$\boxed{3.1}$ $f(x,y) = \sqrt{4 - x^2 - 2xy - y^2}$

$f(x,y) = \sqrt{4 - (x+y)^2}$

$(x-y)^2 \leq 4$

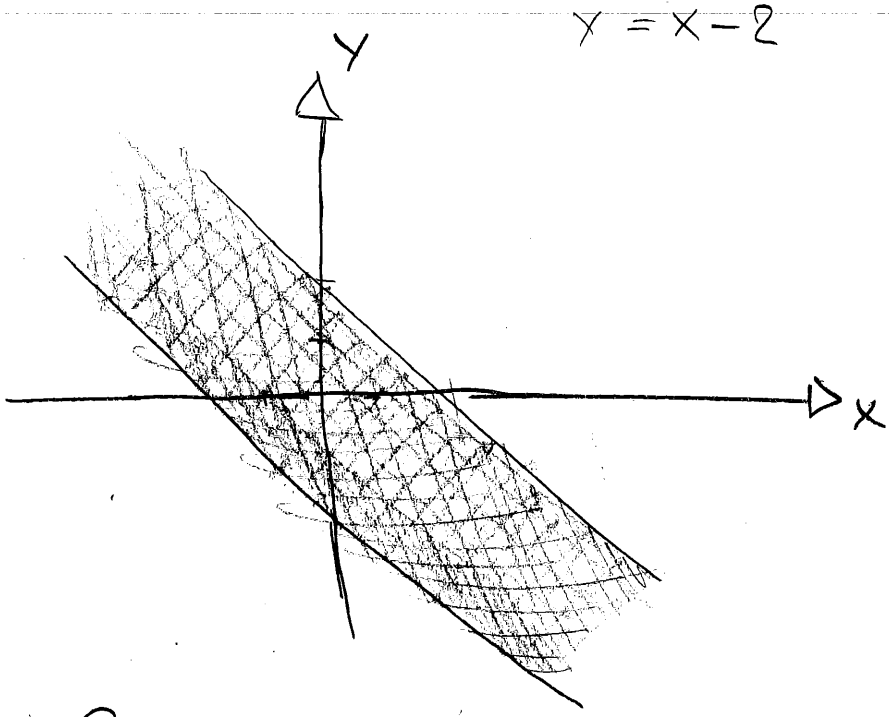
$|x-y| \leq 2$

$y \leq 2 + x$

$y = x - 2$

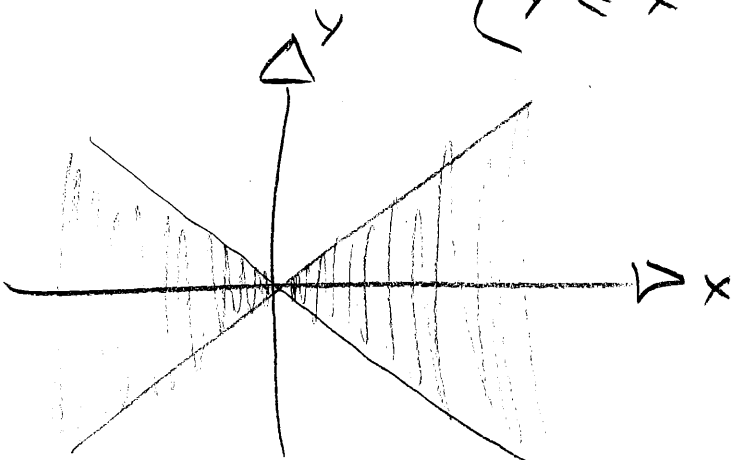
$-x + y = 2$

$x - y = 2$



c) $f(x,y) = \ln\left(\frac{x+y}{x-y}\right)$

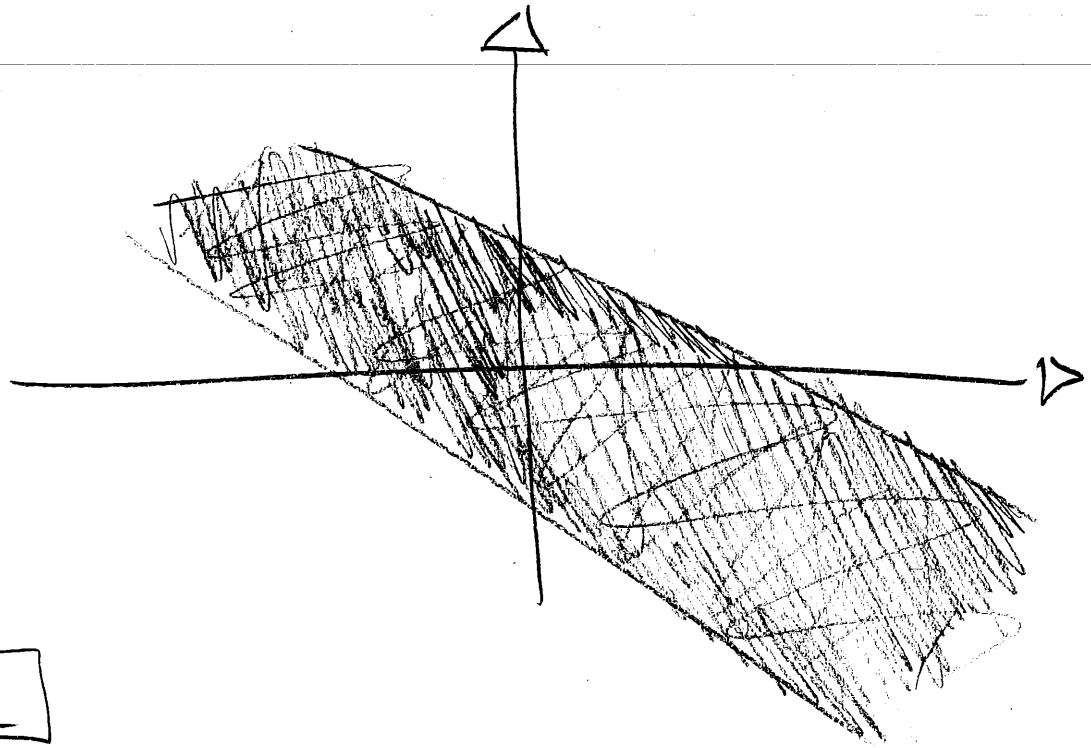
$|y| \leq x \iff \begin{cases} -y \leq x \\ y \leq x \end{cases} \iff \begin{matrix} y \geq -x \\ y \leq x \end{matrix}$



