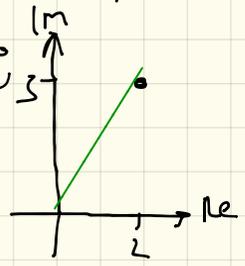


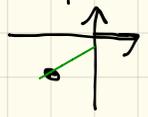
6.1

Re(z), Im(z) & plac. kompl. talpland.

a)  $z = 2 + 3i$     $\text{Re } z = 2$     $\text{Im } z = 3$



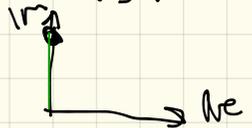
b)  $z = -1 - i$     $\text{Re } z = -1$     $\text{Im } z = -1$



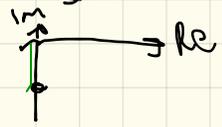
c)  $z = 3$     $\text{Re } z = 3$     $\text{Im } z = 0$



d)  $z = 2i$     $\text{Re } z = 0$     $\text{Im } z = 2$

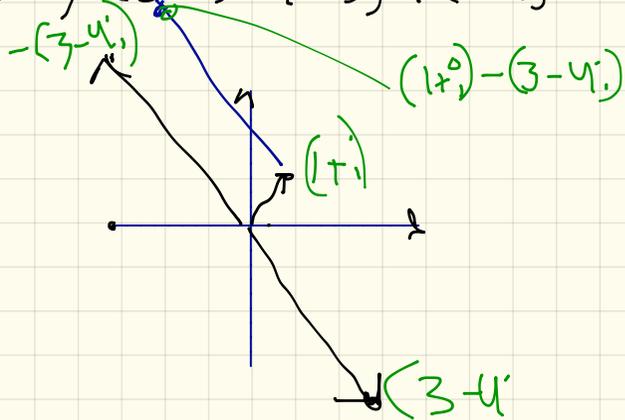


e)  $z = -i$     $\text{Re } z = 0$     $\text{Im } z = -1$



$$6.2 \quad a) (1+i) + (-3-2i) = (1-3) + (1-2)i = -2-i$$

$$b) (1+i) - (-3-4i) = (1-3) + (1+4)i = -2+5i$$



$$c) (1+i)(3-4i) = 3 - 4i + 3i - 4i^2 = 3 + 4 - i = 7 - i$$

$$d) (1-i)^2 = 1 - 2i + i^2 = 1 - 1 - 2i = -2i$$

$$\begin{array}{c} \Delta \\ i \\ 2 \\ 1 \\ 3 \end{array}$$

$$6.2 \quad e) (5-2i)^3$$

### Binomialsatz

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} \cdot b^k$$

### Pascals triangel

$$\begin{array}{cccc} 1 & 2 & 1 & n=2 \\ 1 & 3 & 3 & 1 & n=3 \\ 1 & 4 & 6 & 4 & 1 & n=4 \end{array}$$

$$(5-2i)^3 = 1(5)^3 + 3(5^2 \cdot (-2i)) + 3(5 \cdot (-2i)^2) + (-2i)^3$$

$$\bullet 1(5)^3 = 5 \cdot 5 \cdot 5 = 25 \cdot 5 = 125$$

$$\bullet 3(5^2 \cdot (-2i)) = 3(25 \cdot (-2i)) = 3(-50i) = -150i$$

$$\bullet 3(5 \cdot (-2i)^2) = 3(5 \cdot (-2)^2 (i)^2) = 3(5 \cdot 4 \cdot -1) = 3(-20) = -60$$

$$\bullet (-2i)^3 = (-2i)(-2i)(-2i) = -2^3 \cdot i^3 = -8(i)^3 = 8i$$

$$(5-2i)^3 = (125-60) - (150-1)i = 65 - 142i$$

6.2 f)  $(1-i)^4$  Binomialformel:  $(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$  1 4 6 4 1

$$(1-i)^4 = \underbrace{1}_{a} \binom{4}{0} \underbrace{(1)^3}_{b} \underbrace{(-i)^1}_{c} + \underbrace{4}_{d} \binom{4}{1} \underbrace{(1)^2}_{e} \underbrace{(-i)^2}_{f} + \underbrace{6}_{g} \binom{4}{2} \underbrace{(1)^1}_{h} \underbrace{(-i)^3}_{i} + \underbrace{4}_{j} \binom{4}{3} \underbrace{(1)^0}_{k} \underbrace{(-i)^4}_{l} + \underbrace{1}_{m} \binom{4}{4} \underbrace{(1)^0}_{n} \underbrace{(-i)^4}_{o}$$

