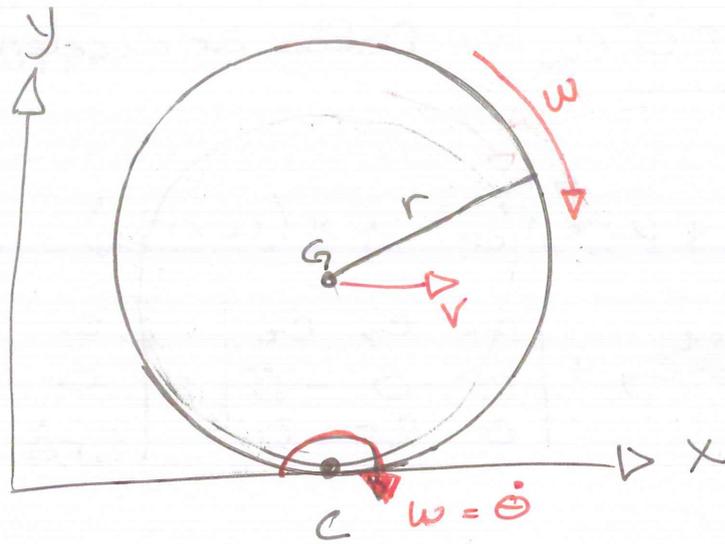


FÖRELÄSNING 11

Rullning:



C (kontaktpunkten) står stilla

$$v_G = v e_x = r \dot{\theta} e_x$$

$$v_Q = v_G + \bar{\omega} \times \bar{r}_{GQ}$$

$$v_C = \bar{0}$$

$$a_G = r \ddot{\theta} e_x$$

$$* \bar{a}_G = \bar{a}_C + \dot{\bar{\omega}} \times \bar{r}_{CG} + \bar{\omega} \times (\bar{\omega} \times \bar{r}_{CG})$$

$$\bar{\omega} = (0, 0, -\dot{\theta}), \quad \bar{r}_{CG} = (0, r, 0)$$

$$\text{ger: } \bar{\omega} \times \bar{r}_{CG} = \begin{vmatrix} e_x & e_y & e_z \\ 0 & 0 & -\dot{\theta} \\ 0 & r & 0 \end{vmatrix} = (r\dot{\theta}, 0, 0)$$

$$\bar{\omega} \times (\bar{\omega} \times \bar{r}_{CG}) = \begin{vmatrix} e_x & e_y & e_z \\ 0 & 0 & -\dot{\theta} \\ r\dot{\theta} & 0 & 0 \end{vmatrix} = (0, -r\dot{\theta}^2, 0)$$

$$* r\ddot{\theta}e_x = \bar{a}_c + r\ddot{\theta}e_x - r\dot{\theta}^2 e_y \neq$$

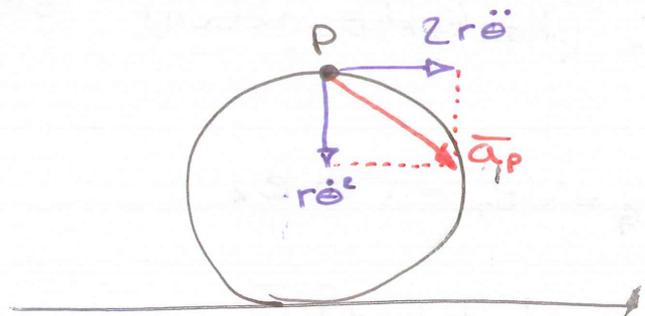
$$\Leftrightarrow \boxed{a_c = r\dot{\theta}^2 e_y} \text{ — Detta är accelerationen i } c.$$

$$\bar{a}_p = a_c + \omega \times \bar{r}_{cp} + \omega \times (\omega \times \bar{r}_{cp}) =$$

$$= r\dot{\theta}^2 e_y + \begin{vmatrix} e_x & e_y & e_z \\ 0 & 0 & -\dot{\theta} \\ 0 & 2r & 0 \end{vmatrix} + \begin{vmatrix} e_x & e_y & e_z \\ 0 & 0 & -\dot{\theta} \\ 2r\dot{\theta} & 0 & 0 \end{vmatrix} =$$