b) $f(x,y) = \arcsin(x+y)$

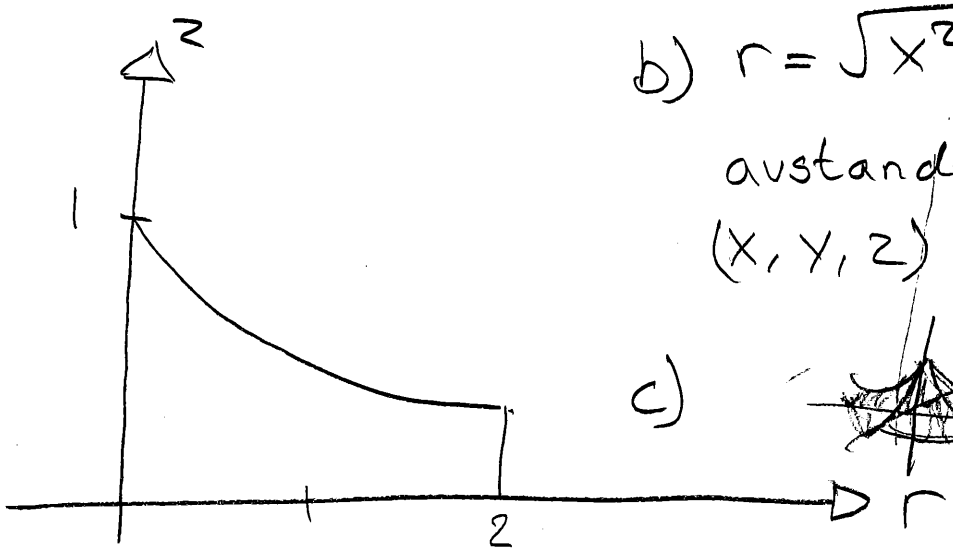
$|x+y| \leq \pi/2$

$$\begin{cases} -x-y \leq \pi/2 \\ x+y \leq \pi/2 \end{cases} \iff \begin{cases} y \geq -x-\pi/2 \\ y \leq -x+\pi/2 \end{cases}$$



$\boxed{3.2}$

a) $z = e^{-r}$, $0 \leq r \leq 2$

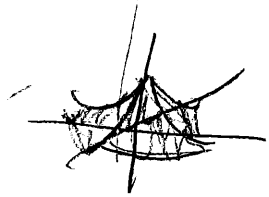


b) $r = \sqrt{x^2+y^2}$

avstandet fr.

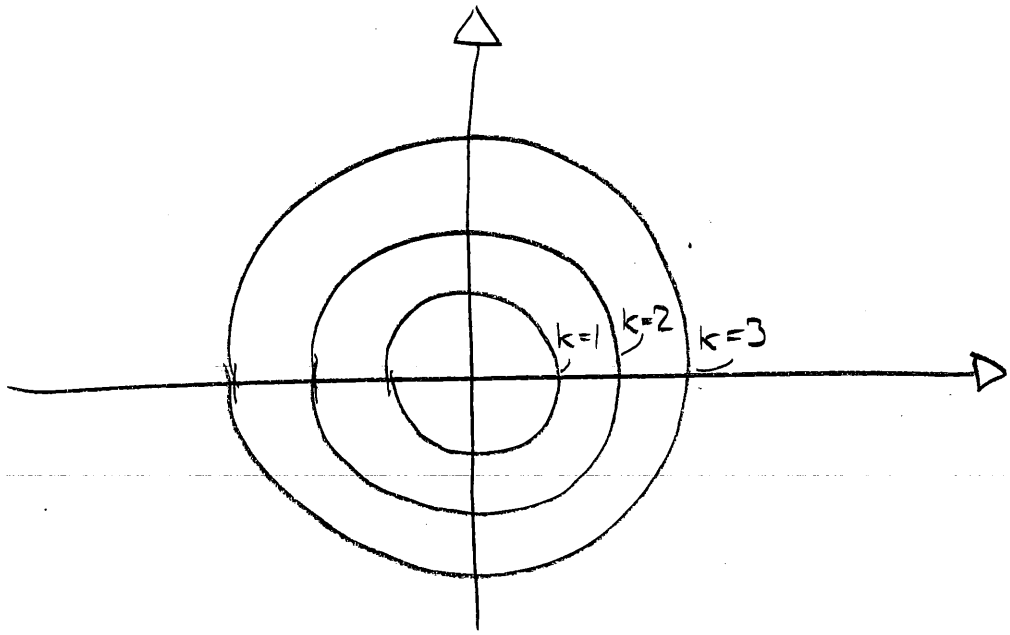
$(x,y,z) \to (0,0,z)$

c)