Term für Term:

a) = 1

b) =  $-4i$

c)  $6(-1) = -6$

d)  $4(-i) = -4i$

e) 1

$(1-i)^4 = -4$

6.3  $z = a+bi$  ;  $\bar{z} = a-bi$  (!"spiegla i re-welk!")

a)  $\overline{1+i} = 1-i$

b)  $\overline{3-5i} = 3+5i$

c)  $\overline{-7} = -7$

d)  $(1+i)(\overline{1+i}) = (1+i)(1-i) = 1-i+i-i^2 = 1+1=2$

e)  $|1+i|$   $z = a+bi$  ;  $|z| = \sqrt{a^2+b^2}$   
 $\ll \sqrt{1^2+1^2} = \sqrt{2}$

f)  $|i| = \sqrt{1^2} = 1$

g)  $|3-4i| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

h)  $|-5i| = \sqrt{5^2} = 5$

6.4

a) Def  $\frac{a}{w} = \frac{w \overline{w}}{|w|^2}$  "Förklaring med nämnarens konjugat."

$$\left[ \frac{1}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i}{(1+i)(1-i)} = \frac{1-i}{|1+i|^2} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i \right]$$

$$b) \frac{1}{3-4i} = \frac{1(3+4i)}{(3-4i)(3+4i)} = \frac{3+4i}{|3-4i|^2} = \frac{3+4i}{25} = \frac{3}{25} + \frac{4}{25}i$$

$$c) \frac{3-4i}{1+i} = \frac{(3-4i)(1-i)}{(1+i)(1-i)} = \frac{3-3i-4i+4i^2}{|1+i|^2} = \frac{3-4-7i}{2} = \frac{-1-7i}{2} = -\frac{1}{2} - \frac{7}{2}i$$

$$d) \frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{(1-i)^2}{|1+i|^2} = \frac{1-2i+i^2}{2} = \frac{1-2i-1}{2} = \frac{-2i}{2} = -i$$

$$e) (1+i)^{-2} = \frac{1}{(1+i)^2} = \frac{1}{1^2+2i+i^2} = \frac{1}{2i} = \frac{-2i}{|2i|^2} = \frac{-2i}{4} = -\frac{1}{2}i$$

$$f) \frac{1}{i} = \frac{1 \cdot (-i)}{i \cdot (-i)} = \frac{-i}{-i^2} = \frac{-i}{-(-1)} = \frac{-i}{1} = -i$$

6.5 a)  $|(-i)^{14}|$  Lehnregel  $|z \cdot w| = |z| \cdot |w|$   $\times$   $\left| \frac{z}{w} \right| = \frac{|z|}{|w|}$

$$|(a+bi)^n| = |a+bi|^n = \sqrt{a^2+b^2}^n$$

$$|(-i)^{14}| = |1-i|^{14} = \sqrt{1^2+1^2}^{14} = \sqrt{2}^{14} = 2^7 = 1, 4, 8, 16, 32, 64, 128$$

b)  $\left| \frac{3+i}{4+3i} \right|$  Fall  $\left| \frac{z}{w} \right| = \frac{|z|}{|w|}$

$$= \frac{|3+i|}{|4+3i|} = \frac{\sqrt{3^2+1^2}}{\sqrt{4^2+3^2}} = \frac{\sqrt{10}}{5} = \frac{\sqrt{5 \cdot 2}}{5} = \frac{5^{1/2} \cdot 2^{1/2}}{5} =$$

$$= 5^{1/2} \cdot 2^{1/2} \cdot 5^{-1} = 5^{1/2-1} \cdot 2^{1/2} = \frac{\sqrt{2}}{\sqrt{5}} = \sqrt{2/5}$$

