

6.50  $p(x) = x^4 + 2x^3 + 3x^2 + dx + 2$ . Fakt.  $x^2 + 2x + 2$ . Best. d & lös  $p(x)=0$

Pol. div.

$$\begin{array}{r} x^2 + 1 \\ \hline x^4 + 2x^3 + 3x^2 + dx + 2 \quad | \quad x^2 + 2x + 2 \\ -x^4 - 2x^3 - 2x^2 \\ \hline x^2 + dx + 2 \\ -x^2 - 2x - 2 \\ \hline (d-2)x \end{array}$$

$$(d-2)x = 0 \Rightarrow d = 2$$

Für nu:  $x^4 + 2x^3 + 3x^2 + 2x + 2 = 0 \Leftrightarrow (x^2 + 1)(x^2 + 2x + 2) = 0$

$$x^2 + 1 = 0 \Leftrightarrow x = \pm i$$

$$x^2 + 2x + 2 = 0 \Leftrightarrow x = -1 \pm \sqrt{1-2} = -1 \pm i$$

$$6.52a) p(x) = x^3 - x$$

$$x^3 - x = 0 \Leftrightarrow x(x^2 - 1) = 0$$

$$x=0 \quad x=\pm 1$$

$$(x+1)(x-1) = x^2 - 1 \quad \text{mult. Polst.}$$

$$\begin{array}{r} x \\ \hline x^3 - x \quad | \quad x^2 - 1 \\ -x^2 + x \\ \hline 0 \end{array}$$

$$x^3 - x = x(x+1)(x-1)$$

$$652b) \quad x^3 + x = x(x^2 + 1)$$

$$x(x^2 + 1) = 0 \Rightarrow x_1 = 0 ; x_{2,3} = \pm i$$

Komp

Sicher eindeutig reelle Fakt:  $x(x^2 + 1)$

b.32c)  $x^4 - 1$ ; reelle Fakt.  $x = \pm 1$ .

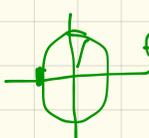
$$(x+1)(x-1) = x^2 - 1$$

$$\begin{array}{r} x^2 + 1 \\ x^4 - 1 \overline{) x^2 - 1} \\ \underline{-x^4 + x^2} \\ x^2 - 1 \\ \underline{-x^2 + 1} \\ 0 \end{array}$$

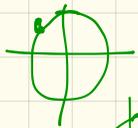
$$x^4 - 1 = (x+1)(x-1)(x^2+1)$$

6.52d)  $x^4 + 1$  Hitta 0-stärken:  $x^4 + 1 = 0 \Leftrightarrow x^4 = -1$

Ans  $x = a + bi = re^{i\theta}$  då förs  $x^4 = -1 \Leftrightarrow (re^{i\theta})^4 = -1 \Leftrightarrow re^{i4\theta} = -1$

  $\theta = \pi$   $4\theta = \pi + 2k\pi$ ,  $k = \{0, 1, 2, 3\} \Leftrightarrow \theta = \frac{\pi}{4} + \frac{1}{2}k\pi$

$k=0$   $\frac{\pi}{4}$   $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$   $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$   $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$   $E_1$  reell!



$k=1$   $\frac{\pi}{4} + \frac{1}{2}\pi = \frac{3\pi}{4}$   $\cos \frac{3\pi}{4} = \frac{1}{\sqrt{2}}$   $\sin \frac{3\pi}{4} = \frac{1}{\sqrt{2}}$   $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$   $E_2$  reell!



$k=2$   $\frac{\pi}{4} + \pi = \frac{5\pi}{4}$   $\cos \frac{5\pi}{4} = \frac{1}{\sqrt{2}}$   $\sin \frac{5\pi}{4} = \frac{1}{\sqrt{2}}$   $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$   $E_3$  reell!



$k=3$   $\frac{\pi}{4} + \frac{3}{2}\pi = \frac{7\pi}{4}$   $\cos \frac{7\pi}{4} = \frac{1}{\sqrt{2}}$ ,  $\sin \frac{7\pi}{4} = \frac{1}{\sqrt{2}}$   $E_4$  reell.

