9.1 a) $\begin{vmatrix} 1 & -2 \\ -2 & 4 \end{vmatrix} = 1\cdot4-(-2)\cdot(-2) = 0$

b) $\begin{vmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{vmatrix} = \cos\varphi\cdot\cos\varphi - \sin\varphi\cdot(-\sin\varphi) = \cos^2\varphi+\sin^2\varphi = 1$

c) $\begin{vmatrix} 1 & 2 & 0 \\ 3 & 0 & 2 \\ 0 & 0 & 1 \end{vmatrix} = \overset{0}{1\cdot0\cdot1} + \overset{0}{2\cdot2\cdot0} + \overset{0}{0\cdot3\cdot0} - \overset{0}{0\cdot0\cdot0} - \overset{0}{0\cdot2\cdot1} - \overset{-6}{1\cdot3\cdot2} = -6$

d) $\begin{vmatrix} 3 & 2 & 2 \\ 3 & 1 & 3 \\ 1 & 0 & 1 \end{vmatrix} = \overset{3}{3\cdot1\cdot1} + \overset{6}{2\cdot3\cdot1} + \overset{0}{2\cdot3\cdot0} - \overset{-2}{1\cdot1\cdot2} - \overset{0}{0\cdot3\cdot3} - \overset{-6}{1\cdot3\cdot2} = 1$

e) $\begin{vmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{vmatrix} = 2\cdot5\cdot7 = 70.$

9.2 a) $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = b(a+b)+a(c+a)+c(b+c)-c(c+a)-a(a+b)-b(b+c) =$

$= \cancel{ab}+\cancel{b^2}+\cancel{ac}+\cancel{a^2}+\cancel{cb}+\cancel{c^2}-\cancel{c^2}-\cancel{ac}-\cancel{a^2}-\cancel{ab}-\cancel{b^2}-\cancel{bc} = 0$

b) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \overbrace{(1+a)(1+b)}^{1+a+b+ab}(1+c)+1+1-(1+b)-(1+a)-(1+c) =$

$= \cancel{1}+\cancel{a}+\cancel{b}+ab+\cancel{c}+ac+bc+abc-\cancel{1}-\cancel{a}-\cancel{b}-\cancel{c} =$

$= abc+ab+bc+ac$

9.3) $A+B = \begin{pmatrix} 4 & 3 \\ 1 & 3 \end{pmatrix}$

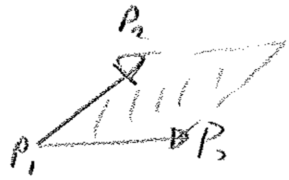
$\det(A+B) = 12-3 = 9$

$\det A = 1-6 = -5$, $\det B = 6+2 = 8$

Slutsats: $\det(A+B) \neq \det A + \det B$

9.4) $u\cdot(v\times w) = \begin{vmatrix} 1 & 1 & -1 \\ 3 & 0 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 0+1-6-0-2-3 = -10$

9.5) $P_1:(1,1)$, $P_2:(3,7)$; $P_3:(2,3)$



$\overline{P_1P_2} = (2,6)$, $\overline{P_1P_3} = (1,2)$

$\begin{vmatrix} 2 & 1 & 0 \\ 6 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 4-6 = -2$ ← Volymen på epipeden över parallellogrammen med höjd 1

Area triangel $= \frac{|-2|}{2} = 1$ a.e.

9.6 b) $\overline{P_1P_2} = (0,1,2)$, $\overline{P_1P_3} = (2,0,3)$, $\overline{P_1P_4} = (1,1,1)$

$\begin{vmatrix} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 4+3-2 = 5 \Rightarrow V = \frac{5}{6}$

a) $V = 6$ (se b) ovan)

9.10a) A 3x3-matris, $A = \begin{pmatrix} | & | & | \\ A_1 & A_2 & A_3 \\ | & | & | \end{pmatrix}$, $2A = \begin{pmatrix} | & | & | \\ 2A_1 & 2A_2 & 2A_3 \\ | & | & | \end{pmatrix}$

Determinanten linjär i varje kolonn ger

$$\det 2A = 2^3 \cdot \det A = 8 \det A$$

b) $A^T = -A$. Visa $\det A = 0$ (A 3x3)

(i) $\det A^T = \det A$ (känd sats)

(ii) $\det(-A) = -\det A$ (linj i kolonnerna)

(iii) $\det A^T = \det(-A)$

Tillsammans ger detta $\det A = -\det A \Rightarrow \det A = 0$.

