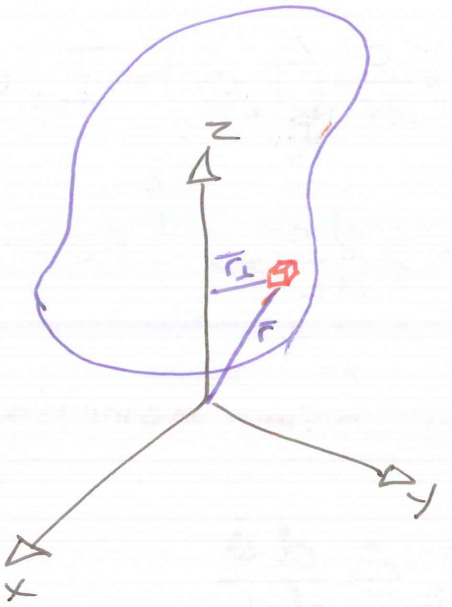
FÖRELÄSNING 12

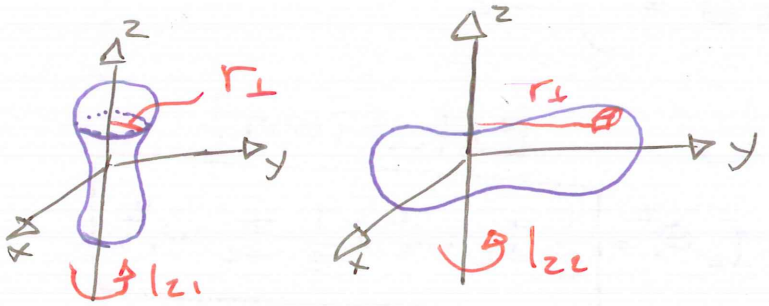


$$T = \frac{1}{2}\int r_\perp^2 \, dm \, \dot{\theta}^2$$

I_z = tröghetsmom. m.a.p z-axeln.

$$H_{oz} = \left(\int r_\perp^2 \, dm\right)\dot{\theta} = I_z \dot{\theta}$$

Exempel på två jordnötter



$$I_{z_1} < I_{z_2} \Rightarrow T_1 < T_2 \Rightarrow H_{oz_1} < H_{oz_2}$$

$$I_z = \int r_\perp^2 \, dm$$

Nu vill vi att I ska vara lika stor som följande:

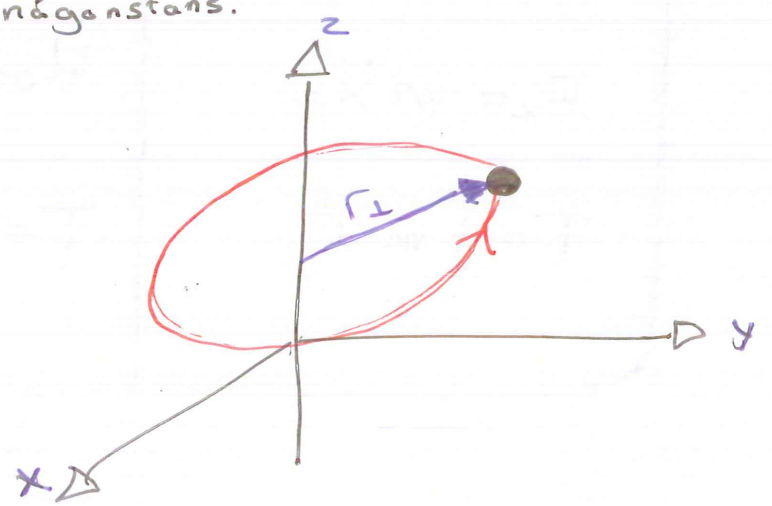
"Jag klumpar ihop all massa och lägger den någonstans."

DEF AV k:

$$I_z = \int r_\perp^2 \, dm = k^2 m$$

$$k = \sqrt{I_z / m} = k_z$$

TRÖGHETSRADIEN



O = fix punkt $\quad \overline{M}_O = \dot{\overline{H}}_O$

Rot. kring z: $M_{Oz} = \dot{H}_{Oz} = \frac{d}{dt}(I_z\dot{\theta}) = I_z\ddot{\theta}$

$\left(F = ma = m\ddot{x} = m\frac{dx}{dt}\cdot\frac{d\dot{x}}{dx} = m\dot{x}\frac{d\dot{x}}{dx} \Rightarrow \int_1^2 F\,dx = m\int_1^2 \dot{x}\,d\dot{x}\right)$

Nu gör vi samma grej för momentekvationen.

