1.1 $U = \langle -2, 3, 5, 1, 1, \ldots \rangle$ $V = \langle 1, 3, -2, 5, 4 \rangle$

a) $U+V = \langle -1, 6, 3, 6, 5, \ldots \rangle$

b) $-2U = \langle -4, 6, 10, 2, 2, \ldots \rangle$

c) $2U - 3V = \langle -1, -3, 16, -13, -10 \rangle$

d) $U \cdot V = \langle -2, 9, -10, 5, 4, \ldots \rangle$

e) $\sum U = \langle -2, 3, 5, 1, 1, \ldots \rangle$

1.2

$$S = \sum U = \langle k^3 \rangle \qquad S_n = \sum_{k=0}^{n} U_k = n^3$$

$$U_n = S_n - S_{n-1} = n^3 - (n-1)^3 = \boxed{3n^2 - 3n + 1}$$

1.3 $S = \sum U = \langle k^2 (k+1)^2 \rangle = \langle 0, 4, 100, 126 \rangle$

$$U_n = S_n - S_{n-1} = n^2 (n+1)^2 - (n-1)^2 \cdot n^2 =$$
$$= n^2 (n^2 + 2n + 1 - (n^2 - 2n + 1)) = n^2 \, 4n = \boxed{4n^3}$$

1.5 $U = \langle 72, 65, 58, \ldots \rangle$ $U_n = 72 - 7 \cdot n$

$$U_{75} = 72 - 7 \cdot 75 = \boxed{-453}$$

$\boxed{1.6}$

$6r = -2 \Rightarrow \boxed{r = -\frac{1}{3}}$

$U_0 \cdot r^5 = 6$

$U_0 = \frac{6}{(-\frac{1}{3})^5} = \boxed{-1458}$

$\boxed{1.7}$

$\boxed{1.7}$

a) $U_0 = 150$

$U_1 = 150 - 1{,}5$

$U_2 = U_1 - 1{,}5 \cdot \left(\frac{U_1}{150}\right)$

$U_n = U_{n-1} - 1{,}5 \left(\frac{U_{n-1}}{150}\right)$

$U_n = U_{n-1}\left(1 - 1{,}5\left(\frac{1}{150}\right)\right) = 0{,}99\, U_{n-1}$

$U_n = 0{,}99^n \cdot 150$

$U_{30} = 0{,}99^{30} \cdot 150 = \boxed{111}$

$\Rightarrow$

b)

hälften: $75 = 150 \cdot 0{,}99^x \Rightarrow x =$

$15 = 150 \cdot 0{,}99^x \Rightarrow x =$

$1 = 150 \cdot 0{,}99^x \Rightarrow x =$

c) $\frac{x}{2} = x \cdot \left(1 - \frac{k}{150}\right)^{365} \Rightarrow \boxed{k = 0{,}28}$

$\boxed{1.9}$

$$1^2+3^2+5^2+\dots+(2n-1)^2=\boxed{\sum_{k=0}^{n}(2k-1)^2}$$

$\boxed{1.11(i)}$

$$s=v_0t+\frac{at^2}{2}$$

$$U_0=0$$

$$U_1=\frac{1}{2}g$$

$$U_2=U_1+g=\frac{1}{2}g+g$$

$$U_3=U_2+g$$

$$U_n=\frac{1}{2}g+(n-1)g=-\frac{1}{2}g+ng$$

$$\sum_{k=0}^{n}U=-\frac{1}{2}g+kg=\sum g\left(-\frac{1}{2}+\right)=$$

$+kg-g$

$$=g\left(-\frac{1}{2}n+\frac{n(n+1)}{2}\right)=g\left(\frac{n^2}{2}\right)=\boxed{\frac{gn^2}{2}}$$

$S_0 = 100$

$S_1 = 100 \cdot 1,11 + 100$

$S_2 = 100 \cdot 1,11^2 + 100 \cdot 1,11 + 100$

$$S_n = \sum_{k=0}^{n} 1,11^k \cdot 100 = 100 \sum_{k=0}^{n} 1,11^k$$

$$S_n = 100\,(1 + 1,11 + 1,11^n$$

$$S_n \cdot 1,11 = 100\,(1,11^2 + \dots + 1,11^{n+1})$$

$$= 100\,\frac{1,11^{n+1} - 1}{1,11 - 1}$$

$$\boxed{S_{14} = 3440\,kr}$$

Ingen

SKATT

$\boxed{1.17}$

$$\sum \left(\frac{1}{k^2-1}\right), \quad k \geq 2$$

$$\frac{1}{(k+1)(k-1)} = \frac{A}{k+1} + \frac{B}{k-1}$$

$$1 = A(k-1) + B(k+1)$$

$$1 = Ak - A + Bk + B$$

$$1 = k(A+B) + B - A$$

$$\begin{cases} A+B=0 \Rightarrow A=-B \\ B-A=1 \Rightarrow B-(-B)=1 \Leftrightarrow \boxed{B=\frac{1}{2}} \end{cases}$$