$$= r\dot{\theta}^2 e_y + 2r\dot{\theta}e_x - 2r\dot{\theta}^2 e_y =$$

$$= \boxed{2r\dot{\theta}e_x - r\dot{\theta}^2 e_y}$$



RULLNING PÅ KRÖKT YTA

$$\boxed{b = \psi r = \psi R}$$

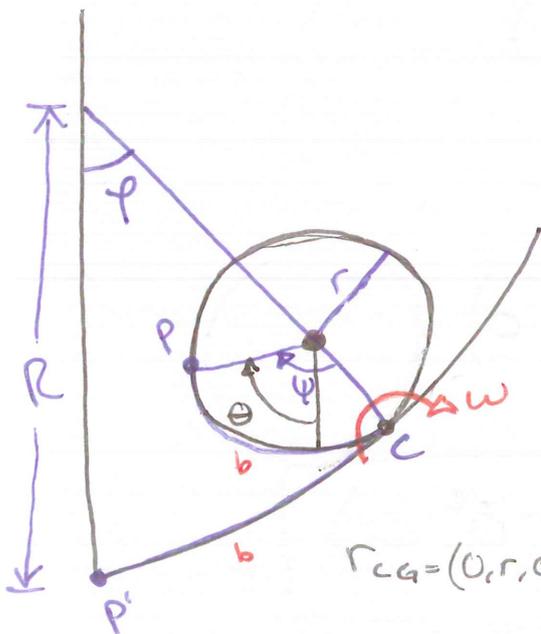
$$\theta = \psi - \varphi, \quad \dot{\theta} = \dot{\psi} - \dot{\varphi} = \left(\frac{R}{r} - 1\right) \dot{\varphi}$$

Med naturliga koordinater: (e_t, e_n, e_o)
Om masscentrum kan ses som en partikel som rör sig i en cirkelbana.

$$V_G = V_c + \bar{\omega} \times \bar{r}_{cG}$$

$$\bar{r}_{cG} = (0, r, 0), V_c = \bar{0}, \bar{\omega} = (0, 0, -\omega)$$

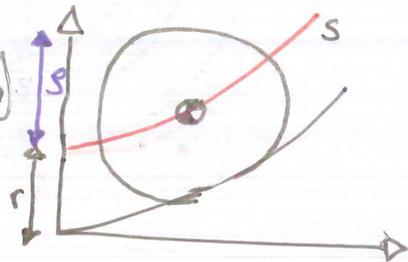
forts



$$V_G = \bar{0} + \begin{vmatrix} e_t & e_n & e_b \\ 0 & 0 & -\omega \\ 0 & r & 0 \end{vmatrix} = \omega r \bar{e}_t$$

$$\omega r = \dot{\theta} r = \left(\frac{R}{r} - 1\right) \dot{\varphi} \cdot r = (R-r) \dot{\varphi} = \boxed{s \dot{\varphi}}$$

$R-r = s$ (avståndet till tyngdpunkten)



Accelerationerna

MC rör sig längs banan S som en partikel.

$$\bar{V}_G = \dot{s} \bar{e}_t = r \dot{\theta} \bar{e}_t$$

$$a_G = \ddot{s} \bar{e}_t + \frac{\dot{s}^2}{s} \bar{e}_n = r \ddot{\theta} \bar{e}_t + \frac{(r \dot{\theta})^2}{R-r} \bar{e}_n$$

$$a_c = \bar{a}_G + \omega \times r_{cG} + \dot{\omega} \times (r_{cG}) =$$

$$= a_G + \begin{vmatrix} e_t & e_n & e_b \\ 0 & 0 & -\ddot{\theta} \\ 0 & -r & 0 \end{vmatrix} + \begin{vmatrix} e_t & e_n & e_b \\ 0 & 0 & -\dot{\theta} \\ -r \dot{\theta} & 0 & 0 \end{vmatrix} =$$