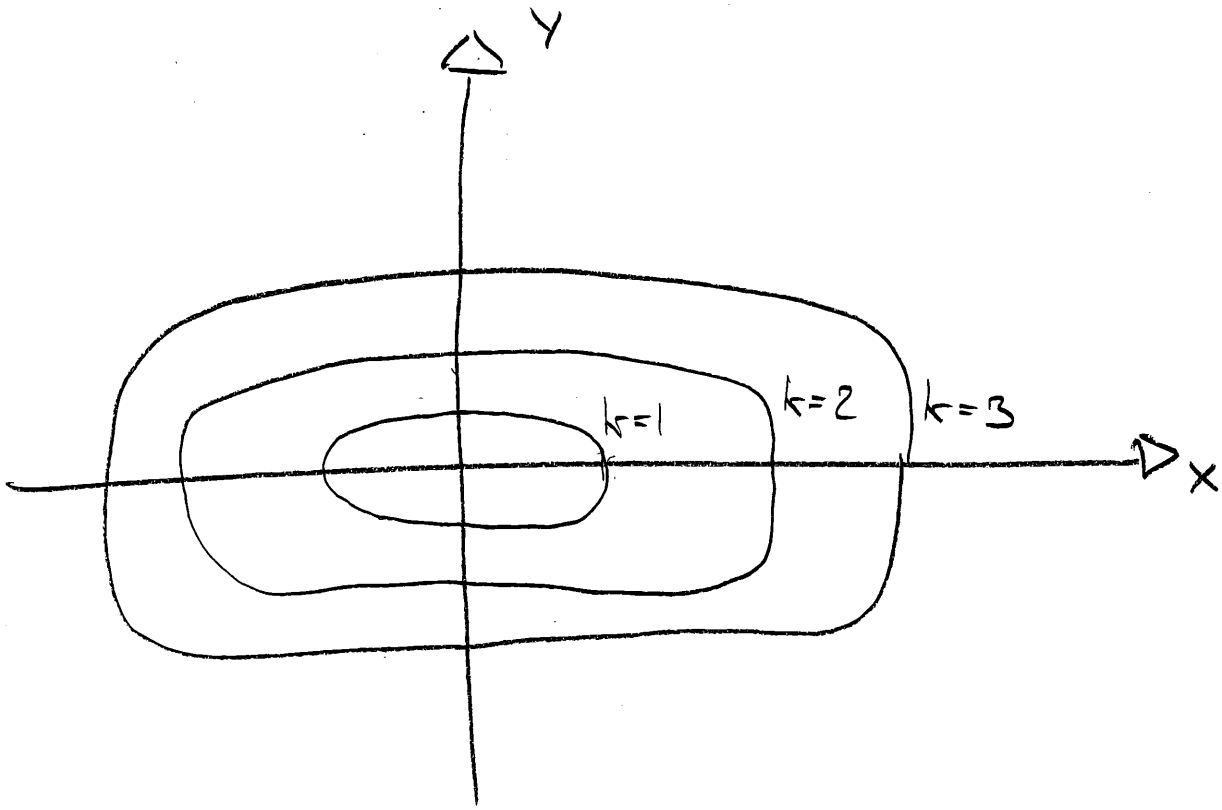


$\boxed{3.5}$

a) $f(x,y) = \sqrt{x^2+y^2}$, $k = 1,2,3$

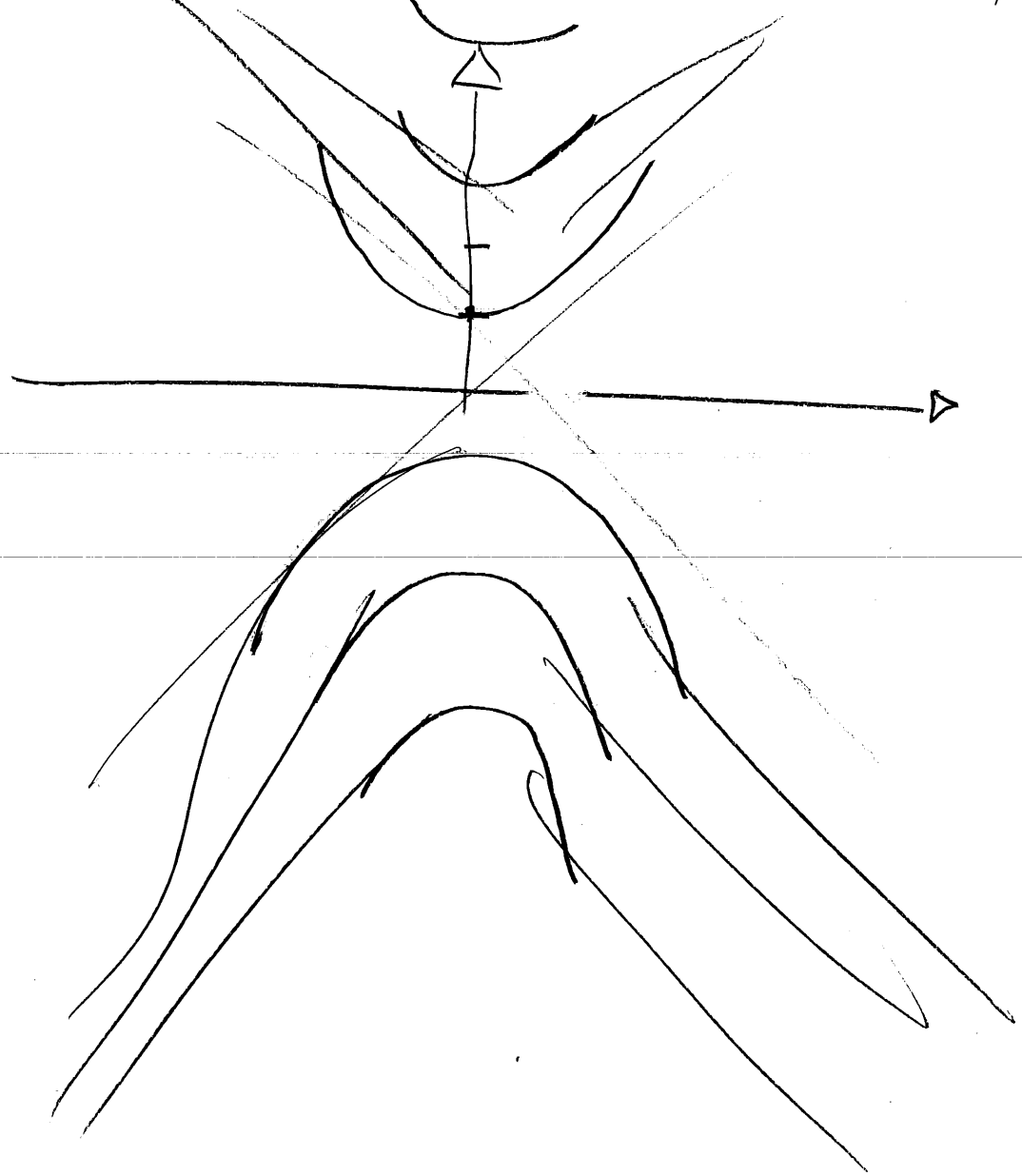


b) $f(x,y) = x^2 + 4y^2$, $k = 1,2,3$

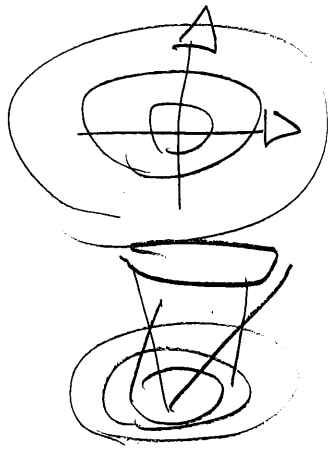


$\boxed{3.5}$

c) $f(x,y) = x^2 - y^2$ $\quad k = -1, 1, 2$



$\boxed{3.7}$ a) $f(x,y) = (x^2+y^2)^{1/2}$, $(x,y) \in \mathbb{R}^2$



