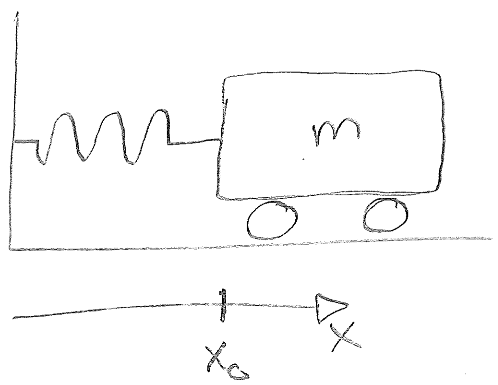
Föreläsning 2

Svängningsrörelse (kap. 12)

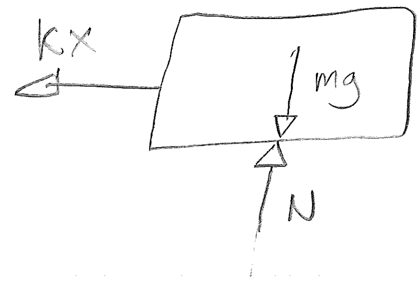
- Fria eller påtvingade svängningar.
  - Odämpade eller dämpade

Vi ska formulera samband för dessa fyra situationer.

Fria odämp. svängn.



frilägg $\Rightarrow$



$$\boxed{x\text{-led: } m\ddot{x} = -kx} \iff \ddot{x} + \frac{k}{m}x = 0$$

$$y\text{-led: } 0 = N - mg \qquad \text{dim: } \frac{m}{s^2} + \underbrace{\left[\frac{k}{m}\right]}_{\frac{1}{s^2}} m$$

▷ Def: $\frac{k}{m} = \omega_n^2$

Vi vill lösa $\ddot{x} + \omega_n^2 x = 0$

Ansats: $x = Ae^{\lambda t}$ $[\lambda t] = 1$ $[\lambda] = \frac{1}{[t]} = \frac{1}{s}$

$\Rightarrow \ddot{x} = \lambda^2 A e^{\lambda t}$

sätt in: $\lambda^2 A e^{\lambda t} + \omega_n^2 A e^{\lambda t} = 0$

$\Leftrightarrow A e^{\lambda t}(\lambda^2 + \omega_n^2) = 0$

kar pol: $\lambda^2 + \omega_n^2 = 0 \Rightarrow \boxed{\lambda = i\omega_n}$

$x = A_1 e^{i\omega_n t} + A_2 e^{-i\omega_n t} =$

$= C_1 \cos(\omega_n t) + C_2 \sin(\omega_n t) = C \sin(\omega_n t + \beta) =$

$= C\left(\sin(\omega_n t)\cos(\beta) + \sin(\beta)\cos(\omega_n t)\right) =$

<u>Häng med nu</u> verifiera likamedtecknet

$x = \cos\omega_n t\,[C_1] + \sin\omega_n t\,[C_2] = \cos\omega_n t\,[C\sin\beta] + \sin\omega_n t\,[C\cos\beta]$

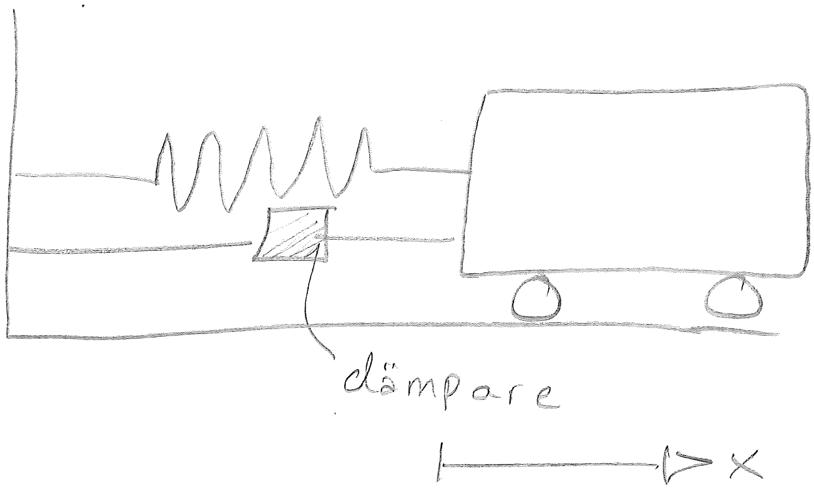
$\Rightarrow \left.\begin{array}{l} C_1 = C\sin\beta \\ C_2 = C\cos\beta \end{array}\right\} \begin{array}{l} \tan\beta = \frac{C_1}{C_2} \\ C^2(\cos^2\beta + \sin^2\beta) = C_1^2 + C_2^2 \end{array}$