Har roten  $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$  ;  $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$  ;  $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$  ;  $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

652d-forts

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i; \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i; \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i; \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$$

För följande rotställen:  $(x - (\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)) (x - (\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)) (x - (\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i)) (x - (\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i)) =$

$$\cancel{\left(x - \frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}i} \left| \cancel{\left(x + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)} \left( \cancel{x - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i} \right) \left( \cancel{x + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i} \right) \right.$$

$(x+y)(x-y) = x^2 - y^2$ . Parar ihop:

$$\underbrace{\left(x - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) \left(x - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)} \cdot \underbrace{\left(x + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) \left(x + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)}$$

$$\underbrace{\left(x - \frac{1}{\sqrt{2}}\right)^2 - \left(\frac{i}{\sqrt{2}}\right)^2}$$

$$\underbrace{\left(x + \frac{1}{\sqrt{2}}\right)^2 - \left(\frac{i}{\sqrt{2}}\right)^2}$$

$$\underbrace{x^2 - \frac{2x}{\sqrt{2}} + \frac{1}{2} - \frac{(-1)}{2}}$$

$$\underbrace{x^2 + \frac{2x}{\sqrt{2}} + \frac{1}{2} - \frac{(-1)}{2}}$$

$$x^2 - \frac{2x}{\sqrt{2}} + 1$$

$$x^2 + \frac{2x}{\sqrt{2}} + 1$$

2.2.1.1

$$x^2 - \frac{2x}{\sqrt{2}} + 1$$

0

$$x^2 + \frac{2x}{\sqrt{2}} + 1$$

$$\frac{2x}{\sqrt{2}} = 2x \cdot 2^{-1/2} = x \cdot 2^{1-1/2} = \sqrt{2}x$$

$$\text{Für: } (x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)$$

$$[63] \quad z^3 = 7i \quad ; \quad z^3 = -7i$$

$$z^3 = 7i \quad \text{har} \quad \text{nodstallet} \quad (z^3 - 7i)$$

$$z^3 = -7i \quad \text{har} \quad \text{nodstallet} \quad (z^3 - (-7i))$$

Ska ha alla nodställen

$$(z^3 - 7i)(z^3 + 7i) = 0 \quad \Leftrightarrow \quad (z^3)^2 - (7i)^2 = 0$$

$$\text{konj. regel: } (x+y)(x-y) = x^2 - y^2$$

$$\Leftrightarrow z^6 - (-49) = 0 \quad \Rightarrow \quad z^6 = -49$$

12.] "Primitiv fun - vad har jag derivert?"

a)  $x^4$   $\int x^4 dx = \frac{x^5}{5} + C$ ,  $C$  godtycklig konst.

b)  $\frac{1}{x}$ ,  $\int \frac{1}{x} dx = \ln|x| + C$

c)  $e^x$ ,  $\int e^x dx = e^x + C$

d)  $\cos x$ ,  $\int \cos x dx = \sin x + C$

e)  $\int \sin x dx = -\cos x + C$

f)  $\int \frac{1}{\cos^2 x} dx = \tan x + C$

g)  $\int \frac{1}{\sin^2 x} = -\cot x + C$

$$\begin{aligned} D \tan &= D \frac{\sin x}{\cos x} = \frac{\cos x \cdot \cos x - (-\sin x) \sin x}{\cos^2 x} \\ \left(\frac{f}{g}\right)' &= \frac{f'g - g'f}{g^2} \quad \Bigg| \quad = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \# \end{aligned}$$

$$D \cot = D \frac{1}{\tan} = D \frac{\cos x}{\sin x} =$$

$$= \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x} = \frac{-1}{\sin^2 x} \#$$

12.)