$$A = -\frac{1}{2}$$

$$\sum \left(-\frac{1/2}{k+1} + \frac{1/2}{k-1}\right) = \frac{1}{2}\left(\frac{-1}{3}+\frac{1}{1}\right) +$$

$$+\frac{1}{2}\left(\frac{-1}{4}+\frac{1}{2}\right)+\frac{1}{2}\left(\frac{-1}{5}+\frac{1}{3}\right)+\frac{1}{2}\left(\frac{-1}{6}+\frac{1}{4}\right)=$$

$$= \frac{1}{2}\left(1 + \frac{1}{2} + \cancel{\frac{1}{(n-2)+1}} + \frac{-1}{(n-1)+1} + \frac{-1}{n+1}\right) =$$

$$= \frac{1}{2}\left(\frac{3}{2} - \frac{1}{n} - \frac{1}{n+1}\right) = \boxed{\frac{3n^2 - n - 2}{4n(n-1)}}$$

$\boxed{1.18}$ $U = \langle k(k+3) \rangle \quad k \geq 0$

$U = \langle k^2 + 3k \rangle$

$$S = \sum_{k=1}^{n} k^2 + 3\sum_{k=1}^{n} k = \boxed{\frac{n(n+1)(2n+1)}{6} + 3\,\frac{n(n+1)}{2}}$$

1.14

$$\left\langle \ln\left(1-\frac{1}{k}\right)\right\rangle \quad k \geq 2$$

$$\ln\left(1-\frac{1}{k}\right) = \ln\left(\frac{k-1}{k}\right) = \ln(k-1) - \ln(k)$$

$$\sum_{k=2}^{n}\left(\ln(k-1)-\ln(k)\right) = \left(\ln(2-1)-\ln(2)\right)+\left(\ln(3-1)-\ln(3)\right)$$

$$+\left(\ln(n-1)-\ln(n)\right) = \ln(1) - \ln(n) = \boxed{-\ln(n)}$$

1.15

$$S = \sum_{k=1}^{n}\frac{1}{k(k+1)} = \sum_{k=1}^{n}\frac{1}{k} - \frac{1}{k+1} = \left(\frac{1}{1}-\frac{1}{2}\right)+$$

$$+\left(\frac{1}{2}-\frac{1}{3}\right)+\dots+\left(\frac{1}{n-1}-\frac{1}{n}\right)+\left(\frac{1}{n}-\frac{1}{n+1}\right) =$$

$$\frac{1}{1}-\frac{1}{n+1} = \frac{n+1}{n+1} - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \boxed{\frac{n}{n+1}}$$

$\boxed{1.16}$

$$\sum \frac{k-1}{k!} = \sum_{k=1}^{n} \frac{k}{k!} - \frac{1}{k!} = \left(\frac{1}{1} - \frac{1}{1}\right) + \left(\frac{2}{2} - \frac{1}{2}\right) +$$

$$+ \left(\frac{3}{3\cdot 2} - \frac{1}{3\cdot 2}\right) + \left(\frac{4}{4\cdot 3\cdot 2} - \frac{1}{4\cdot 3\cdot 2}\right) + \dots + \left(\frac{n}{n!} - \frac{1}{n!}\right) =$$

$$= \boxed{1 - \frac{1}{n!}}$$

$\boxed{1.19}$

$$S = \sum_{k=1}^{n} (2k-1)^2 = (2-1)^2 + (4-1)^2 + (6-1)^2 + (8-1)^2 +$$

$$+ \dots + (2n-1)^2 = 1^2 + 3^2 + 5^2 + 7^2 + \dots + (2n-1)^2 =$$

$\boxed{1.38}$

$\boxed{1.40}$

a) $P(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4$

$\langle 1, 2, 3, 4, 5, 0, 0, \dots, 0 \rangle$

b) $0 + 0x + 0x^2 + x^3 + 0x^4 + 0x^5 + 0x^6 + x^7 + 0 + 0 \dots$

$\langle 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, \dots, 0 \rangle$

c) $P(x) = (1+2x)^5 = \sum_{k=0}^{5} 1^{5} \cdot (2x)^{k} \binom{5}{k} =$

$= * \boxed{\langle 1, -10, 40, -80, 80, -32, 0, \dots \rangle}$