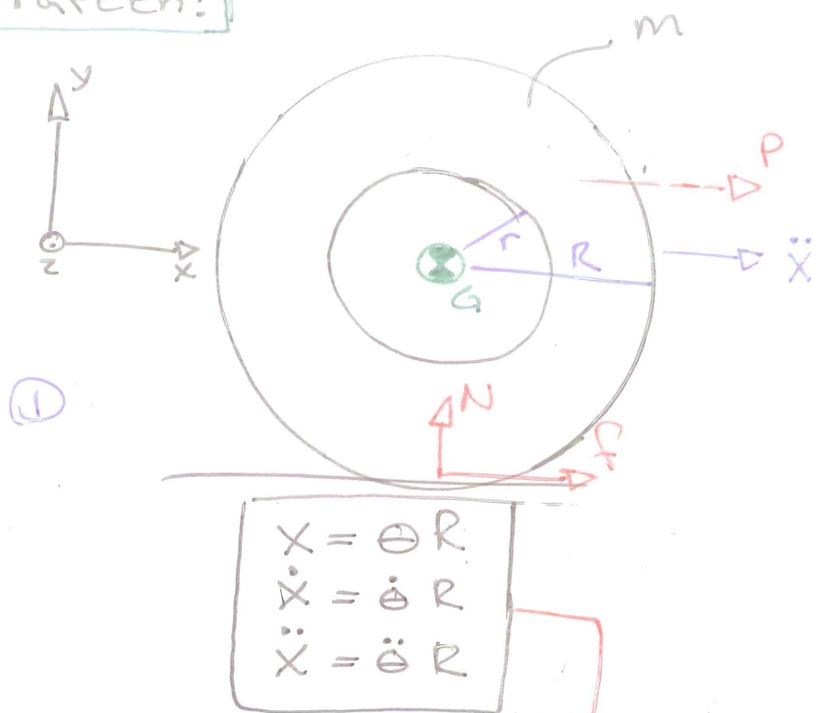
$$\begin{cases} P + f = m\ddot{\theta}R \\ fR - Pr = -md^2\ddot{\theta} \end{cases} \Leftrightarrow fR = Pr - d^2 \frac{P+f}{R}$$

$$\Leftrightarrow f = \frac{Pr}{R} - d^2 \frac{P+f}{R^2} = \frac{P}{R} \left(r - \frac{d^2}{R} \right) - \frac{d^2 f}{R^2}$$

$$\Leftrightarrow f \left(1 + \frac{d^2}{R^2} \right) = \frac{P}{R} \left(r - \frac{d^2}{R} \right)$$

$$\Leftrightarrow f = \frac{P}{R} \cdot \frac{r - \frac{d^2}{R}}{1 + \frac{d^2}{R^2}}$$

$$\Leftrightarrow f = P \cdot \frac{Rr - d^2}{R^2 + d^2}$$



4.57

Bestäm den vinkelacceleration som kroppen får och bestäm det friktionstal μ som krävs för rollning utan glidning



$$I_G = mr^2$$

$$x = r\theta$$

$$\dot{x} = r\dot{\theta}$$

$$\ddot{x} = r\ddot{\theta}$$

Kraftekvationen

$$\swarrow : mgsin\beta - f = m\ddot{x}_G \quad (1)$$

$$\nwarrow : N - mg\cos\beta = 0 \quad (2)$$

Momentekvationen

$$\overline{M}_G = \dot{H}_G : \swarrow : fr = I_G \cdot \ddot{\theta} = mr^2\ddot{\theta} \Leftrightarrow f = mr\ddot{\theta} \quad (3)$$

$$(3) \text{ i } (1) \Rightarrow mgsin\beta - mr\ddot{\theta} = mr\ddot{\theta} \Leftrightarrow gsin\beta = 2r\ddot{\theta}$$

$$\Leftrightarrow \ddot{\theta} = \frac{gsin\beta}{2r}$$

$$(2) : N = mg\cos\beta$$

$$f = \frac{mgsin\beta}{2}$$

friktionsvillkor: $\frac{|f|}{|N|} \leq \mu \Leftrightarrow \frac{\tan\beta}{2} \leq \mu$