6.7  $z + 2\bar{z} = 2 - i$ . Sätter  $z = a + bi$  & får  $(a + bi) + 2(a - bi) = 2 - i \Leftrightarrow$

$$(a + bi) + 2a - 2bi = 2 - i \Leftrightarrow 3a - bi = 2 - i$$

Her:  $\begin{cases} 3a = 2 \\ -b = -1 \end{cases} \Leftrightarrow \begin{cases} a = 2/3 \\ b = 1 \end{cases}$

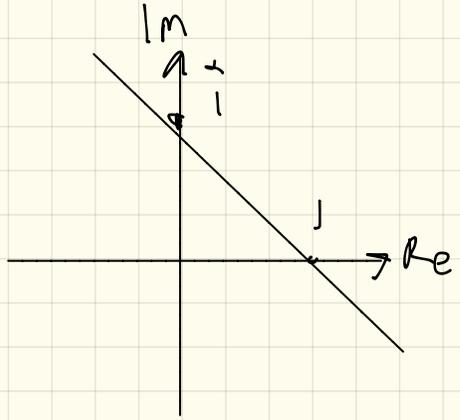
Ater till  $z$ :  $2/3 + i$

$$6.8b \quad z \cdot 2\bar{z} = 1+i \Leftrightarrow 2z\bar{z} = 1+i \Leftrightarrow 2|z|^2 = 1+i, \text{ f\u00fcr } z=a+bi$$

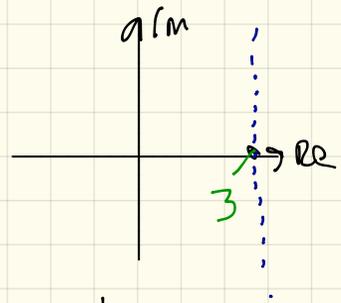
$$2|a+bi|^2 = 1+i \Leftrightarrow 2(a^2+b^2) = 1+i \Leftrightarrow 2a^2+2b^2 = 1+i$$

5-Min, i pa VL \u00c4j tasbar.

6.9  $\operatorname{Re} z + \operatorname{Im} z = 1$ . Setze  $z = a + bi$  & für  $a + b = 1 \Leftrightarrow b = 1 - a$



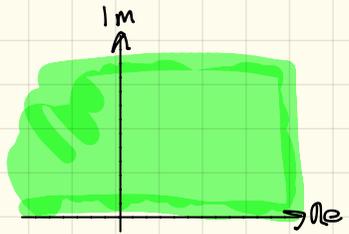
5.10 a)  $\operatorname{Re} z = 3$



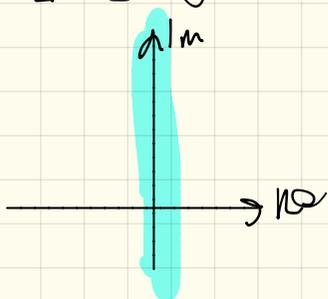
b)  $\operatorname{Im} z = -1$



c)  $\operatorname{Im} z \geq 0$



5.10 d)  $z + \bar{z} = 0 \quad a + bi + a - bi = 0 \Leftrightarrow 2a = 0 \Leftrightarrow a = 0$

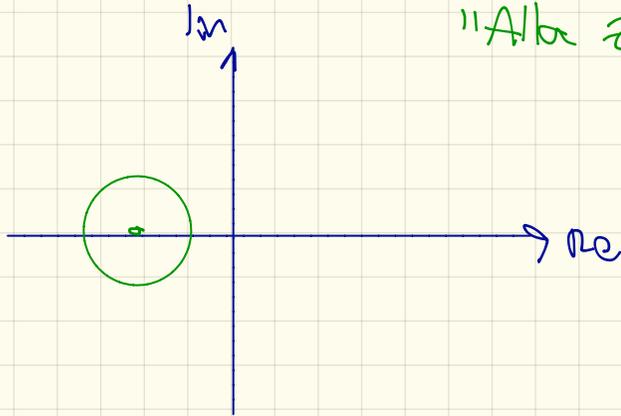


e)  $z = \bar{z} \Leftrightarrow a + bi = a - bi \Leftrightarrow \begin{cases} a = a \\ b_i = -b_i \end{cases} \Leftrightarrow \begin{cases} a = c \\ b = 0 \end{cases}, c \in \mathbb{R}$