Fri odämpad svängning
$x = C_1\cos(\omega t) + C_2\sin(\omega t)$
$\dot{x} = -C_1\omega\sin(\omega_n t) + C_2\omega\cos(\omega_n t)$
$\ddot{x} = -C_1\omega_n^2\cos(\omega t) - C_2\omega_n^2\sin(\omega_n t)$

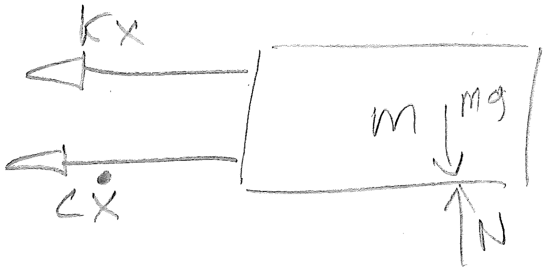
$\leftarrow$ OKEJ

#e2p2

Fri dämpad svängning



friläggning



massax acc

x-led: $m\ddot{x} = -kx - c\dot{x}$

$$\ddot{x} + \underbrace{\frac{c}{m}}_{2\zeta\omega_n}\dot{x} + \frac{k}{m}x = 0$$

$[\zeta]$ är dimensionslös

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x$$

Ansats: $x = Ae^{\lambda t}$

$\dot{x} = \lambda Ae^{\lambda t}$

$\ddot{x} = \lambda^2 Ae^{\lambda t}$

Insättning ~~ger~~ i $\ddot{x}+2\zeta\omega_n\dot{x}+\omega_n^2 x=0$

$$Ae^{\lambda t}(\lambda^2+2\zeta\omega_n\lambda+\omega_n^2)=0$$

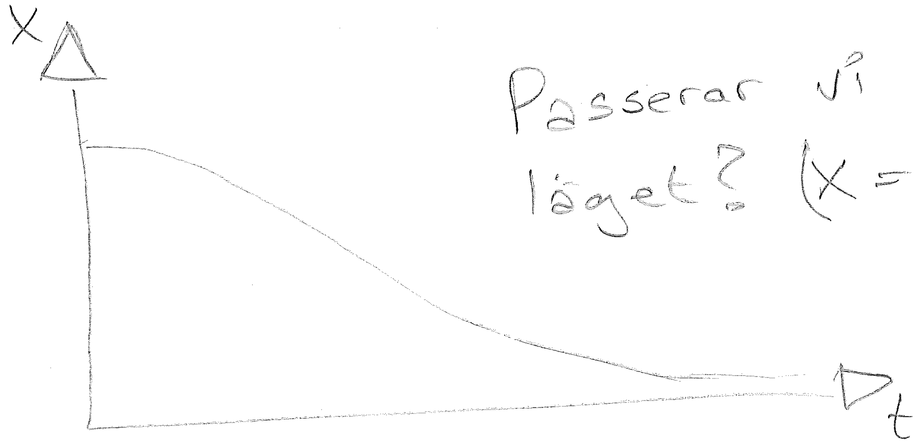
kar. pol.

$$\lambda = -\zeta\omega_n \pm\sqrt{\zeta^2\omega_n^2-\omega_n^2} = \boxed{\omega_n\left(-\zeta\pm\sqrt{\zeta-1}\right)}$$

Om $\zeta>1$: Stark dämpning

$\lambda_1, \lambda_2 \in \mathbb{R}$

$x = A_1e^{\lambda_1 t}+A_2e^{\lambda_2 t}$ (ingen svängning)



Passerar vi jämvikts-läget? $(x=0)$

$$x=A_1e^{\lambda_1 t}+A_2e^{\lambda_2 t}=0 \Rightarrow e^{(\lambda_1-\lambda_2)t} = -\frac{A_2}{A_1}$$

bara strängt avtagande.

Om $\zeta = 1$: Kritisk dämpning

$\lambda_1 = \lambda_2 = -\zeta\omega_n$ (dubbelrot)

$$X = (A_1 + A_2 t)e^{\lambda t}$$

$$X = 0 \Rightarrow t = -\frac{A_1}{A_2}$$ (Jämviktsläget)

$$\dot{X} = 0 = A_2 e^{\lambda t} + (A_1 + A_2 t)\lambda e^{\lambda t}$$

$$A_2 + \lambda(A_1 + A_2 t) = 0$$

$$\Rightarrow \boxed{t = \frac{1}{A_2}\left(-\frac{A_2}{\lambda} - A_1\right)}$$ (Vändpunkt)

Vid kritisk dämpning återskapas jämviktsläget snabbast möjligt

↳ Dimensionering av våg, bilsitsfjädrar osv. sånt vi inte vill att ska pendla.

$$\zeta = 1 \Rightarrow 2\zeta\omega_n = 2\omega_n = \frac{c}{m}, \quad \omega_n = \sqrt{\frac{k}{m}} \quad \Bigg\} \quad 2\sqrt{\frac{k}{m}} = \frac{c}{m}, \quad 2\sqrt{km} = c$$