$$\Rightarrow \mu_{\min} = \frac{\tan\beta}{2}$$

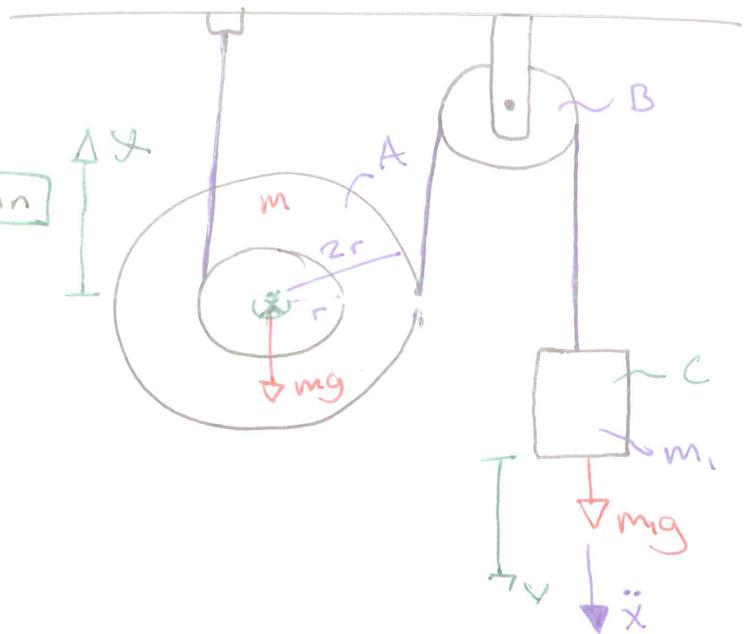
4.63

Trådrulle har trögh.m.s.m. I .

Bestäm rullens vinkelacceleration

Kraftekvationen

$$\begin{aligned} \text{A: } \uparrow \quad Q + s - mg &= m\ddot{x} & \text{①} \\ \text{C: } \downarrow \quad s - m_1g &= m_1\ddot{y} & \text{②} \end{aligned}$$



Momentekvationen

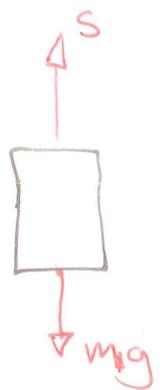
$$\vec{M}_G = 2rs - Qr$$

$$\vec{H} = I \cdot \ddot{\theta}$$

känd!

$$2rs - Qr = I\ddot{\theta} \quad \text{③}$$

Friläggning



Vad är förhållandet mellan \ddot{x} , \ddot{y} & $\ddot{\theta}$?

$$\ddot{x} = r\ddot{\theta}, \quad \ddot{y} = 3r\ddot{\theta} \quad *$$

3 eftersom P är momentancentrum på rulle A.

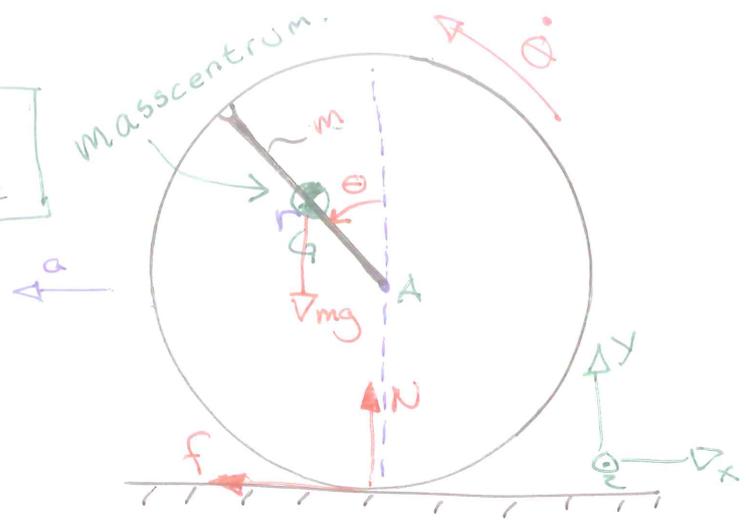
sätter in * i ① och ②!

$$\begin{cases} Q + s - mg = m r \ddot{\theta} \\ s - m_1g = m_1 \cdot 3r \ddot{\theta} \\ 2rs - Qr = I \ddot{\theta} \end{cases}$$

$$\ddot{\theta} = \frac{(3m_1 - m)gr}{I + r^2(m + 9m_1)}$$

4.76

Bestäm rörelseekvationen för kroppen som rullar på bordet



Kraftekvationen

$$\uparrow N - mg = 0$$

$$\rightarrow -f = -m\ddot{x}$$

Tröghetsmoment

$$I_G = \frac{mr^2}{12}, \quad I_A = I_G + \left(\frac{r}{2}\right)^2 m = \quad (\text{Steiners sats})$$