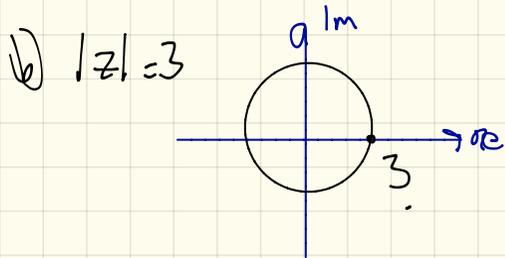
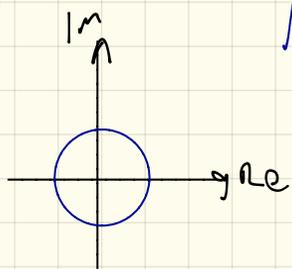
Ain reella tal

$$6.11 \quad |z+2|=1 \Leftrightarrow |z-(-2)|=1$$

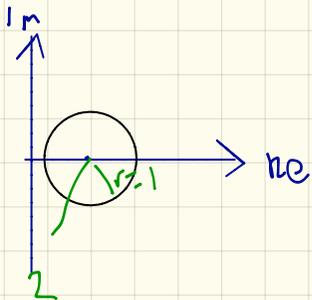
"Alla  $z$  med avst 1 till  $-2$ "



6.12 a)  $|z|=1$   $|z| = \text{Avståndet till origo}$

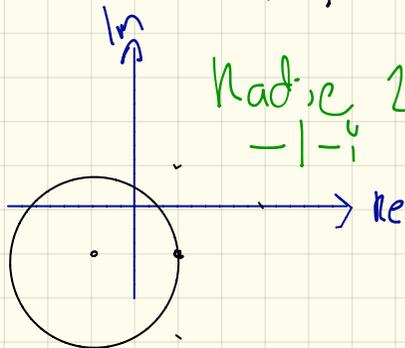


c)  $|z-2|=1$  "Avst från  $z$  till  $2$ !"



6.12

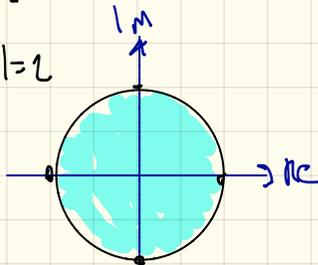
$$d) |z+1+i|=2 \Leftrightarrow |z-(-1-i)|=2$$



Rad: 2 & reitel  
-1-i

$$e) |z| \leq 2$$

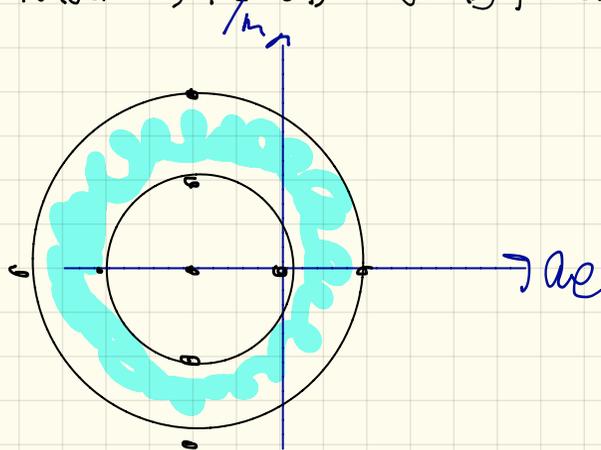
Border  $|z|=2$



6.12 f)  $|z| > 2$  Allt utanför cirkeln, medelpunkt origo & radie 2.

