

figur1 Från sist:

$$A_2 = 2A_1$$

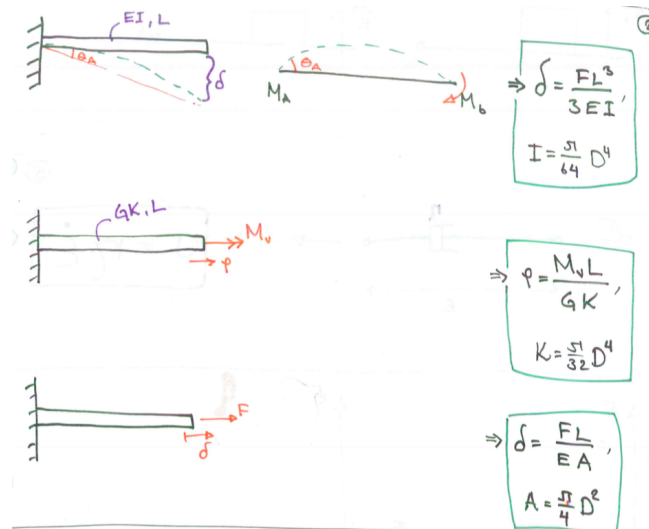
Jämvikt: $F = S_2 - S_1$

$$\Leftrightarrow F = S_2 - S_1 = 2S_2 = 2\sigma_2 A_2 = 2 \frac{\delta}{2L} EA_2 \frac{\delta EA_2}{L}$$

$$\Rightarrow \delta = \frac{FL}{EA_2}$$

Summering av vad vi lärt oss

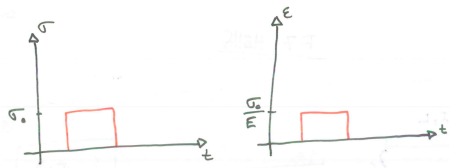
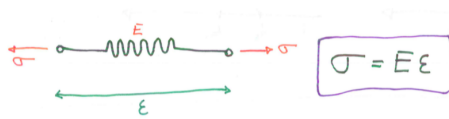
	Differentialekvation	Kroppskr.	Styvhet	Belastning	Motstånd
Böjning	$q = -\frac{dT}{dx}, \quad T = \frac{dM}{dx}, \quad M = -EI \frac{d^2W}{dx^2}$	q	EI	$\sigma_{max} = \frac{M}{W_b}$	W_b
Vridning	$\frac{dM_v}{dx} = -\kappa, \quad M_v = GK \frac{d\Phi}{dx}$	κ	GK	$\tau_{max} = \frac{M_v}{W_v}$	W_v
Dragning	$\frac{dF}{dx} = -X, \quad F = EA \frac{du}{dx}$	X	EA	$\sigma_{max} = \frac{F}{A}$	A



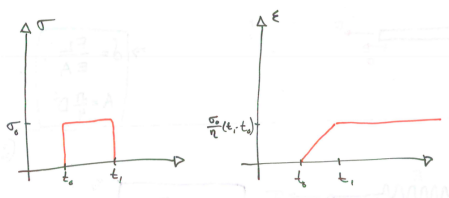
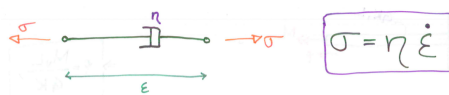
Viskoelastiska material

1. Bygg modell
2. Formulera en differentialekvation
3. Laplacetransformera tur och retur

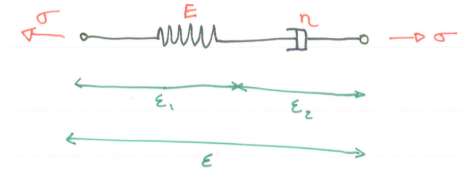
Linjärt elastiska element



Viskosa element



Maxwell-material



$$\begin{cases} \epsilon = \epsilon_1 + \epsilon_2 \\ \sigma = E\epsilon_1 \\ \sigma = \eta\dot{\epsilon}_2 \end{cases} \Rightarrow \begin{cases} \dot{\epsilon} = \dot{\epsilon}_1 + \dot{\epsilon}_2 \\ \dot{\epsilon} \frac{\sigma}{E} + \frac{\sigma}{\eta} \end{cases}$$

$$\sigma(t) = \sigma_0(\theta(t - t_0) - \theta(t - t_1))$$

Laplacetransform

$$\mathcal{L}(\theta(t), s) = \tilde{\sigma}, \quad \mathcal{L}(\sigma(t), s) = s\tilde{\sigma} - \sigma(t_0)$$

$$\mathcal{L}(\epsilon(t), s) = \tilde{\epsilon}, \quad \mathcal{L}(\dot{\epsilon}(t), s) = s\tilde{\epsilon}$$

Vår differentialekvation blir:

$$s\tilde{\epsilon} = \frac{s}{E}\tilde{\sigma} + \frac{1}{\eta}\tilde{\sigma} = \left(\frac{s}{E} + \frac{1}{\eta}\right)\tilde{\sigma}$$

$$\theta(t) \rightarrow \frac{1}{s}$$

$$\theta(t - t_0) \rightarrow \frac{1}{s}e^{-t_0s}$$

$$s\tilde{\epsilon} = \left(\frac{s}{E} + \frac{1}{\eta}\right)\sigma_0(e^{-t_0s} - e^{-t_1s})\frac{1}{s} = \left(\frac{1}{E} + \frac{1}{s\eta}\right)(e^{-t_0s} - e^{-t_1s})\sigma_0$$

Vi vill lösa ut $\tilde{\epsilon}$:

$$\tilde{\epsilon} = \left(\frac{1}{SE} + \frac{1}{s^2\eta} \right) (e^{-t_0s} - e^{t_1s}) \sigma_0$$

$$\mathcal{L}^{-1}(\tilde{\epsilon}) = \epsilon(t) = \frac{1}{E} (\theta(t - t_0) - \theta(t - t_1)) \sigma_0 + \frac{1}{\eta} ((t - t_0)\theta(t - t_0) - (t - t_1)\theta(t - t_1)) \sigma_0$$

$$\begin{cases} 0, & t < t_0 \\ \sigma_0 \left(\frac{1}{E} + \frac{1}{\eta} \right) (t - t_0), & t_0 < t < t_1 \\ \sigma_0 \frac{1}{\eta} (t_1 - t_0), & t_1 < t \end{cases}$$

