Föreläsning 7

Sannolikhetsström

kort Schrödinger

$V(x) = V_0 = \text{const.}$ $\qquad \boxed{\phi'' + k^2\phi = 0}\ , \quad k = \sqrt{\frac{2m(E-V_0)}{\hbar}}$

WTF?

VAD ÄR DETTA?

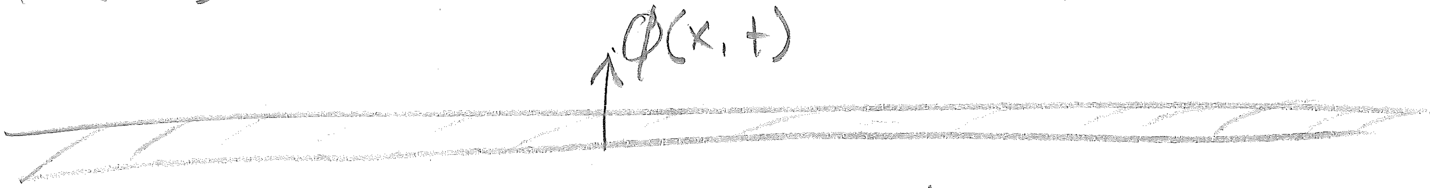
$\phi(x) = Ae^{ikx}$

$\phi(-x) = Ae^{-ikx}$

$\Psi(x,t) = \phi(x)e^{-i\omega t} = \phi(x)\cdot e^{-\frac{E}{\hbar}t} \qquad E = \hbar\omega$

$\phi_+(x,t) = |\Psi_+(x,t)|^2 = |A|^2 \cdot \underbrace{|e^{ikx}|^2}_{1} \cdot \underbrace{|e^{-\frac{iE}{\hbar}t}|^2}_{1} = |A|^2$

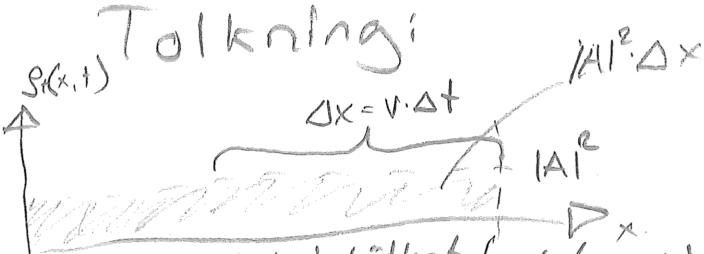
$\phi_-(x,t) = |B|^2 = \text{const.}$



Men: $p = \hbar k$ (rörelsemängd)

$V = \frac{\hbar k}{m}$ (hastighet)

Tolkning:



$\rho = \frac{N}{\Delta x} = \frac{\text{sannolikhetstäthet (antal part.)}}{\text{längd}}$

$$\phi(x) = \begin{cases} Ae^{ik_I x} + Be^{-ik_I x} & x<0 \\ Ce^{ik_{II} x} & x>0 \end{cases}$$

Passningsvillkor vid $x=0$

① $\phi_I(x=0) = \phi_{II}(x=0)$

② $\phi_I'(x=0) = \phi_{II}'(x=0)$

Vid $x=0$

$A\cdot 1 + B\cdot 1 = C \Leftrightarrow A+B=0$

$A ik\cdot e^{ik0} - B ik\cdot e^{-ik0} = ik_{II}\, C\cdot e^{ik_{II}0}$

$\Leftrightarrow Aik - Bik = Cik_{II} \Leftrightarrow Ak - Bk = Ck_{II}$

$$\begin{cases} B = \dfrac{1 - k_{II}/k}{1 + k_{II}/k} \\[2ex] C = \dfrac{2}{1 + k_{II}/k} \end{cases}$$

$\boxed{\text{Amp. hos reflekt våg}}$ (B)

$\boxed{\text{Amp. transm. våg}}$ (C)

$\boxed{\text{Gör egen räkning!}}$

$$S_{in} = \frac{\hbar k}{m} |A|^2$$

$$S_{refl} = \frac{\hbar k}{m} \cdot |B|^2$$

$$S_{transm.} = \frac{\hbar k}{m} \cdot |C|^2$$

Reflekt.-koeff.: $R = \frac{S_{refl}}{S_{infall}} = \frac{|B|^2}{|A|^2} = \frac{k_{II} - k_I}{k_{II} + k_I}$

Transmission $T \doteq \frac{S_{transm.}}{S_{infall}} = \frac{|C|^2 \cdot k_{II}}{|A|^2 \cdot k_I} = \frac{4 k_I k_{II}}{(k_{II} + k_I)^2}$

När $k_{II} \sim k$ Reflekteras inget, detta händer då E e stort.