$$I_G = \frac{4mr^2}{12} = \boxed{\frac{mr^2}{3}}$$

$$I_2^C = I_G$$

4.95

Bestäm skivans vinkelhastighet som en funktion av vinkeln θ

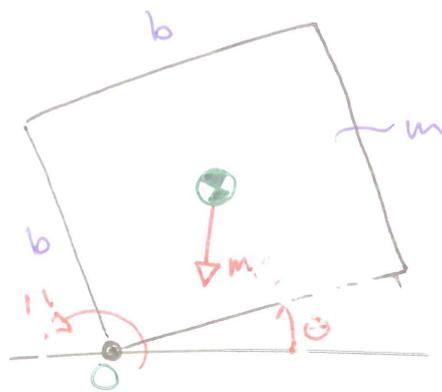
Momentekvationen

$$\vec{M}_O = mg \cos \theta \frac{b}{2} - mg \sin \theta \frac{b}{2} - M$$

$$\vec{H}_O = I_z^O \ddot{\theta} = \frac{2}{3} m b^2 \ddot{\theta} \quad \#$$

Tröghetsmoment

$$I_z^O = \frac{m(b^2 + b^2)}{3} = \frac{2}{3} m b^2$$



$$\# \vec{H}_O = \vec{M}_O \Rightarrow \frac{bmg}{2} (\cos \theta - \sin \theta) - M = \frac{2}{3} b m \ddot{\theta}$$

$$\Leftrightarrow \ddot{\theta} = \frac{3g}{4b} (\cos \theta - \sin \theta) - \frac{3M}{2b} \Leftrightarrow \ddot{\theta} = \frac{3g}{4b} (\cos \theta - \sin \theta) - \frac{3M}{2bm}$$

Vi vill bestämma $\dot{\theta}$!

Integrering m.p. t ger:

$$\frac{1}{2} \dot{\theta}^2 = \frac{3g}{4b} (\sin \theta + \cos \theta) - \frac{3M}{2mb^2} \theta + \text{konstant}$$

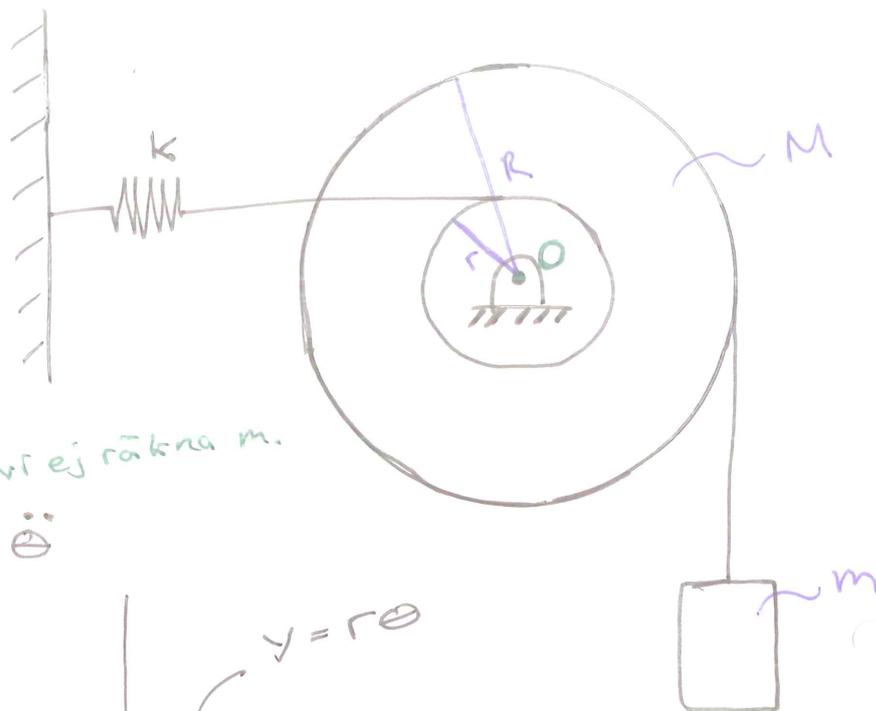
$$\theta = 0 \Rightarrow \dot{\theta} = 0 \Rightarrow 0 = \frac{3g}{4b} + \text{konstant} \Leftrightarrow \text{konstant} = -\frac{3g}{4b}$$

$$\Rightarrow \frac{1}{2} \dot{\theta}^2 = \frac{3g}{4b} (\sin \theta + \cos \theta - 1) - \frac{3M}{2mb^2} \theta$$

$$\Leftrightarrow \dot{\theta} = \sqrt{\frac{3g}{2b} (\sin \theta + \cos \theta - 1) - \frac{3M}{mb^2} \theta}$$

4.98

Bestäm svängningstiden för systemets små svängningar



$$P = k \cdot y = k r \theta$$

behöver vi ej räkna m.