$x^2 + y^2 = k^2$

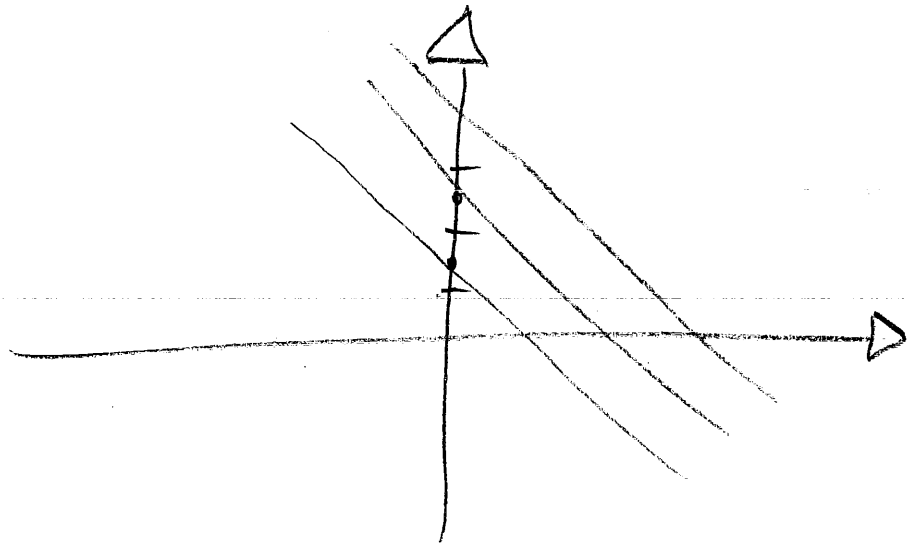
↳ cirklar

$\boxed{\text{KON}}$

$\boxed{3.7}$

b) $f(x,y) = x + 2y - 2 \quad , \quad (x,y) \in \mathbb{R}^2$

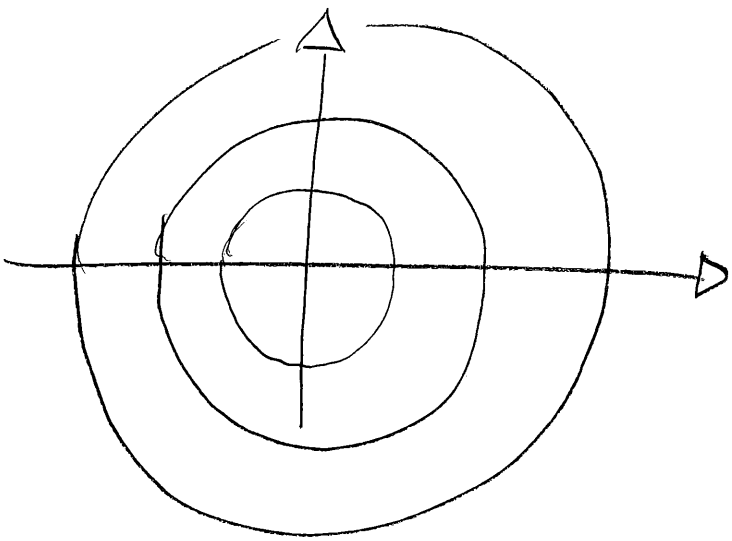
$$x + 2y - 2 = k \Rightarrow y = \frac{-x + 2 + k}{2}$$



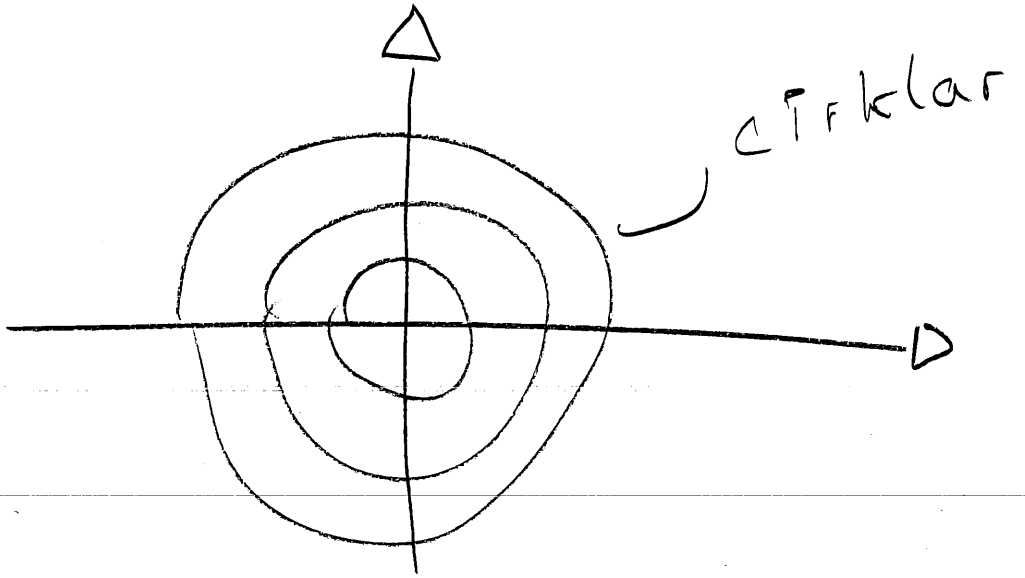
Linjer $\Rightarrow$ Plan!