$$M_{Oz} - I_z\ddot{\theta} = I_z\frac{d\dot{\theta}}{dt}\cdot\frac{d\theta}{d\theta} = I_z\dot{\theta}\frac{d\dot{\theta}}{d\theta}$$

$$\int_1^2 M_{Oz}\,d\theta = \int_1^2 I_z\dot{\theta}\,d\dot{\theta} = \int_1^2 I_z\,\omega\,d\omega$$

Arbetet $U_2 - U_1$

$\frac{1}{2}I_z\dot{\theta}_2^2 - \frac{1}{2}I_z\dot{\theta}_1^2 =$ Rotation! $\boxed{\frac{1}{2}I_z\omega_2^2 - \frac{1}{2}I_z\omega_1^2}$

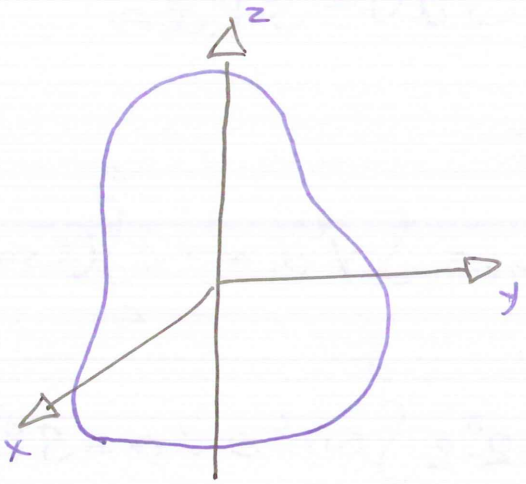
Rätlinjig rörelse	Rotation kring fix axel
$x, \dot{x}, \ddot{x}$	$\theta\ \dot{\theta}\ \ddot{\theta}$
m	I_{zz}
$F_x = m\ddot{x}_G$	$M_{Oz} = I_{zz}\ddot{\theta}$
$T = \frac{1}{2}mv^2$	$T = \frac{1}{2}I_{zz}\omega^2$

"Nu kanske jag vill rotera kring x-axeln istället."

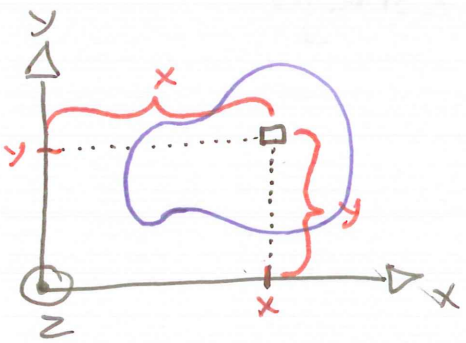
$$I_{zz} = \int (x^2+y^2)\,dm$$

$$I_{xx} = \int (y^2+z^2)\,dm$$

$$I_{yy} = \int (x^2+z^2)\,dm$$



Betrakta en tunn plan skiva med z = konstant



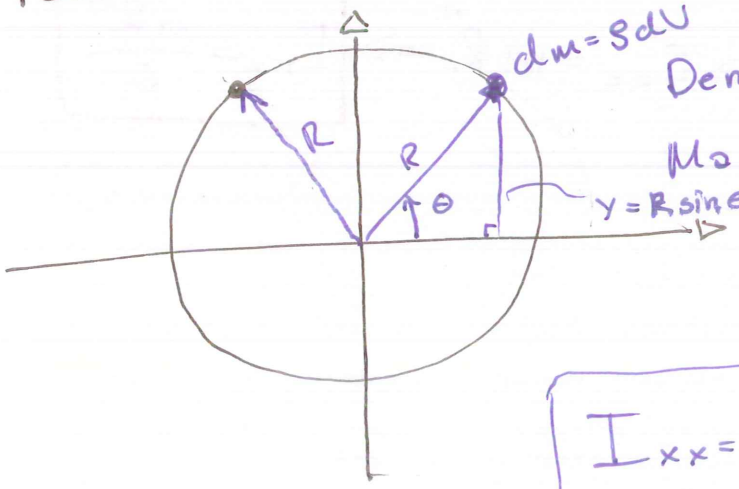
Rot. kring x-axel: $I_{xx} = \int (y^2+z^2)\,dm$

Rot. kring y-axel: $I_{yy} = \int (x^2+z^2)\,dm$

$$\Rightarrow I_{xx} + I_{yy} = \int (x^2+y^2)\,dm = I_{zz}$$

SATSEN OM EN TUNN PLAN SKIVA.

"Ibland kan det vara lite besvärligare"



$dm = \rho\,dV$

Densitet ρ

Massa m

$$I_{xx} = I_x = \int (y^2+z^2)\,dm = \int y^2\,dm$$

"Vad har jag för storlek på elementet?"