$$\textcircled{2} \cdot S + mg = m \ddot{x} = m R \ddot{\theta}$$

$$\Rightarrow S = -m R \ddot{\theta}$$

Momentekvationen

$$M_o = \overset{\circ}{H}_o$$

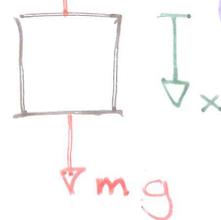
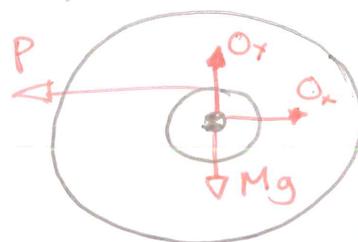
$$-r^2 \theta k - m R^2 \ddot{\theta} = I \ddot{\theta}$$

$$\Leftrightarrow \ddot{\theta} (I + m R^2) + r^2 k \theta = 0$$

$$\ddot{\theta} + \underbrace{\frac{r^2 k}{I + m R^2}}_{\omega_n^2} \theta = 0$$

$$T = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{I + m R^2}{r^2 k}}$$

$y = r\theta$
Friläggning

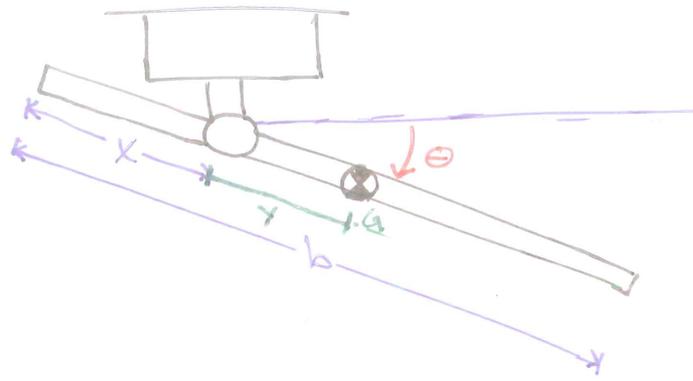


$$x = -r\theta$$

$$\ddot{x} = R \ddot{\theta}$$

4.106

Bestäm x så att $\dot{\theta}$ blir maximal då stängeln släpps från horisontellt läge.



Energiprincipen

$$T_1 + V_1 - T_0 - V_0 = 0 \quad (1)$$

$$x + y = \frac{b}{2}$$

$$\Leftrightarrow y = \frac{b}{2} - x$$

$$T_0 = 0, \quad T_1 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

$$V_0 = 0, \quad V_1 = -mgy \sin \theta = -mg\left(\frac{b}{2} - x\right) \sin \theta$$

$$I_G = \frac{mb^2}{12}, \quad v_G^2 = (y\dot{\theta})^2 = \left(\frac{b}{2} - x\right)^2 \dot{\theta}^2$$

Sätt in i (1)

$$\frac{1}{2} m \left(\frac{b}{2} - x\right)^2 \dot{\theta}^2 + \frac{1}{2} \frac{mb^2}{12} \dot{\theta}^2 - mg\left(\frac{b}{2} - x\right) \sin \theta = 0$$

$$\Leftrightarrow \dot{\theta}^2 = \frac{mg\left(\frac{b}{2} - x\right) \sin \theta}{\frac{1}{2} m \left(\left(\frac{b}{2} - x\right)^2 + \frac{b^2}{12}\right)} = \frac{(4b - 8x) \sin \theta \cdot g}{b^2 + 4x^2 - 4bx + \frac{b^2}{3}} =$$

$$= \frac{3(b - 2x) \sin \theta g}{b^2 - 3bx + 3x^2}, \quad \frac{d\dot{\theta}^2}{dx} = 0 \Rightarrow x = \frac{1}{2} \left(1 - \frac{1}{\sqrt{3}}\right) b$$

$$\dot{\theta}_{\max} = \dot{\theta}(\theta = \frac{\pi}{2}) = \sqrt{\frac{6g}{\sqrt{3}b}}$$

4.107

Bestäm kinetiska energin för A och B

A

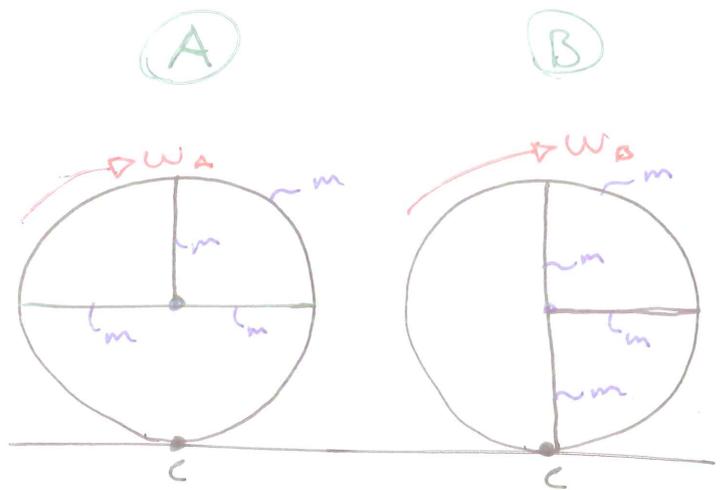
$$T_A = \frac{1}{2} m v_{G_A}^2 + \frac{1}{2} I_C \omega_A^2$$