c) $f(x,y) = (1 - x^2 - y^2)^{1/2}, \quad x^2 + y^2 \leq 1$

$$= (1 - (x^2 + y^2))^{1/2}$$

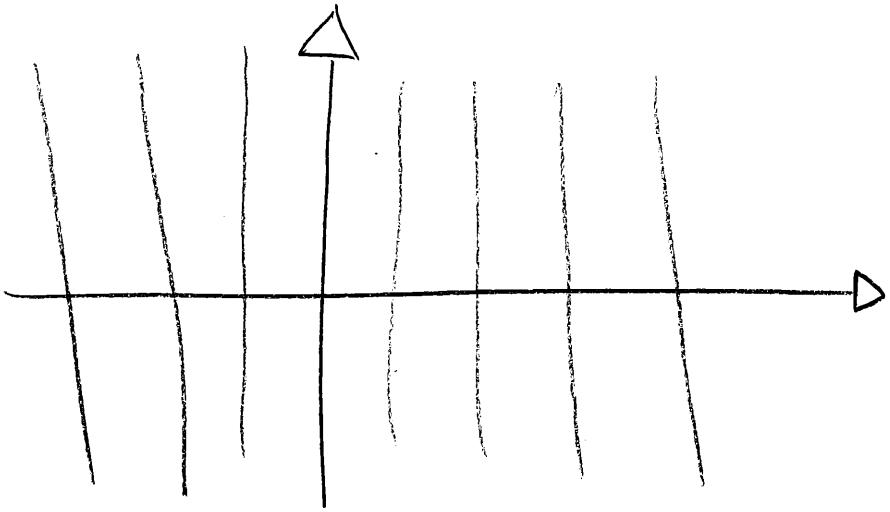


$\boxed{3.7}$ d) $f(x,y) = x^2 + y^2$, $(x,y) \in \mathbb{R}^2$



Paraboloid

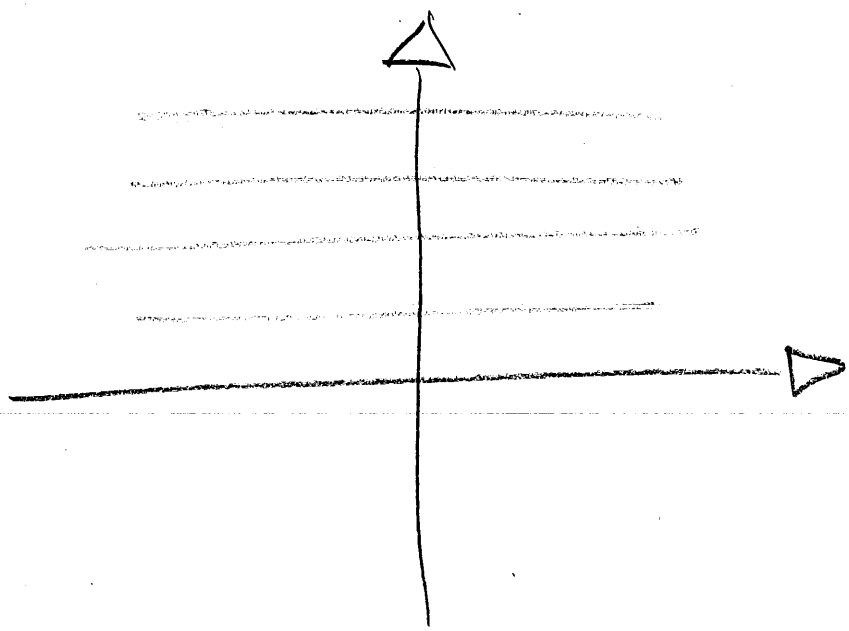
e) $f(x,y) = (1-y^2)^{1/2}$, $x \in \mathbb{R}$, $|y| \leq 1$



Halvcylinder

$\boxed{3.7}$ f) $f(x,y) = x \quad , (x,y) \in \mathbb{R}^2$

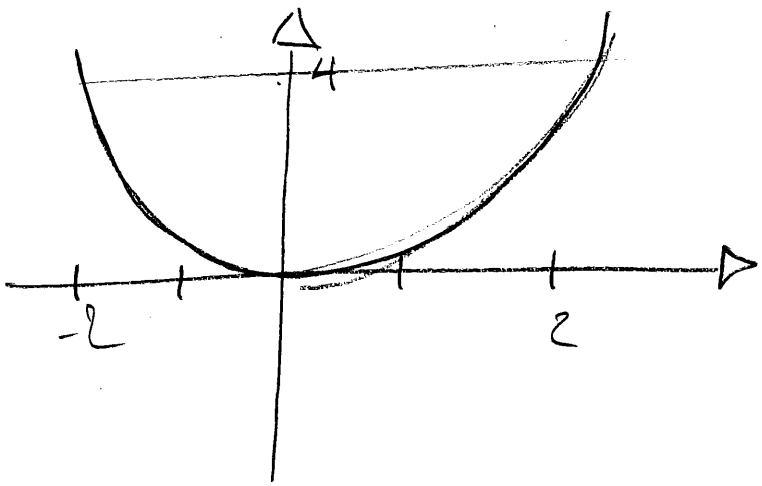
$x = k$



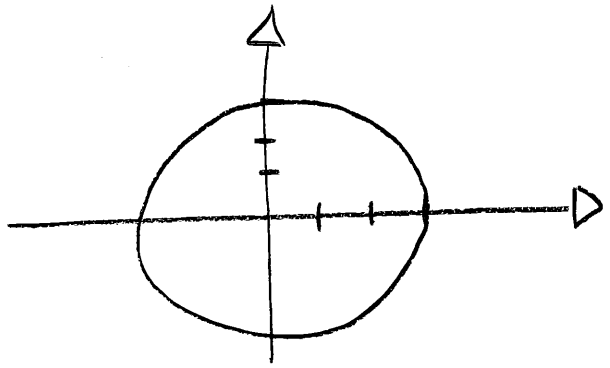
$\boxed{\text{Plan}}$

$\boxed{3.12}$

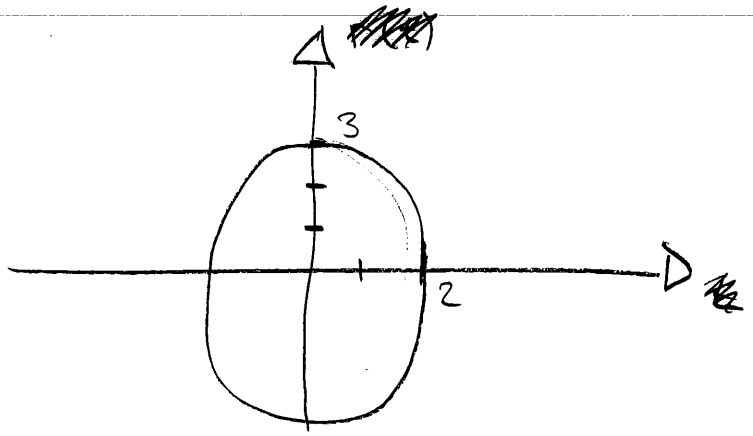
a) $r(t) = (t, t^2) \quad , -2 \leq t \leq 2$