9.16) $\begin{vmatrix} 2 & 2 & a \\ 1 & 2 & 0 \\ -1 & 2 & 1 \end{vmatrix} = 4 + 2a + 2a - 2 = 4a + 2$

$4a + 2 = 0 \Leftrightarrow a = -\frac{1}{2}$

Matrisen inv. bar om $a \neq -\frac{1}{2}$

$$\left(\begin{array}{ccc|ccc} 2 & 2 & a & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & 2 & a & 1 & 0 & 0 \\ 0 & 2 & -a & -1 & 2 & 0 \\ 0 & 6 & 2+a & 1 & 0 & 2 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 2 & 0 & 2a & 2 & -2 & 0 \\ 0 & 2 & -a & -1 & 2 & 0 \\ 0 & 0 & 2+4a & 4 & -6 & 2 \end{array}\right) \underset{(1+2a)II + \frac{a}{2}III}{\overset{(1+2a)I - aIII}{\sim}} \left(\begin{array}{ccc|ccc} 2+4a & 0 & 0 & 2 & -2+2a & -2a \\ 0 & 2+4a & 0 & -1 & 2+a & a \\ 0 & 0 & 2+4a & 4 & -6 & 2 \end{array}\right)$$

så $A^{-1} = \dfrac{1}{2+4a} \begin{pmatrix} 2 & 2a-2 & -2a \\ -1 & a+2 & a \\ 4 & -6 & 2 \end{pmatrix}$

9.11) $\det(AB) = \det A \cdot \det B$

och $\det A = 3$, $\det B = -3$

så $\det(AB) = -9$

9.12 →

9.13) $\det(S^{-1}AS) = \det S^{-1} \det A \cdot \det S = \frac{1}{\det S} \cdot \det A \cdot \det S =$

$= \det A$.

$$\det A = \begin{vmatrix} 2 & 3 & 4 \\ 0 & 5 & 0 \\ 6 & 7 & 8 \end{vmatrix} = 80 - 120 = \underline{\underline{-40}}$$

9.14)

$$\begin{array}{c} \downarrow \\ \begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 2 \\ 2 & 4 & 0 \end{vmatrix} \end{array} = -2 \begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix} + 1 \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} - 4 \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} =$$

$$= (-2) \cdot (-4) + 2 - 4(2+3) = 8 + 2 - 20 = \underline{\underline{-10}}$$

$$\begin{array}{r} \\ \\ \rightarrow \end{array} \begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 2 \\ 2 & 4 & 0 \end{vmatrix} = 2 \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} - 4 \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} + 0 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} =$$

$$= 2(4+1) - 4(2+3) = \underline{\underline{-10}}$$

9.12 a) $A \cdot A^{-1} = I$; Om A^{-1} existerar så $\det A \neq 0$

$$\Rightarrow$$

$$\det(A A^{-1}) = 1$$

$$\Leftrightarrow$$

$$\det A \cdot \det(A^{-1}) = 1$$

$$\Leftrightarrow$$

$$\det A^{-1} = \frac{1}{\det A}$$

b) $A \cdot A^{T} = I$

$$\Rightarrow$$

$$\det(A A^{T}) = 1$$

$$\Leftrightarrow$$

$$\det A \cdot \det A^{T} = 1$$

$$\Leftrightarrow$$

$$(\det A)^2 = 1$$

$$\Leftrightarrow$$

$$\det A = \pm 1$$

c) A inv.bar $\Leftrightarrow$ $\det A \neq 0$ $\Leftrightarrow$ $\det A^2 \neq 0$ $\Leftrightarrow$

$\Leftrightarrow$ A^2 inverterbar.