$$I_{O_A} = m r^2 + 3 \cdot \frac{m r^2}{3} = 2 m r^2$$

$$I_{G_A} = I_{O_A} - r_{O_G}^2 \cdot (4m) = 2 m r^2 - \left(\frac{r}{8}\right)^2 \cdot (4m) = \frac{31}{16} m r^2$$

$$I_{C_A} = I_{G_A} + r_{C_G_A}^2 \cdot (4m) = \frac{31}{16} m r^2 + \left(r + \left(\frac{r}{8}\right)\right)^2 \cdot 4m = \boxed{7 m r^2}$$

$$T_A = \frac{1}{2} m v_{G_A}^2 + \frac{7}{2} m r^2 \omega_A^2$$



B

$$T_B = \frac{1}{2} m v_{G_B}^2 + \frac{1}{2} I_C \omega_B^2$$

$$I_{G_B} = I_{G_A} = \frac{31}{16} m r^2$$

$$I_{C_B} = I_{G_B} + r_{C_G_B}^2 \cdot 4m = \frac{31}{16} m r^2 + \sqrt{r^2 + \left(\frac{r}{8}\right)^2}^2 \cdot 4m = \boxed{6 m r^2}$$

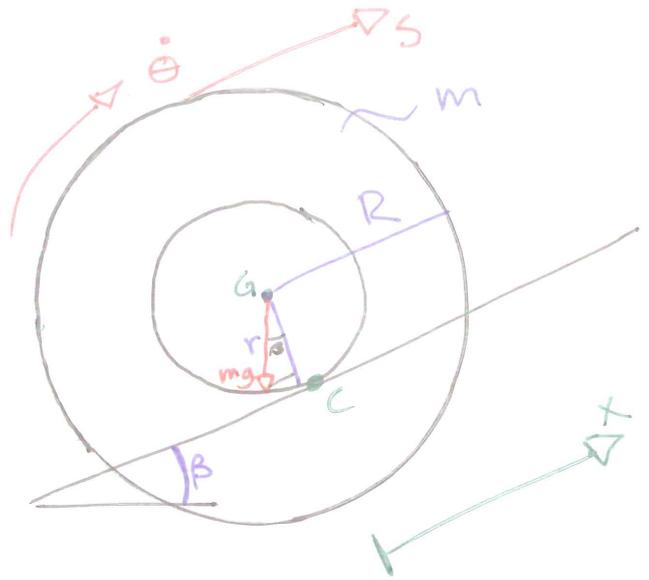
$$T_B = \frac{1}{2} m v_{G_B}^2 + 3 m r^2 \omega_B^2$$

Detta är logiskt eftersom potentiella energin ändras med höjden av ekrarna och $T + V = \text{konstant}$.

4.122

tröghetsradie d .

Bestäm vinkelhastigheten och effekten som kraften S ger som funktion av förflyttningen.



$\vec{M}_C = \dot{\vec{H}}_C$, Momentekvationen

$$\vec{M}_C = S \cdot (R+r) - mgr \sin \beta$$

$$\dot{\vec{H}}_C = I_C \ddot{\theta} = m(d^2 + r^2) \ddot{\theta}$$

$$I_G = md^2$$

$$I_C = I_G + r_{ac}^2 \cdot m = md^2 + mr^2$$

$$\Rightarrow S(R+r) - mgr \sin \beta = m(d^2 + r^2) \ddot{\theta}$$

$$\Leftrightarrow \int \ddot{\theta} \cdot \dot{\theta} = \left(\frac{S(R+r)}{m(d^2+r^2)} - \frac{gr \sin \beta}{(d^2+r^2)} \right) \cdot \dot{\theta}$$

$$\Leftrightarrow \frac{1}{2} \dot{\theta}^2 = \frac{S(R+r) - gr \sin \beta m}{m(d^2+r^2)} \cdot \frac{x}{r}$$

$$\Leftrightarrow \dot{\theta} = \sqrt{2 \frac{S(R+r) - gr \sin \beta m}{m(d^2+r^2) r} x}$$

$$\begin{aligned} \dot{\theta} r &= \dot{x} \\ \ddot{\theta} r &= \ddot{x} \end{aligned}$$

Beräkning av P

$$P = \dot{T}$$

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_c \dot{\theta}^2$$

$$\dot{T} = 0 + \frac{1}{2} I_c \ddot{\theta} \cdot 2 \dot{\theta} = I_c \dot{\theta} \ddot{\theta}$$