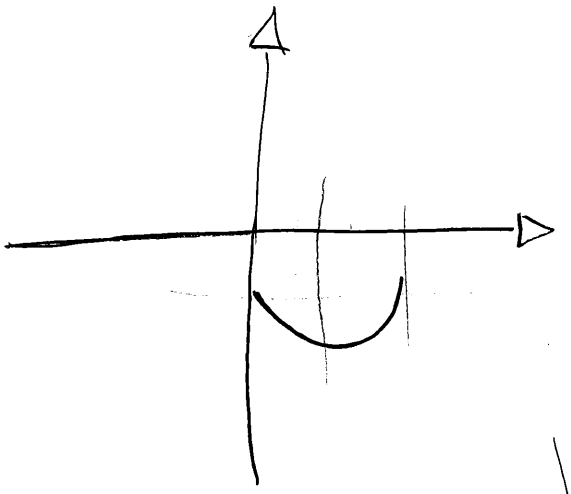
b) $r(t) = (3\cos t, 3\sin t)$, $0 \leq t \leq 2\pi$



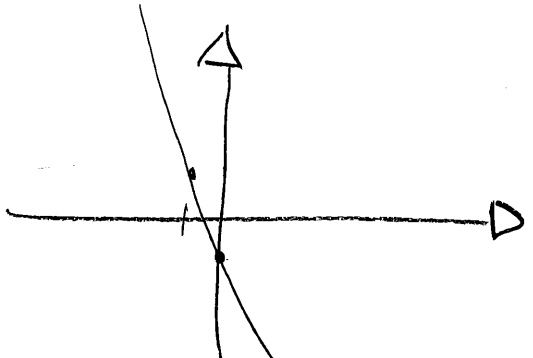
c) $r(t) = (2\cos t, 3\sin t)$, $0 \leq t \leq 2\pi$



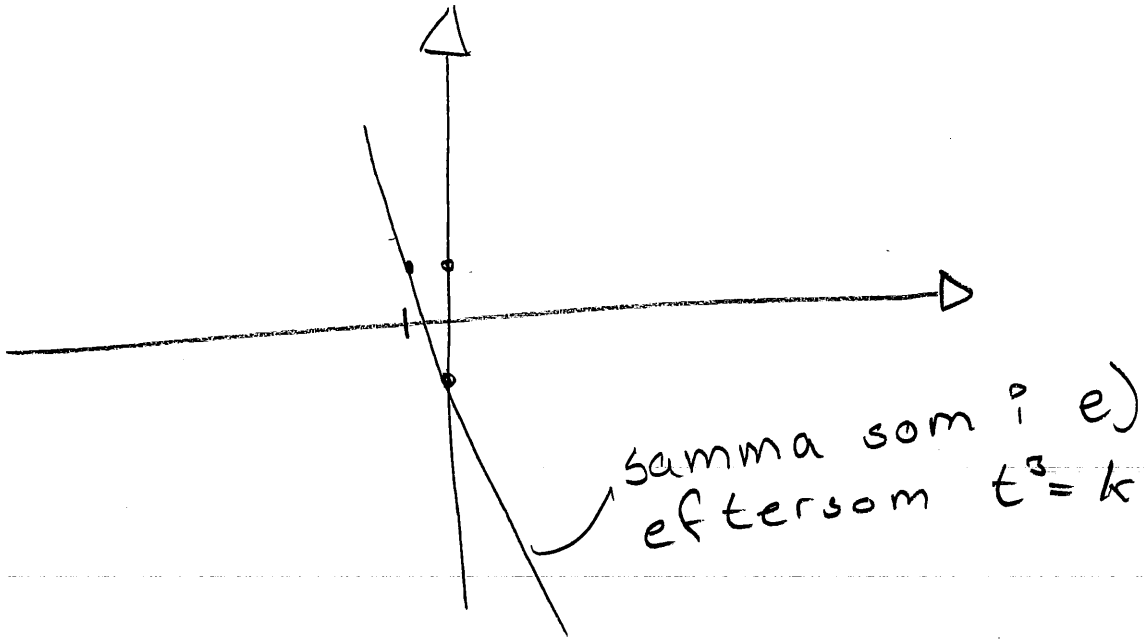
d) $r(t) = (1+\cos t, -1+\sin t)$, $\pi \leq t \leq 2\pi$