9.15 a) $(A^{-1})^T = \frac{1}{11}\begin{pmatrix}4 & -3\\ 1 & 2\end{pmatrix} \Rightarrow A^{-1} = \frac{1}{11}\begin{pmatrix}4 & 1\\ -3 & 2\end{pmatrix}$

b)

$\det A = 1 - 2 - 1 = -2$

$$(A^{-1})^T = \frac{1}{-2}\begin{pmatrix} \begin{vmatrix}1 & 0\\ -1 & 1\end{vmatrix} & -\begin{vmatrix}2 & 0\\ 1 & 1\end{vmatrix} & \begin{vmatrix}2 & 1\\ 1 & -1\end{vmatrix} \\ -\begin{vmatrix}0 & 1\\ -1 & 1\end{vmatrix} & \begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix} & -\begin{vmatrix}1 & 0\\ 1 & -1\end{vmatrix} \\ \begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix} & -\begin{vmatrix}1 & 1\\ 2 & 0\end{vmatrix} & \begin{vmatrix}1 & 0\\ 2 & 1\end{vmatrix} \end{pmatrix} = -\frac{1}{2}\begin{pmatrix}1 & -2 & -3\\ 1 & 0 & 1\\ -1 & 2 & 1\end{pmatrix}$$

så

$$A^{-1} = -\frac{1}{2}\begin{pmatrix}1 & 1 & -1\\ -2 & 0 & 2\\ -3 & 1 & 1\end{pmatrix}$$

9.17 a) $\begin{cases} 3x_1 - x_2 = 1 \\ x_1 + 2x_2 = 2 \end{cases} \Leftrightarrow \underbrace{\begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix}}_{A}\begin{pmatrix}x_1\\ x_2\end{pmatrix} = \begin{pmatrix}1\\ 2\end{pmatrix}$

$\det A = 7$

$x_1 = \frac{1}{7}\begin{vmatrix}1 & -1\\ 2 & 2\end{vmatrix} = \frac{4}{7} \qquad x_2 = \frac{1}{7}\begin{vmatrix}3 & 1\\ 1 & 2\end{vmatrix} = \frac{5}{7}$

b) $\begin{pmatrix}2 & 3\\ 3 & 2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} = \begin{pmatrix}2\\ -5\end{pmatrix}$, $\det A = -5$

$x_1 = -\frac{1}{5}\begin{vmatrix}2 & 3\\ -5 & 2\end{vmatrix} = -\frac{19}{5} \qquad x_2 = -\frac{1}{5}\begin{vmatrix}2 & 2\\ 3 & -5\end{vmatrix} = \frac{16}{5}$

9.17c) $\rightarrow$

9.18a) $\begin{vmatrix} 3 & 0 & 1 \\ 1 & 4 & 2 \\ 2 & 5 & 3 \end{vmatrix} = 36+5-8-30 = 3 \neq 0$ linj ober.

b) $\begin{vmatrix} 2 & 1 & 3 \\ 1 & -4 & -3 \\ -1 & 2 & 1 \end{vmatrix} = -8+3+6-12+12-1 = 0$ lin ber.

9.19) $\begin{vmatrix} 1 & 2 & 2a \\ 1 & a & 1 \\ 1 & 2-a & a-2 \end{vmatrix} = a(a-2)+2+2a(2-a)-2a^2-(2-a)-2(a-2) =$

$= a^2-2a+2+4a-2a^2-2a^2-2+a-2a+4 = -3a^2+a+4$

$-3a^2+a+4=0 \iff a^2-\frac{1}{3}a-\frac{4}{3}=0$

$a = \frac{1}{6} \pm \sqrt{\frac{1}{36}+\frac{4}{3}}$

$a = \frac{1}{6} \pm \frac{7}{6}$

$a=-1 \ , \ a=\frac{4}{3}$

För dessa a linjärt beroende

9.22a) $\begin{vmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 3 & 6 & 4 \end{vmatrix} = 8-6 = 2 \neq 0$ inv bar

b) $\begin{vmatrix} 2 & 1 & 1 \\ -1 & 2 & 3 \\ 0 & 5 & 7 \end{vmatrix} = 28-5-30+7 = 0$ ej inv bar.