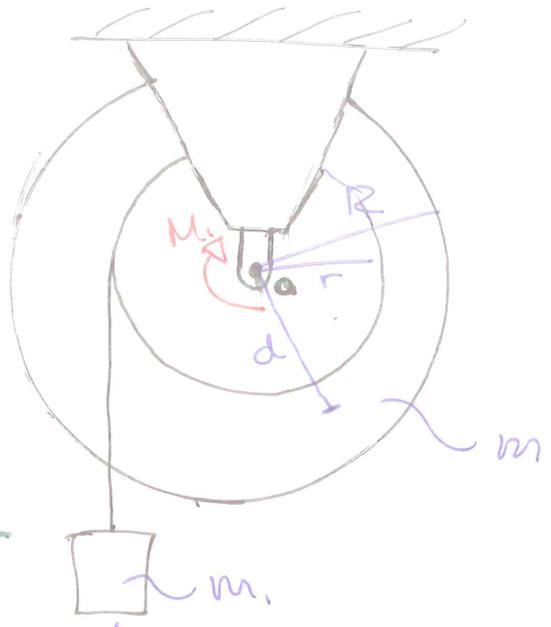
Derivering av $\dot{\theta}$ ger

$$P = (S(R+r)) \cdot \sqrt{2 \frac{S(R+r) - mgr \sin \beta}{mr(d^2 + r^2)}} \cdot x$$



4.112

Bestäm tyngdens fart som en funktion av dess förflyttning h från motsvarande läge.



Momentekvation (A)

$$\vec{M}_0 = \dot{\vec{H}}_0, \quad \vec{H}^A = -md\ddot{\theta}$$

$$\vec{M}_0^A = M_1 - Sr = 0$$

$$\Leftrightarrow Sr - M_1 = md\ddot{\theta} = md^2 \frac{\ddot{h}}{r} \quad \textcircled{1}$$

$$h = r\theta$$

$$\dot{h} = r\dot{\theta}$$

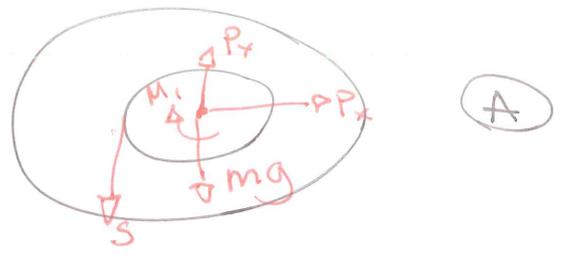
$$\Leftrightarrow \ddot{\theta} = \frac{\ddot{h}}{r}$$

Friläggning

Kraftekvation (B)

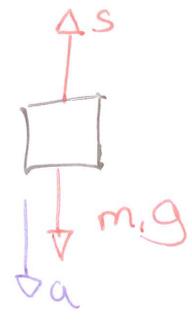
$$\vec{F} = m\ddot{h}$$

$$m_1g - S = m\ddot{h} \quad \textcircled{2}$$



① och ②:

$$m_1g - \left(\frac{M_1}{r} + md\ddot{h}\right) = m\ddot{h}d^2$$



$$\Leftrightarrow \ddot{h} = \frac{m_1gr^2 - M_1r}{md^2 + m_1r}$$

Tidsderivering

$$\frac{1}{2}\dot{h}^2 = \frac{m_1gr^2 - M_1r}{md^2 + m_1r} h + C$$

$$C = \frac{1}{2}v_0^2 \quad (\dot{h}(0) = v_0)$$

$$\dot{h} = \dot{v}(h) = \sqrt{2 \frac{m_1gr^2 - M_1r}{md^2 + m_1r} h + v_0^2}$$

4.116

Bestäm hastigheten hos trådrollens centrum som funktion av läget.

Energin bevaras

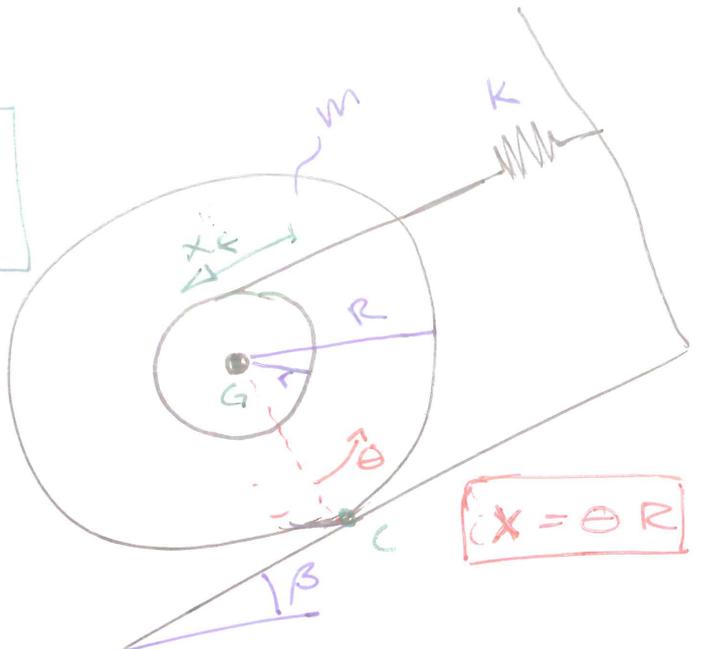
$$T_1 + V_1 - T_0 - V_0 = 0 \quad *$$

$$T_0 = 0 \quad (\text{"släpps från vila"})$$

$$V_0 = \frac{1}{2} k d^2 \quad (\text{"fjäderförlängning } d \text{"})$$

$$T_1 = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I_c \dot{\theta}^2 = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m d^2 \frac{\dot{x}^2}{R^2}$$