$dm = \sigma dV = \sigma R d\theta$

Så

$$I_{xx} = \int y^2 dm = 2\int_0^{\pi} (R\sin\theta)^2 \sigma R d\theta =$$

$$= \sigma R^3 \cdot 2 \int_0^{\pi} \sin^2\theta \, d\theta = \sigma R^3 2 \int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta =$$

$$= \sigma R^3 2 \left[\frac{\theta - \frac{1}{2}\sin 2\theta}{2} \right]_0^{\pi} = \sigma R^3 \pi = \left[m = \sigma 2\pi R \right] =$$

$$= \frac{m}{2\pi R} R^3 \pi = \boxed{\frac{mR^2}{2}}$$

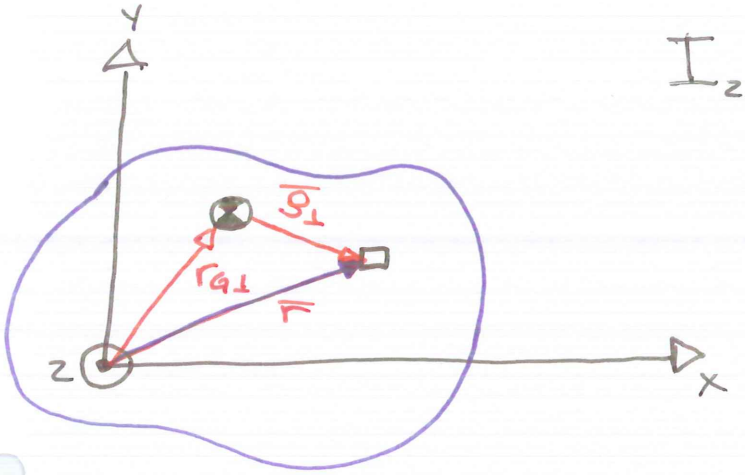
SATSEN OM EN TUNN PLAN SKIVA: $I_{zz} = I_{xx} + I_{yy}$

$$I_z = \int_0^{2\pi} R^2 dm = R^2 \int \sigma R d\theta = R^3 \sigma \int_0^{2\pi} d\theta =$$

$$= \sigma R^3 \cdot 2\pi = mR^2, \quad I_z = \int_m R^2 dm = R^2 \int dm = \boxed{R^2 m}$$

Symmetri: $I_{xx} = I_{yy} \Rightarrow I_{zz} = mR^2 \Rightarrow \boxed{I_{xx} = \frac{mR^2}{2}}$

STEINERS SATS



Motivering

$$I_z = \int r_\perp^2 \, dm = \int (\bar{r}_{G\perp} + \bar{\rho}_\perp)^2 \, dm =$$

$$= \int (\bar{r}_{G\perp} + \bar{\rho})(\bar{r}_{G\perp} + \bar{\rho}_\perp) \, dm =$$

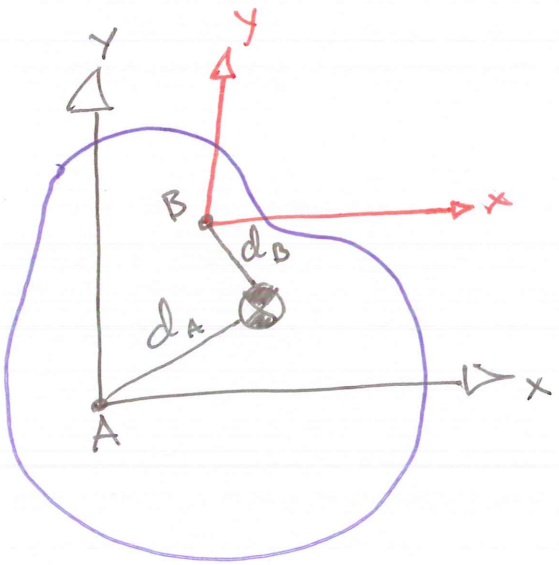
$$= \underbrace{\int r_{G\perp}^2 \, dm}_{r_{G\perp}^2 \int dm = r_{G\perp}^2 m} + \underbrace{\int 2 \bar{r}_{G\perp} \bar{\rho}_\perp \, dm}_{2 r_{G\perp} \int \bar{\rho}_\perp dm = 0} + \underbrace{\int \rho_\perp^2 \, dm}_{I_G}$$

$\rho_\perp m = 0$ statiska momentet map G

$$\boxed{I_z = r_{G\perp}^2 m + I_G}$$

Gäller om z-axeln är parallell genom G

För parallella axlar gäller:



$$I_A = I_G + m d_A^2$$

$$I_B = I_G + m d_B^2$$

$$\Rightarrow \boxed{I_A - I_B = m(d_A^2 - d_B^2)}$$