9.17c) $\begin{pmatrix} s & s+1 \\ -s+1 & s \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$\det A = s^2 - (-s+1)(s+1) = s^2 - (-s^2 - s + s + 1) = 2s^2 - 1$$

$$x_1 = \frac{1}{2s^2-1} \begin{vmatrix} 1 & s+1 \\ -1 & s \end{vmatrix} = \frac{s+(s+1)}{2s^2-1} = \frac{2s+1}{2s^2-1}$$

$$x_2 = \frac{1}{2s^2-1} \begin{vmatrix} s & 1 \\ -s+1 & -1 \end{vmatrix} = \frac{-s-(-s+1)}{2s^2-1} = \frac{-1}{2s^2-1}$$

om $s \neq \pm \frac{1}{\sqrt{2}}$ (annars blir $2s^2-1=0$)

Om $s = \pm\frac{1}{\sqrt{2}}$ fungerar inte Cramers regel. Vi får

$s = \frac{1}{\sqrt{2}}$; $\begin{cases} \frac{1}{\sqrt{2}} x_1 + \left(1+\frac{1}{\sqrt{2}}\right)x_2 = 1 \\ \left(-\frac{1}{\sqrt{2}}+1\right)x_1 + \frac{1}{\sqrt{2}} x_2 = -1 \end{cases} \Leftrightarrow \begin{cases} x_1 + (\sqrt{2}+1)x_2 = \sqrt{2} \\ (-1+\sqrt{2})x_1 + x_2 = -\sqrt{2} \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x_1 + (\sqrt{2}+1)x_2 = \sqrt{2} \\ (\sqrt{2}+1)\cdot \mathrm{II} \quad x_1 + (\sqrt{2}+1)x_2 = -\sqrt{2}(\sqrt{2}+1) \end{cases}$ lösning saknas

$s = -\frac{1}{\sqrt{2}}$; $\begin{cases} -\frac{1}{\sqrt{2}} x_1 + \left(1-\frac{1}{\sqrt{2}}\right)x_2 = 1 \\ \left(\frac{1}{\sqrt{2}}+1\right)x_1 - \frac{1}{\sqrt{2}} x_2 = -1 \end{cases} \Leftrightarrow \begin{cases} -x_1 + (\sqrt{2}-1)x_2 = \sqrt{2} \\ (1+\sqrt{2})x_1 - x_2 = -\sqrt{2} \end{cases}$

... lösning saknas

9.23 $\underbrace{\begin{pmatrix} a & 2 & 2a \\ 2a & 1 & 2 \\ 2a & 2 & a \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\det A \neq 0 \Leftrightarrow$ entydig lösning

$$\begin{vmatrix} a & 2 & 2a \\ 2a & 1 & 2 \\ 2a & 2 & a \end{vmatrix} = a^2 + 8a + 8a^2 - 4a^2 - 4a - 4a^2 = a^2 + 4a = 0$$

$\Leftrightarrow a = 0$ eller $a = -4$

Om $a \neq 0$, $a \neq -4$ så entydig lösning. Eftersom systemet homogent (0:or i högerledet) så är denna entydiga lösning

$x = y = z = 0$

Om $a = 0$;

$$\begin{cases} 2y = 0 \\ y + 2z = 0 \\ 2y = 0 \end{cases} \Leftrightarrow \begin{cases} x = t \\ y = 0 \\ z = 0 \end{cases}$$

Om $a = -4$,

$$\begin{cases} -4x + 2y - 8z = 0 \\ -8x + y + 2z = 0 \\ -8x + 2y - 4z = 0 \end{cases} \Leftrightarrow \begin{cases} -4x + 2y - 8z = 0 \\ -3y + 18z = 0 \\ -2y + 12y = 0 \end{cases} \Leftrightarrow \begin{cases} -4x + 2y + 8z = 0 \\ -y + 6z = 0 \end{cases}$$

$12t - 8t$

$$\Leftrightarrow \begin{cases} x = t \\ y = 6t \\ z = t \end{cases}$$

9.25 $\underbrace{\begin{pmatrix} 1 & -1 & a \\ 2 & -1 & 1 \\ a & 1 & -1 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$\det A = 1 - a + 2a + a^2 - 1 - 2 = a^2 + a - 2 = 0 \Leftrightarrow a = 1$ eller $a = -2$

Om $a \neq 1, a \neq -2$ <u>en</u> lösning.