Om $\zeta < 1$: Svag dämpning

Imaginärt $\lambda \Rightarrow$ cos & sin $\Rightarrow$ svängningar!

$$\lambda_{1,2} = \omega_n\left(-\zeta \pm i\sqrt{1-\zeta^2}\right)$$

Sätt in

$$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} = A_1 e^{\omega_n\left(-\zeta + i\sqrt{1-\zeta^2}\right)t} + A_2 e^{\omega_n\left(-\zeta - i\sqrt{\ }\right)t} =$$

$$= e^{-\omega_n t \zeta}\left(A_1 e^{i\omega_n\sqrt{\ }t} + A_2 e^{-i\omega_n\sqrt{\ }t}\right)$$

Def $\boxed{\omega_n\sqrt{1-\zeta^2} = \omega_d}$ (dämpad egenvinkelfrekvens)

$$\omega_d < \omega_n$$

$$\omega_d = 2\pi f_d = 2\pi \frac{1}{T_d}$$

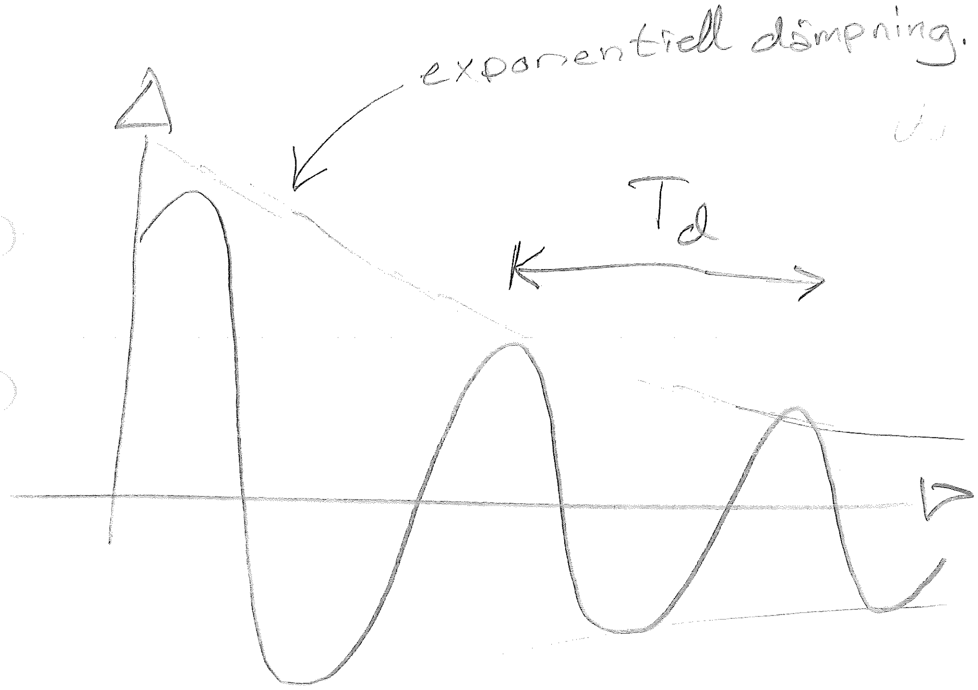
$$T_d > T_n$$

Så $\boxed{x = e^{-\omega_n \zeta t}\left(A_1 e^{i\omega_d t} + A_2 e^{-i\omega_d t}\right)}$

Derivera två ggr!

$$x = e^{-\omega_n \zeta t}\left(A_1 e^{i\omega_d t} + A_2 e^{-i\omega_d t}\right)$$

$$\dot{x} = e^{-\omega_n \zeta t}\left[B_1 \sin(\omega_d t + \alpha) + B_2 \cos(\omega_d t + \beta)\right]$$



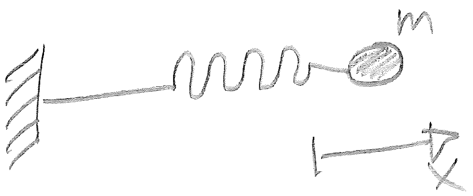
$$\boxed{\omega_d = \omega_n\sqrt{1-\zeta^2}}$$ **Definition**

Odämpat: $c = 0 \Rightarrow \zeta = \frac{c}{m}\cdot\frac{1}{2\omega_n} = 0$

$$\Rightarrow \boxed{\omega_d = \omega_n}$$

6666...

Exempel



fjäderkraften konservativ $\Rightarrow$ mekaniska energilagen gäller.

$$T_1 + V_1 = T_0 + V_0 = \text{konstant.}$$

$$\Leftrightarrow \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 = \text{konstant}$$

$$\frac{d}{dt} = \frac{1}{2} m 2 \dot{x} \ddot{x} + \frac{1}{2} k 2 x \dot{x} = 0$$

$$\Leftrightarrow \boxed{m\ddot{x} + kx = 0}$$