g)  $1 \leq |z+i| \leq 2 \Leftrightarrow 1 \leq |z-(-i)| \leq 2$

Börjar med att rita raderna: i)  $|z-(-i)| = 1$  & ii)  $|z-(-i)| = 2$



## 6.18 Polar form.

$$z = a + bi = r \cos \theta + ri \sin \theta, \text{ där } r = |z| = \sqrt{a^2 + b^2}$$

$\theta$  kallas argumentet av  $z$

Def  $e^{i\theta} = \cos \theta + i \sin \theta$ . Där då:  $z = re^{i\theta}$

Metod  $z = a + bi$

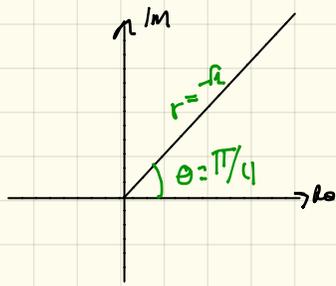
$$r = |z| = \sqrt{a^2 + b^2}$$

$$\text{Arg } z = \sqrt{a^2 + b^2} \left( \frac{a}{\sqrt{a^2 + b^2}} + \frac{b}{\sqrt{a^2 + b^2}} i \right)$$

Så här på  $\left( \frac{a}{\sqrt{a^2 + b^2}}, \frac{b}{\sqrt{a^2 + b^2}} \right)$

$$\cos \theta = \frac{a}{\sqrt{a^2 + b^2}} \quad \& \quad \sin \theta = \frac{b}{\sqrt{a^2 + b^2}}$$

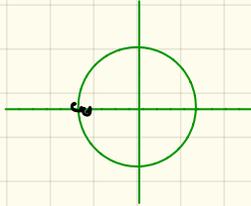
6.18 a)



$$z = \sqrt{2} \cos \frac{\pi}{4} + i \sqrt{2} \sin \frac{\pi}{4} =$$

$$= \cancel{\sqrt{2}} \cdot \cancel{\frac{1}{\sqrt{2}}} + i \cdot \cancel{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 1 + i$$

b)  $1, \pi$ .  $z = 1 \cdot \underbrace{\cos \pi}_{-1} + i \underbrace{\sin \pi}_0 = -1$



618

$$c) \sqrt{2}, \frac{\pi}{4}, \frac{\sqrt{2}}{4} = \sqrt{2} \frac{\pi}{4} \cdot \frac{1}{\sqrt{2}}$$

$$\sqrt{2} \left( \frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \right) = 1 + j$$

$$d) \downarrow, \pi/2 \quad \cos \pi/2 = 0, \sin \pi/2 = 1$$

$$z = 1$$

$$e) 1, 2\pi \quad \cos 2\pi = 1, \sin 2\pi = 0 \quad z = 1$$

$$f) \frac{1}{\sqrt{2}}, -\pi/4 \quad \cos -\pi/4 = \frac{1}{\sqrt{2}}, \sin -\pi/4 = -\frac{1}{\sqrt{2}}$$

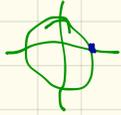
$$\frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \right) = \frac{1}{2} - \frac{1}{2} j$$

6.18

g)  $\downarrow -100j\pi$

$$\frac{-100j\pi}{2\pi} = -50$$

$$-100j\pi = -50 \cdot 2\pi$$



$$\cos 2\pi = 1 \quad \& \quad \sin 2\pi = 0$$

$$z = 1$$

6.19 a)  $z=1$    $\arg z=0$   $|z|=1$  polar form:  $r \cos \theta + i \sin \theta =$   
 $e^{i\pi} = 1e^{i0}$

b)  $-13$    $\arg z = \pi$ ;  $|z|=13$   $z = 13(\cos \pi + i \sin \pi) = 13e^{i\pi}$

c)  $i$    $\arg z = \pi/2$ ;  $|z|=1$   $z = 1e^{i(\pi/2)}$

d)  $-1+i$    $\tan \theta = \frac{\text{y-axis}}{\text{x-axis}} = \frac{1}{-1} = -1$   $\tan \theta = -1 \Rightarrow \theta = 3\pi/4$   
 $\theta = \pi - \pi/4 = \frac{3\pi}{4}$