Om $a = 1$

$\begin{cases} x - y + z = 1 \\ 2x - y + z = -1 \\ x + y - z = 1 \end{cases} \Leftrightarrow \begin{cases} x - y + z = 1 \\ y - z = -3 \\ 2y - 2y = 0 \end{cases}$ omöjligt lösning saknas

Om $a = -2$

$\begin{cases} x - y - 2z = 1 \\ 2x - y + z = 1 \\ -2x + y - z = 1 \end{cases} \Leftrightarrow \begin{cases} x - y - 2z = 1 \\ y + 5z = -1 \\ -y - 5z = 1 \end{cases}$ samma oändligt många lösningar.

9.26) Planen skär längs linje om ekv. systemetets lösningsmängd är "en-parametrig", speciellt oändligt många lösningar.

$$\begin{vmatrix} 1 & 2 & a \\ 1 & 3 & 2a \\ 2 & a & 8 \end{vmatrix} = 24 + 8a + a^2 - 6a - 2a^2 - 16 = -a^2 + 2a + 8 = 0$$

$$\Leftrightarrow a = 4 \text{ eller } a = -2$$

$a = 4$ ger

$$\begin{cases} x + 2y + 4z = 8 \\ x + 3y + 8z = 12 \\ 2x + 4y + 8z = 16 \end{cases} \Leftrightarrow \begin{cases} x + 2y + 4z = 8 \\ y + 4z = 4 \\ 0 = 0 \end{cases}$$ oändligt många lösningar

$a = -2$ ger

$$\begin{cases} x + 2y - 2z = -4 \\ x + 3y - 4z = 12 \\ 2x - 2y + 8z = 16 \end{cases} \Leftrightarrow \begin{cases} x + 2y - 2z = -4 \\ y - 2z = 16 \\ -6y + 12y = 24 \end{cases}$$ $\Leftarrow$ orimligt lösning saknas

Planen skär längs linje $\Leftrightarrow a = 4$

9.28) Areaskalning $= |\det A| = |6 - 8| = 2$

Ny area: $5 \cdot 2 = 10$ a.e.

9.31 a)
$$\begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 1 \\ 1 & 3 & 1 & 2 \\ 1 & 2 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -2 \\ 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & -1 & -1 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & -2 \\ -1 & -1 \end{vmatrix} = -1-2 = -3$$

9.32)
$$\begin{vmatrix} a & 1 & 1 & 1 \\ 2 & 2a & 5 & 3 \\ 0 & 2 & 3 & -1 \\ 0 & a & 2a & -2a \end{vmatrix} = a \begin{vmatrix} a & 1 & 1 & 1 \\ 2 & 2a & 5 & 3 \\ 0 & 2 & 3 & -1 \\ 0 & 1 & 2 & -2 \end{vmatrix} =$$

$$= a\left(a \cdot \begin{vmatrix} 2a & 5 & 3 \\ 2 & 3 & -1 \\ 1 & 2 & -2 \end{vmatrix} - 2 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ 1 & 2 & -2 \end{vmatrix}\right) =$$

$$= a\left(a \cdot (-12a - 5 + 12 - 9 + 4a + 20) - 2(-6 - 1 + 4 - 3 + 2 + 4)\right) =$$

$$= a\left(a \cdot (-8a + 18) - 2 \cdot (0)\right) = a \cdot (-8a^2 + 18a) =$$

$$= -8a^3 + 18a^2$$

9.35 Utv efter rad n

$$(-1)^{n-1} a \begin{vmatrix} a & 0 & & \\ 1 & a & & \\ 0 & 1 & & \\ \vdots & \vdots & & \\ 0 & b & & \end{vmatrix} + (-1)^{n} \begin{vmatrix} 1 & 0 & 0 & \\ \vdots & a & 0 & \\ \vdots & 1 & a & \\ & \vdots & \vdots & \\ 0 & 0 & 0 & a \end{vmatrix} = (-1)^{n-1} a^{n} + (-1)^{n} a^{n-2}$$

$$= (-1)^{n-1} \left(a^{n} - a^{n-2} \right)$$

9.36)