__SÅ__

$$\boxed{\begin{array}{ll} R \to 1 \text{ då } k_I \to 0 & (E \sim 0) \\ R \to 0 \text{ då } k_I \to k_{II} & (E \gg V_0) \end{array}}$$

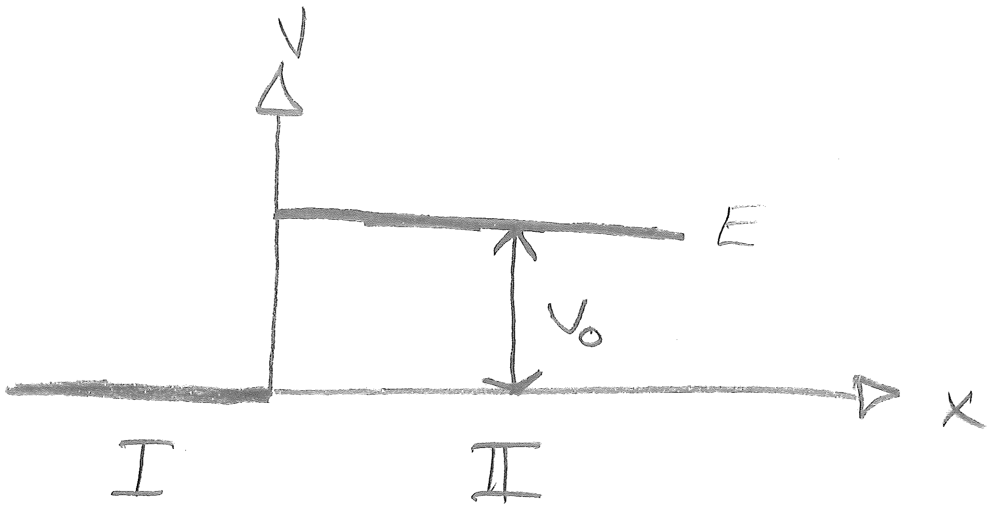
VIKTIGT

TUNNELEFFEKT

(eller: när "i" fick betydelse)

$\varkappa$

$\varkappa$



Vad händer?

I: $\phi_I(x) = Ae^{ikx} + Be^{-ikx}$, $k = \sqrt{\dfrac{2mE}{\hbar}}$

II: fall $E < V_0$ (Nu borde inget ta sig upp)

$$-\frac{\hbar^2}{2m} \cdot \phi_{II}''(x) + V_0 \cdot \phi_{II}(x) = E\phi_{II}(x)$$

$$\phi_{II} + k_{II}^2 \phi = 0 \quad , \quad k_{II} = \frac{\sqrt{2m(E - V_0)}}{\hbar}$$

negativ för $E < V_0$

$$E - V_0 = -(V_0 - E) = i^2 (V_0 - E)$$

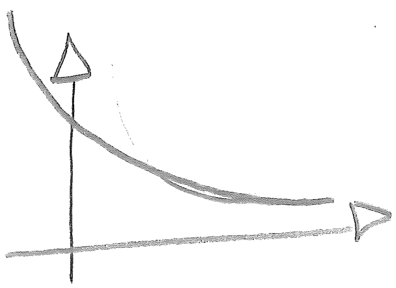
$$\Rightarrow k_{II} \frac{\sqrt{2m i^2 (V_0 - E)}}{\hbar} = \varkappa \Leftarrow \text{kappa}$$

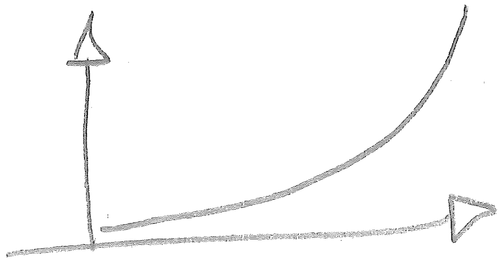
II: $\phi_{II} = C\cdot e^{ik_{II}x} + D\cdot e^{-ik_{II}'x}$

$$= C\cdot e^{i^2\varkappa x} + D\cdot e^{-i^2\varkappa x}$$

$$= \boxed{C\cdot e^{-\varkappa x} + \cancel{De^{+\varkappa x}}}$$

└ Inga vågor då i försvann!