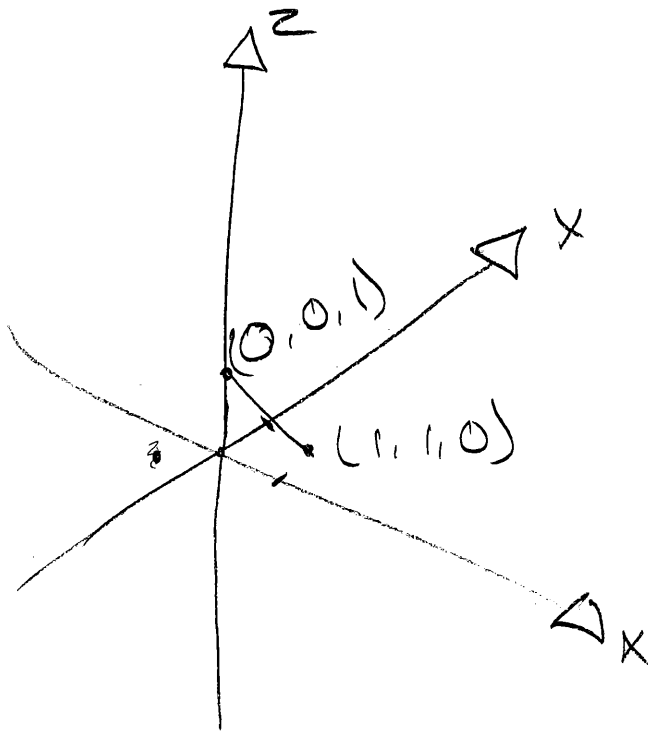
e) $r(t) = (-1+t, 1-2t)$



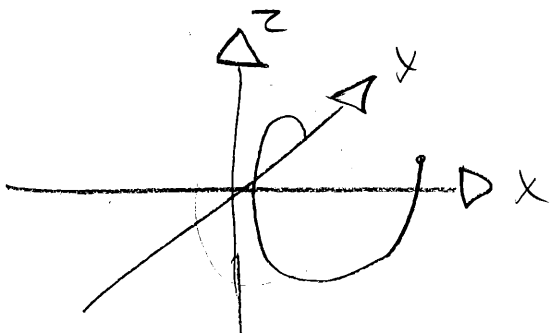
f) $r(t) = (-1+t^3,\ 1-2t^3)$



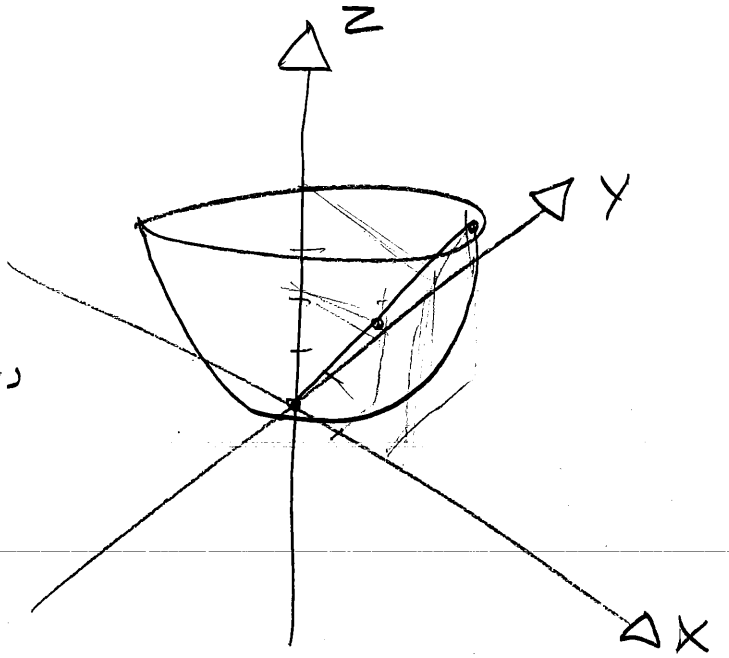
g) $r(t) = (t,\ t,\ 1-t)$, $(0 \leq t \leq 1)$



h) $r(t) = (t,\ 2\cos t,\ 2\sin t)$ $\quad 0 \leq t \leq 1$

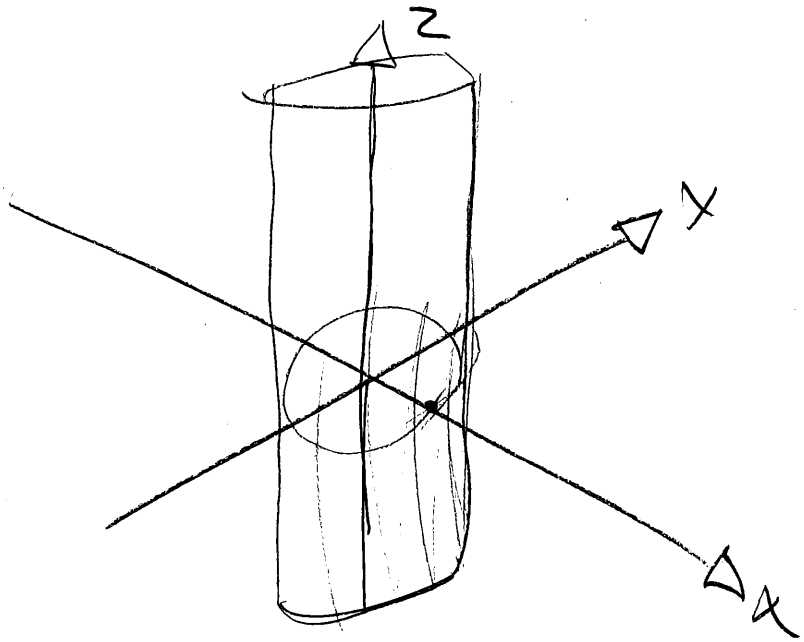


$\boxed{3.13}$ a) $r(s,t) = (s, t, s^2+t^2)$, $0 \leq s^2+t^2 \leq 4$



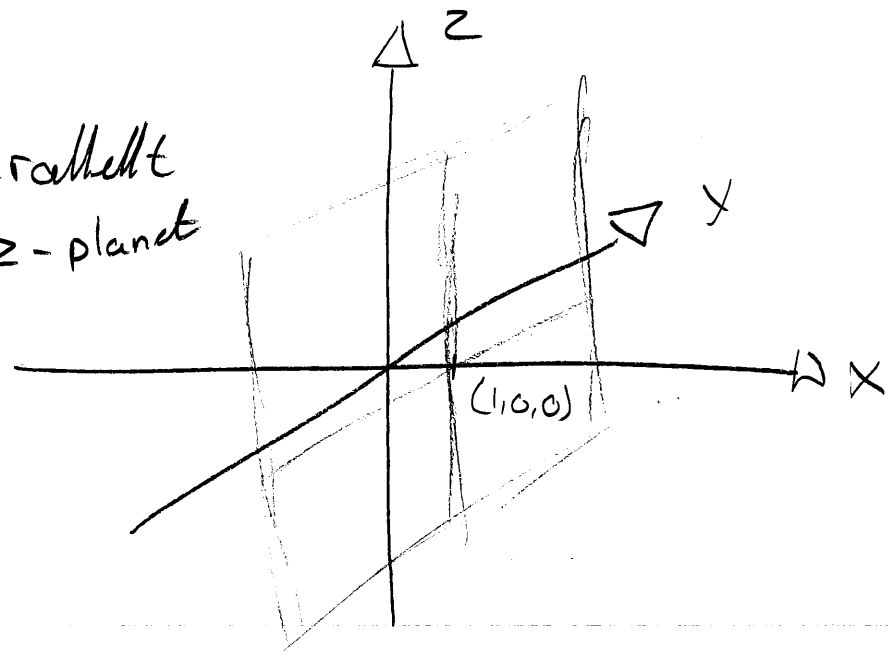
Paraboloid, avskuren.