$$\begin{vmatrix} 5 & 8 & 3 & 9 \\ 5 & 4 & 1 & 6 \\ 7 & 5 & 1 & 3 \\ 2 & 1 & 0 & 1 \end{vmatrix} = -2\begin{vmatrix} 8 & 3 & 9 \\ 4 & 1 & 6 \\ 5 & 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 5 & 3 & 9 \\ 5 & 1 & 6 \\ 7 & 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 5 & 8 & 3 \\ 5 & 4 & 1 \\ 7 & 5 & 1 \end{vmatrix} =$$

$$= -2\cdot\overbrace{(24+90+36-45-48-36)}^{21} + \overbrace{(15+126+45-63-30-45)}^{48}$$

$$+1\underbrace{(20+56+75-84-25-40)}_{2} = -54+11+2 = 8 \neq 0$$

alltså inv.bar

$$\det A^{-1} = \frac{1}{\det A} = \frac{1}{8}$$

9.37)

$$\begin{vmatrix} 0 & a & 1 & 0 \\ a & 0 & a & 1 \\ 1 & a & 0 & a \\ 0 & 1 & a & 0 \end{vmatrix} = -a\begin{vmatrix} a & a & 1 \\ 1 & 0 & a \\ 0 & a & 0 \end{vmatrix} + 1\begin{vmatrix} a & 0 & 1 \\ 1 & a & a \\ 0 & 1 & 0 \end{vmatrix} =$$

$$= -a(a-a^3) + 1(1-a^2) = (1-a^2)(1-a^2) = 0 \iff a = \pm 1$$

Linjärt beroende för $a = \pm 1$.

9.40)

$$\begin{cases} 2x - y - z = b+2 \\ x + y + bz = 1-b \\ x + by + z = 2b+1 \end{cases}$$

$$\det A = \begin{vmatrix} 2 & -1 & -1 \\ 1 & 1 & b \\ 1 & b & 1 \end{vmatrix} = 2-b-b+1-2b^2+1 = -2b^2-2b+4$$

$$\det A = 0 \Leftrightarrow b^2+b-2=0 \Leftrightarrow b=1 \text{ eller } b=-2$$

$b \neq 1,\ b \neq -2$) entydig lösning: Gausse ...

$b=1$)

$$\left.\begin{cases} 2x-y-z=3 \\ x+y+z=0 \\ x+y+z=3 \end{cases}\right\} \text{omöjligt} \Rightarrow \text{lösning saknas}$$

$b=-2$)

$$\begin{cases} 2x-y-z=0 \\ x+y-2z=3 \\ x-2y+z=-3 \end{cases} \Leftrightarrow \begin{cases} 2x-y-z=0 \\ 3y-3z=6 \\ -3y+3z=-6 \end{cases} \Leftrightarrow \begin{cases} 2x-y-z=0 \\ y-z=2 \\ 0=0 \end{cases}$$

osv ...

9.49)

$A = \frac{3}{2} \xrightarrow{\text{spegling}} \frac{3}{2} \xrightarrow{\text{vridning}} \frac{3}{2} \xrightarrow{\text{avb}} 9$

Area av ursprungstriangel: $\overline{P_1P_2} = (3,6)$; $\overline{P_1P_3} = (1,3)$

$$\frac{1}{2}\begin{vmatrix} 3 & 1 \\ 6 & 3 \end{vmatrix} = \frac{3}{2} = A$$

Spegling och vridning förändrar ej A så endast sista avb påverkar.

Avbildningens det: $\begin{vmatrix} 5 & 9 \\ 4 & 6 \end{vmatrix} = -6$

så ny triangelarea: $6 \cdot \frac{3}{2} = 9$ a.e.

9.54 a) $A^2 = -I$ (A; 3×3)

i udda dimension

$\det A^2 = \det A \cdot \det A = (\det A)^2 = \det(-I) = -1$

orimligt, sådant A med reella koefficienter finns ej.

b) Om A 2×2, så

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$ (rot 90° moturs)

uppfylle

$A^2 = -I$

Då har $A = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ samma egenskap.

"Man kan inte rotera 90° i 3D"