$|z| = \sqrt{2}$

$\sqrt{2} e^{i 3\pi/4}$

e)  $z = i\sqrt{3} - 1$    $\tan \psi = \frac{1}{\sqrt{3}} \Rightarrow \psi = \pi/6$   
 $\theta = \pi/2 + \pi/6 = \frac{2\pi}{3} = \frac{4\pi}{6} = \frac{2\pi}{3}$

$|z| = \sqrt{3+1} = 2$

$2e^{i \frac{2\pi}{3}}$

$$6.19f) \quad \sqrt{3} + 3i = z \quad |z| = \sqrt{\sqrt{3}^2 + 3^2} = \sqrt{3+9} = \sqrt{12} = 2\sqrt{3}$$

$$\sqrt{2} \left( \frac{\sqrt{3}}{\sqrt{2}} + \frac{3i}{\sqrt{2}} \right)$$

$$\cos \theta = \frac{\sqrt{3}}{\sqrt{2}} \quad \& \quad \sin \theta = \frac{3}{\sqrt{2}}$$

$$(5.3) \quad \sqrt{3}/\sqrt{12} = \frac{\sqrt{3}}{\sqrt{4 \cdot 3}} \Rightarrow \frac{1}{\sqrt{4}} = \frac{1}{2}. \quad (\cos \theta = \frac{1}{2}) \Rightarrow \theta = \pi/3$$

$$\frac{3}{\sqrt{12}} = \frac{3}{\sqrt{3}} \cdot \frac{1}{\sqrt{4}} = \frac{3}{\sqrt{3}} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pi/3$$

$$\sqrt{2} e^{i\pi/3} = 2\sqrt{3} e^{i\pi/3}$$

6.10 a)  $z = \cos \frac{\pi}{8} + i \sin \frac{\pi}{8}$   $|z| = ?$

Realteil für  $\cos$  & Imag. für  $\sin$ :  $|z| = \sqrt{\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8}} = 1$   
 $\cos^2 \theta + \sin^2 \theta = 1$

b) Sdm a). Not oben  $\cos^2 \theta + \sin^2 \theta = 1$ , aber  $\cos \theta$ !

$$|\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}| = 1$$

$$c) |\cos \theta + i \sin \theta| = 1$$

$$6.7) a) |e^{i\frac{\pi}{8}}| = \left| \cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right| = \sqrt{\underbrace{\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8}}_{\text{Trig. iden.} = 1}} = \sqrt{1} = 1$$

$$b) |e^{i\frac{\pi}{5}}| = \left| \cos \frac{\pi}{5} + i \sin \frac{\pi}{5} \right| = 1$$

$$d) |e^{i\theta}| = \left| \cos \theta + i \sin \theta \right| = 1$$

6.16.

$$\left| \frac{1}{z} - \frac{1}{4} \right| = \frac{1}{4}$$

$$2 \text{ f\"all } \left| \frac{1}{z} - \frac{1}{4} \right| \geq 0 \text{ \& } \left| \frac{1}{z} - \frac{1}{4} \right| < 0.$$

$$\left| \frac{1}{z} - \frac{1}{4} \right| \geq 0 \Rightarrow \left| \frac{1}{z} - \frac{1}{4} \right| = \frac{1}{4} \Leftrightarrow \frac{1}{z} - \frac{1}{4} = \frac{1}{4} \Leftrightarrow \frac{1}{z} = \frac{1}{2} \Leftrightarrow z = 2$$

$$\frac{1}{z} - \frac{1}{4} \geq 0 \text{ ok!}$$

$$\left| \frac{1}{z} - \frac{1}{4} \right| < 0 \Rightarrow \left| \frac{1}{z} - \frac{1}{4} \right| = \frac{1}{4} \Leftrightarrow -\left( \frac{1}{z} - \frac{1}{4} \right) = \frac{1}{4} \Leftrightarrow \frac{1}{4} - \frac{1}{z} = \frac{1}{4} \Leftrightarrow \frac{1}{z} = 0$$

Lsg sein

$z=2$  samt  $\operatorname{Im} z = n, n \in \mathbb{R}$