b) $r(s,t) = (\cos(s), \sin(s), t)$, $0 \leq s \leq 2\pi$

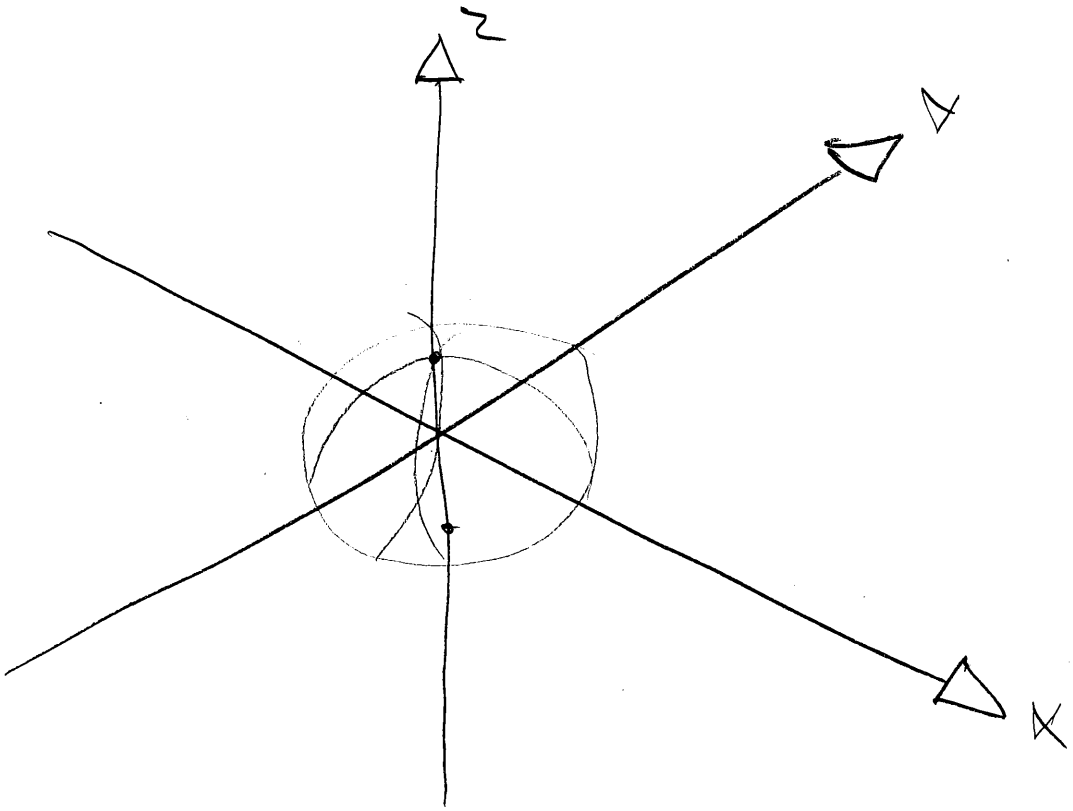


c) $r(s,t) = (1, s, t)$, $(s,t) \in \mathbb{R}^2$

plan, parallellt med yz-planet



d) $r(s,t) = (\sin(s)\cos(t), \sin(s)\sin(t), \cos(s)), (s,t) \in \mathbb{R}$



$\boxed{3.17}$ a) $\lim\limits_{(x,y)\to(1,1)} \dfrac{xy-1}{x-1} = \cancel{\lim\limits_{(x,y)\to(1,1)} \dfrac{r^2\cos\varphi\sin\varphi}{\cos\varphi - 1}}$

$$(x,y) = (1 + r\cos\varphi,\ 1 + r\sin\varphi)$$

$$\frac{xy-1}{x-1} = \frac{r\cancel{\cos\varphi}\sin\varphi + \cancel{\cos\varphi} + \sin\varphi}{\cancel{\cos\varphi}} =$$

$$= \boxed{r\sin\varphi + 1 + \tan\varphi}$$

Inget gränsvärde!

b) $(x^2+y^2)^2 \cdot \ln(x^2+y^2)$, $(x,y) \to (0,0)$

$x = r\cos\varphi$

$y = r\sin\varphi$

$$\left(r^2(\cos^2\varphi + \sin^2\varphi)\right)^2 \cdot \ln\left(r^2(\cos^2\varphi + \sin^2\varphi)\right) =$$

$$= r^4 \cdot \ln r^2 \longrightarrow \boxed{0} \text{ då } r \to 0$$

$\boxed{3.17}$

g) $$\frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = \frac{\sin(r)}{r} \to \boxed{1}$$

$\boxed{3.20}$

d) $x \cdot y \cdot e^{-(x^2+y^2)}$

$$r^2 \sin\varphi \cos\varphi \cdot e^{-r^2} = \frac{r^2 \sin\varphi\cos\varphi}{e^{r^2}} \to \boxed{0}$$

då $r \to \infty$

e) $xy\,e^{-(x+y)^2} =$

Går längs $y = x$

$$x^2 \cdot e^{-4x^2} = \frac{x^2}{e^{4x^2}} \to 0 \text{ då } x \to \infty$$

går längs

$\boxed{3.22}$

c) $f(x,y) = \dfrac{(x+y)^2}{x^2+y^2}$, $(x,y) \neq (0,0)$

$$f(x,y) = \frac{r^2 + 2r^2 \sin\varphi \cos\varphi}{r^2} = 1 + 2\sin\varphi\cos\varphi$$

$\boxed{\text{NEJ}}$

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d) $f(x,y) = \dfrac{(x+y)^4}{x^2+y^2}$, $(x,y) \neq 0$

$$f(x,y) = \frac{(r^2 + 2r^2 \sin\varphi\cos\varphi)^2}{r^2} =$$

$$\frac{r^4 + 4r^4 \sin\varphi\cos\varphi + 4r^4 \sin^2\varphi\cos^2\varphi}{r^2} =$$

$$r^2 + 4r^2 \sin\varphi\cos\varphi(1 + \sin\varphi\cos\varphi) \longrightarrow \boxed{0}$$

$$f(x,y) = \begin{cases} \dfrac{(x+y)^4}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$