$$h) \int \frac{1}{1+x^2} dx = \arctan x + C \quad \text{D } \arctan x: y = \arctan x \Leftrightarrow x = \tan y \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

$$i) \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

$$\text{D } \arctan x = \arctan y = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$

$$j) \int \frac{1}{\sqrt{x^2+a}} dx = \ln |x + \sqrt{x^2+a}| + C$$

### Arkusfunktionen!

$$\text{D } \arcsin: y = \arcsin x \Leftrightarrow x = \sin y, \quad -\pi/2 \leq y \leq \pi/2$$

$$\text{D } \arcsin x = \arcsin y = \frac{1}{\sin y} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$$

*vill ha =*  $\frac{1}{\sqrt{\cos^2 y}}$   
*Sin y ⇒*  $\frac{1}{\cos y}$   
*+ trigonometrische Char*  $\frac{1}{\cos y} = \sec y$   
*Ok. intervall*  $-\pi/2 < y < \pi/2$

$$\text{D } \arccos: y = \arccos x \Leftrightarrow x = \cos y, \quad \text{Intervall...}$$

$$\text{D } \arccos x = \arccos y = \frac{1}{\cos y} = -\frac{1}{\sin y} = \frac{-1}{\sqrt{1-x^2}}$$

$$12.1 \text{ a) } \int x^3 dx = \frac{1}{4} x^4$$

$$\text{b) } \int \frac{1}{x^3} dx = \int x^{-3} dx = -\frac{1}{2} x^{-2} = -\frac{1}{2x^2} \quad \text{D } x^n = n x^{n-1}$$

$$\text{c) } \int \sqrt{x} dx = \int x^{1/2} dx = \frac{2}{3} x^{3/2} = \frac{2}{3} x \sqrt{x}$$

$$\frac{1}{2} + \frac{1}{2} = \frac{3}{2} \quad \frac{2}{3} \cdot \frac{3}{2} = 1$$

$$\text{d) } \int x \sqrt{x} dx = a^n \cdot a^m = a^{n+m} = \int x^1 \cdot x^{1/2} dx = \int x^{3/2} dx = \frac{2}{5} x^{5/2} = \frac{2}{5} x^2 \sqrt{x}$$

$$\frac{3}{2} + \frac{1}{2} = \frac{5}{2}$$

$$\text{e) } \int \frac{1}{\sqrt{x}} dx = \int x^{-1/2} dx = 2x^{1/2} = 2\sqrt{x} \quad \text{D } 2\sqrt{x} = \text{D } 2x^{1/2} = \frac{2}{2} x^{-1/2}$$

$$\text{f) } \int \frac{1}{x\sqrt{x}} dx = \int x^{-1} \cdot x^{-1/2} dx = \int x^{-3/2} dx = -2x^{-1/2} = -\frac{2}{\sqrt{x}}$$

$$123 \text{ a) } \int \frac{2}{x+1} dx = \int \frac{1}{x+1} dx = \ln|x+1| = 02 \ln|x+1| = \frac{2}{x+1} \cdot 1 = \frac{2}{x+1} \rightarrow$$

$$\text{b) } \int \frac{1}{2x+1} dx = 02 \ln|2x+1| = \frac{1}{2x+1} \cdot 2 = \frac{1}{2} \ln|2x+1|$$

$$\text{c) } \int \frac{2}{(1-3x)^2} dx = \int 2(1-3x)^{-2} dx = -2(1-3x)^{-1} \cdot \frac{-1}{3} =$$
$$= \frac{2}{3(1-3x)}$$

$$115 \text{ a) } \int \sin 2x \, dx = -\cos 2x \cdot \frac{1}{2} = \frac{-\cos(2x)}{2}$$

$$\text{b) } \int \sin \frac{x}{3} \, dx = -\cos \frac{x}{3} \cdot 3 = -3 \cos\left(\frac{x}{3}\right)$$

$$\text{c) } \int \sin\left(2x + \frac{\pi}{3}\right) \, dx = -\cos\left(2x + \frac{\pi}{3}\right) \cdot \frac{1}{2} = \frac{-\cos\left(2x + \frac{\pi}{3}\right)}{2}$$