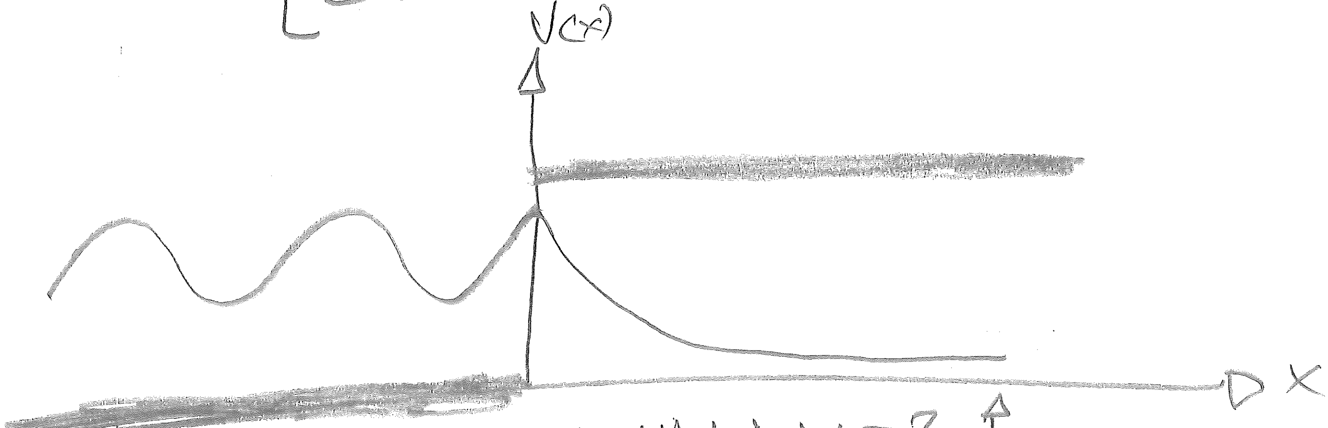




$\boxed{De^{\varkappa x} \text{ "känns" ofysikalisk}}$

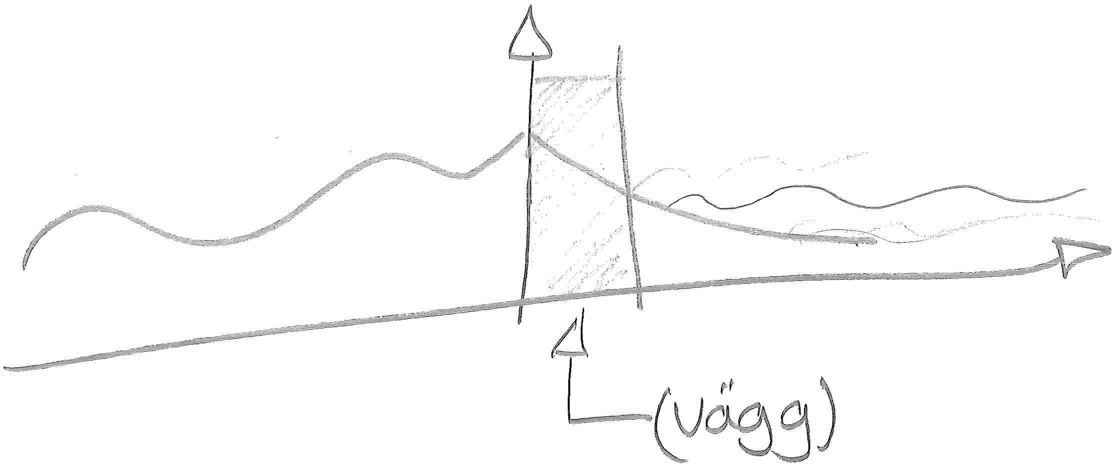
└▷ (leder till mer seriösa problem)

$$\phi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & x < 0 \\ Ce^{-\varkappa x} & x > 0 \end{cases}$$



WHAAAAT? ↑ Bällt som fan

VÄGG



Tar sig partikeln igenom väggen så fortsätter den.

okej, glöm väggen nu

Passningsvillkor:

① $Ae^{ik0} + Be^{-ik0} = Ce^{-\varkappa 0} \Leftrightarrow A + B = C$

② $ikAe^{ik0} - ikBe^{ik0} = -\varkappa Ce^{i\varkappa 0} \Leftrightarrow ik(A-B) = -\varkappa$

SÅ

$$\begin{cases} A + B = C \\ ik(A-B) = -\varkappa \end{cases}$$