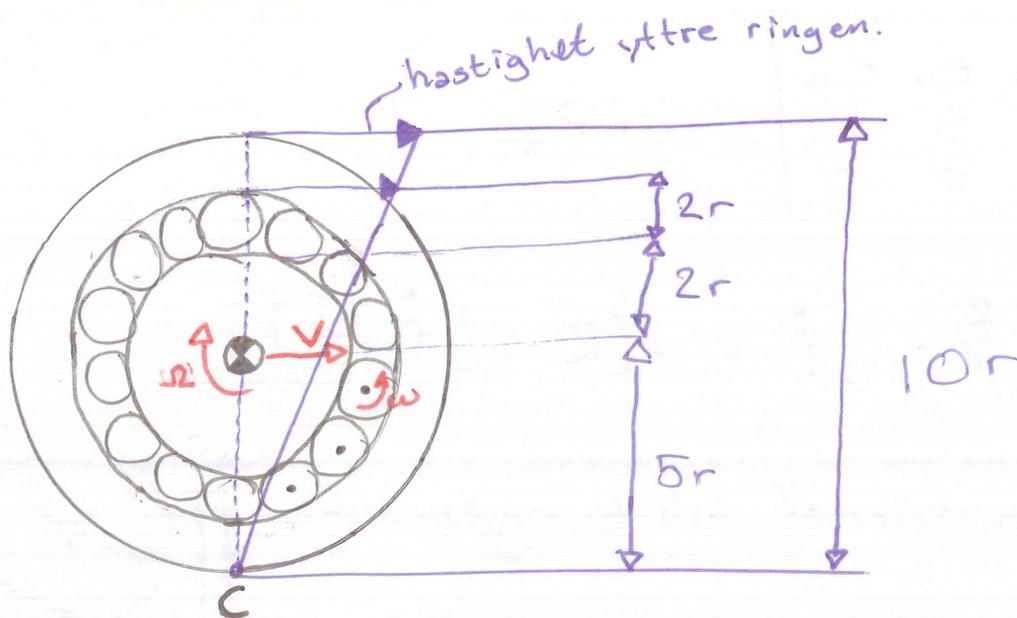
Om $R \rightarrow \infty$ så rullar den på plan mark och andra termen försvinner.

$$= a_G - r \ddot{\theta} \bar{e}_t + r \dot{\theta}^2 \bar{e}_n = \left(0, \frac{(r \dot{\theta})^2}{R-r} + r \dot{\theta}^2, 0\right) =$$

$$= \boxed{r \dot{\theta}^2 \left(\frac{R}{R-r}\right) \bar{e}_n}$$

$$R \rightarrow \infty \Rightarrow \bar{a}_c \rightarrow r \dot{\theta}^2 \bar{e}_n$$

Ex

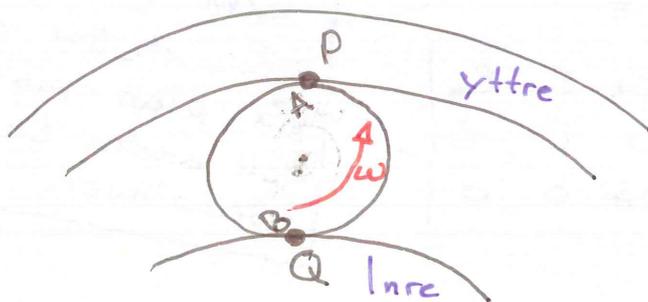


KÄNT: Medelpunktens hastighet = v
Inre delens vinkelhastighet = Ω (medurs)

Antag att det sker rollning i alla kontakter.

Bestäm de små kulornas vinkelh. ω .

Inom en stel kropp: $\vec{v}_A = \vec{v}_B + \omega \times \vec{r}_{BA}$



$$\vec{v}_A = \vec{v}_P$$
$$\vec{v}_B = \vec{v}_Q$$

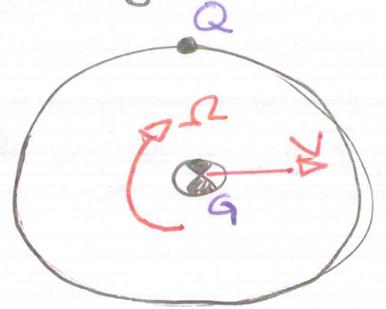
Yttre: Om yttre ringens vinkelhastighet är ω_1 , så är:

$$\left. \begin{aligned} v_p &= 9r\omega_1 \bar{e}_x \\ \otimes \vec{v} &= 5r\omega_1 \bar{e}_x = v \bar{e}_x \end{aligned} \right\}$$

$$\omega_1 = \frac{v}{5r} \Rightarrow v_p = 9r\omega_1 = \frac{9rv}{5r} = \frac{9}{5}v = v_A$$