$$V_1 = -mgx \sin \beta + \frac{1}{2} k \left(d + \left(x + \frac{xr}{R} \right) \right)^2$$



$$x = \theta R$$

(där tröghetsradien)

$$d = \sqrt{\frac{I}{m}}$$

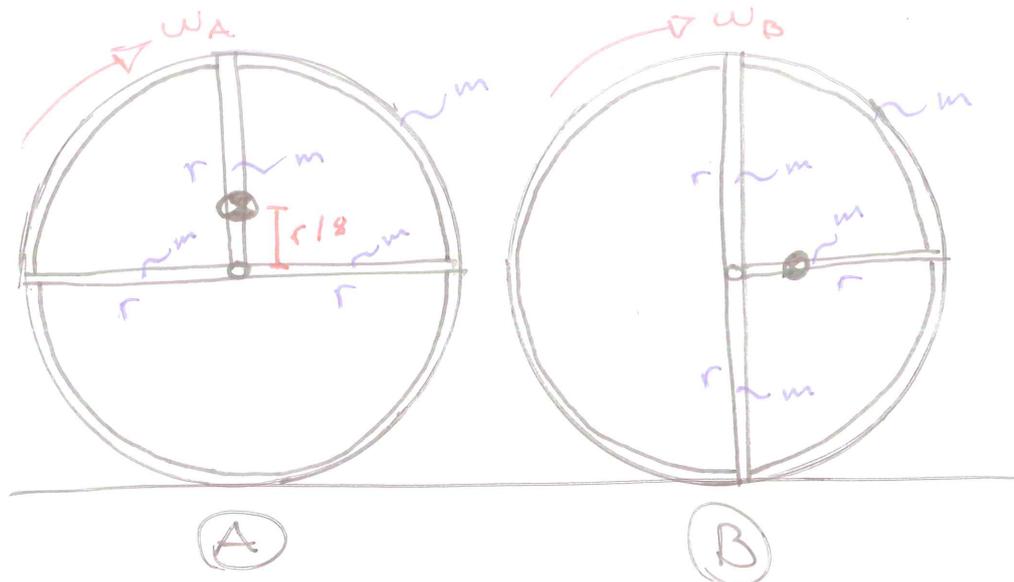
Insättning i * ger

$$\dot{x} = \sqrt{\frac{2R}{m(d^2 + R^2)}} \left(kx \left(\frac{R+r}{R} \right) \left(-\frac{R+r}{2R} x - d \right) + mgx \sin \beta \right)$$

$$\begin{aligned} x_C &= \theta(R+r) = \\ &= \theta R + \theta r = \\ &= x + \frac{xr}{R} \end{aligned}$$

4.129

Bestäm vinkel-
hastigheten i läge
B om den är
 ω_A i läge A.



Jag använder resultatet från 4.107.

$$T_A = \frac{1}{2} m v_G^2 + \frac{7}{2} m r^2 \omega_A^2$$

$$T_B = \frac{1}{2} m v_G^2 + 3 r^2 \omega_B^2 m$$

$$v_B = 0$$

$$v_A = 4 m g \left(\frac{r}{8} \right)$$

Energin bevaras

$$T_A + v_A = T_B + v_B$$

$$\Rightarrow \frac{7}{2} m r^2 \omega_A^2 + 4 m g \left(\frac{r}{8} \right) = 3 m r^2 \omega_B^2$$

$$\Leftrightarrow \omega_B = \sqrt{\frac{7}{6} \omega_A^2 + \frac{g}{6r}}$$

4.134

Bestäm kransens vinkelhastighet $\dot{\theta}$ som en funktion av vridningsvinkeln θ

Tröghetsmoment

$$I_A^z = I_B^z = I_C^z = I_z = \frac{mr^2}{2}$$

(för kugghjulen)

$$I_{O_z} = 2m(3r)^2 = 18mr^2$$

(för kransen)

$$T = 3 \cdot \frac{1}{2} I_z^2 (3\dot{\theta})^2 + \frac{1}{2} I_{O_z}^2 \cdot \dot{\theta}^2 =$$

(Under tiden som kransen roterar ett varv hinner skivorna rotera tre varv.)