Inre: $\vec{v}_Q = \vec{v}_G + \vec{\Omega} \times \vec{r}_{GQ} = v\vec{e}_x + \begin{vmatrix} e_x & e_y & e_z \\ 0 & 0 & -\Omega \\ 0 & 2r & 0 \end{vmatrix} =$

$$= v\vec{e}_x + 2r\Omega\vec{e}_x = (v + 2r\Omega)\vec{e}_x = v_B$$



Kulan är en stel kropp

$$\Rightarrow \vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{BA}$$

$$\frac{9}{5} v\vec{e}_x = (v + 2r\Omega)\vec{e}_x + \underbrace{\begin{vmatrix} e_x & e_y & e_z \\ 0 & 0 & \omega \\ 0 & 2r & 0 \end{vmatrix}}_{-2r\omega\vec{e}_x}$$

$$\frac{9}{5} v = v + 2r\Omega - 2r\omega$$

$$\omega = \Omega - \frac{2}{5} \cdot \frac{v}{r}$$

Nu är vi färdiga med

Kapitel 2!

KINETISK ENERGİ

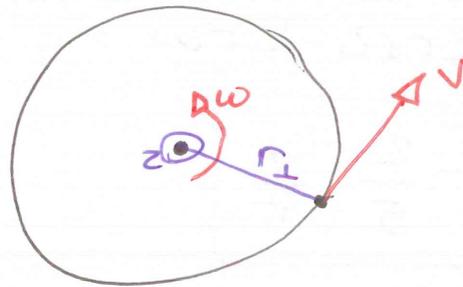
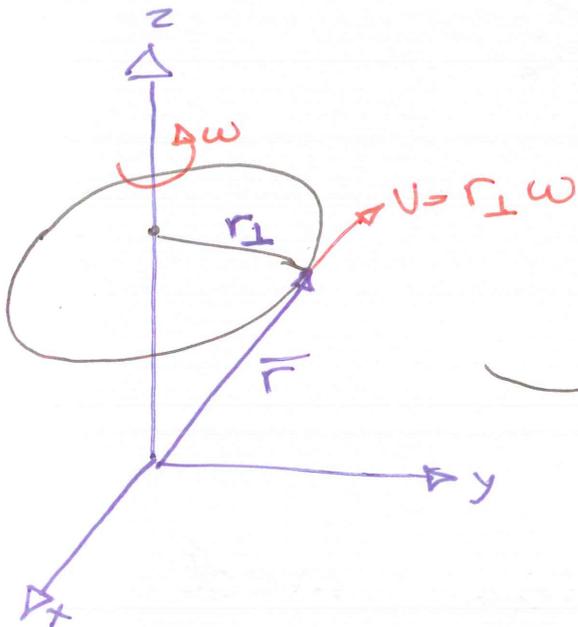
$$F = ma = m \frac{dv}{dt} \cdot \frac{ds}{ds} = m v \frac{dv}{ds}$$

$$\int F ds = m \int v dv = \left[\frac{1}{2} m v^2 \right]_1^2 = T_2 - T_1$$

Dessa två ska vi alltid kunna räkna ut.

RÖRELSEMÄNGDSMOMENT

$$\vec{H}_0 = \vec{r} \times m \vec{v} \quad , \quad \dot{\vec{H}}_0 = \vec{r} \times \vec{F} = \vec{M}_0$$



tröghetsmoment

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m (r_{\perp} \omega)^2 = \frac{1}{2} (m r_{\perp}^2) \cdot \omega^2$$

$$\vec{H}_0 = \vec{r} \times m \vec{v} \quad , \quad \vec{v} = \dot{\vec{r}} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ 0 & 0 & \omega \\ x & y & z \end{vmatrix} = (-\omega y, \omega x, 0)$$

$$\vec{H}_0 = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ x & y & z \\ -m\omega y & m\omega x & 0 \end{vmatrix} = \underbrace{(-m\omega x z, -m\omega y z)}_{\text{tröghetsprodukter}} \underbrace{, m\omega (x^2 + y^2)}_{\text{tröghetsmoment}}$$

tröghetsprodukter tröghetsmoment

Rotation kring z: $T = \sum T_i = \frac{1}{2} \left(\sum m_k r_{k\perp}^2 \right) \dot{\theta}$

$$T = \int \frac{1}{2} v^2 dm = \frac{1}{2} \int (r_{\perp} \dot{\theta})^2 dm = \frac{1}{2} \int r_{\perp}^2 dm \cdot \dot{\theta}^2$$

$$H_{0z} = \left(\sum m_k r_{k\perp}^2 \right) \dot{\theta}$$

$$H_{0z} = \int r_{\perp}^2 dm \cdot \dot{\theta}$$