$$= \frac{27}{4} mr^2 \dot{\theta}^2 + 9mr^2 \dot{\theta}^2 =$$

$$= \frac{63}{4} mr^2 \dot{\theta}^2$$

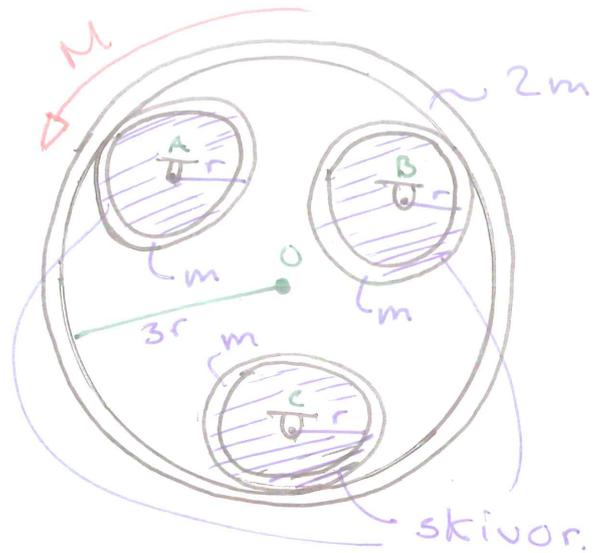
$$\dot{T} = P = M\dot{\theta}$$

$$\dot{T} = \frac{63}{2} mr^2 \dot{\theta} \ddot{\theta} = M\dot{\theta}$$

$$\Leftrightarrow \int \frac{63}{2} mr^2 \dot{\theta} \ddot{\theta} dt = \int M \dot{\theta} dt$$

$$\Leftrightarrow \frac{1}{2} \dot{\theta}^2 \frac{63}{2} mr^2 = M\theta + C$$

$$t=0 \Rightarrow \left\{ \begin{array}{l} \theta=0 \\ \dot{\theta}=0 \end{array} \right. \Rightarrow C=0$$

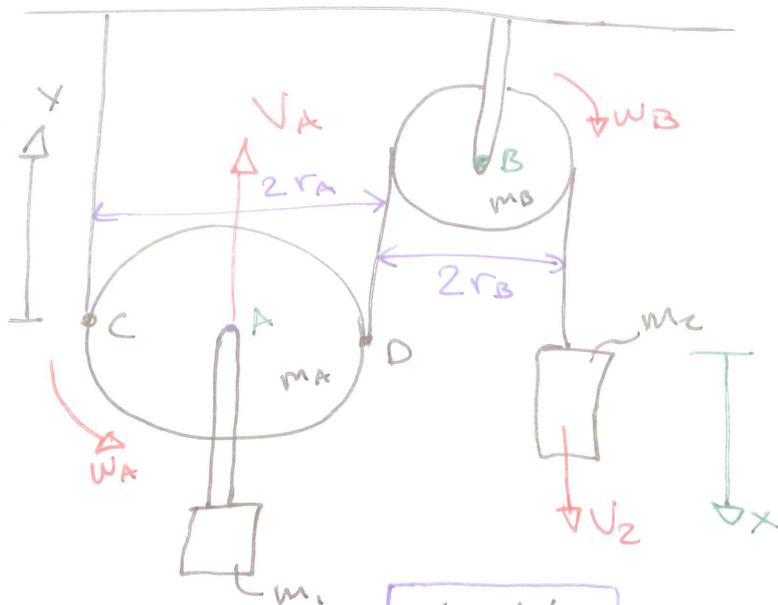


$$\dot{\theta} = \sqrt{\frac{4M\theta}{63mr^2}}$$

4.139

Bestäm för m_1 farten som en funktion av förflyttningen x hos m_2

(Systemet släpps fr. vila)



$$v_2 = 2v_A$$

$$v_A = r_A \omega_A$$

$$v_D = 2r \omega_A$$

eftersom vi roterar kring C!

$$y = x/2$$

Mekaniska energilagen

$$T_i + V_i = T_o + V_o$$

$$T_o = 0$$

$$T_i = \frac{1}{2}(m_1 + m_A)v_A^2 + \frac{1}{2}I_C \omega_A^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}I_B \omega_B^2$$

$$V_o = 0$$

$$V_i = (m_A + m_1)gy - m_2 gx$$

(y = x/2)

$$\Rightarrow \frac{1}{2}(I_A + m_A r_A^2) \omega_A^2 + \frac{1}{2}I_B \omega_B^2 + \frac{1}{2}m_2 v_2^2 + (m_A + m_1)g \frac{x}{2} = m_2 gx$$

$$\Leftrightarrow v_A = \sqrt{\frac{(2m_2 - m_A - m_1)gx}{\frac{I_A}{r_A^2} + m_A + \frac{4I_B}{r_B^2} + m_1 + 4m_2}}$$

