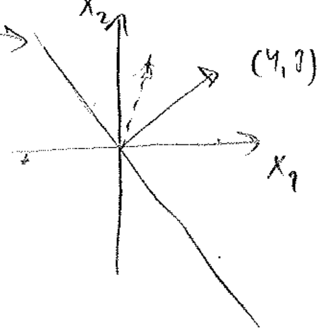
8.1) $4x_1 + 3x_2 = 0$



a,b) $$(y_1, y_2) = (x_1, x_2) - 2\,\frac{(x_1, x_2)\cdot(4,3)}{4^2+3^2}\,(4,3) =$$

$$= (x_1, x_2) - \frac{8x_1 + 6x_2}{25}(4,3) = \frac{1}{25}\left(-7x_1 - 24x_2,\ -24x_1 + 7x_2\right)$$

$(1,0) \mapsto \frac{1}{25}(-7, -24)$

$(0,1) \mapsto \frac{1}{25}(-24, 7)$

$(2,5) \mapsto \frac{1}{25}(-7\cdot 2 - 24\cdot 5,\ -24\cdot 2 + 7\cdot 5) = \frac{1}{25}(-134, -13)$

c) $$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \frac{1}{25}\begin{pmatrix} -7 & -24 \\ -24 & 7 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

d) Samme som i a) ...

8.2 a,b) $$(y_1, y_2) = (x_1, x_2) - \frac{(x_1, x_2)\cdot(4,3)}{4^2+3^2}\,(4,3) =$$

$$= (x_1, x_2) - \frac{4x_1 + 3x_2}{25}(4,3) = \frac{1}{25}\left(9x_1 - 12x_2,\ -12x_1 + 16x_2\right)$$

$(1,0) \mapsto \frac{1}{25}(9, -12)$ $\quad (0,1) \mapsto \frac{1}{25}(-12, 16)$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \frac{1}{25}\begin{pmatrix} 9 & -12 \\ -12 & 16 \end{pmatrix}$$

8.2 a) $\frac{1}{25}\begin{pmatrix} 9 & -12 \\ -12 & 16 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \frac{1}{25}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\frac{1}{25}\begin{pmatrix} 9 & -12 \\ -12 & 16 \end{pmatrix}\begin{pmatrix} 3 \\ -4 \end{pmatrix} = \frac{1}{25}\begin{pmatrix} 75 \\ -100 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$\frac{1}{25}\begin{pmatrix} 9 & -12 \\ -12 & 16 \end{pmatrix}\begin{pmatrix} 11 \\ 2 \end{pmatrix} = \frac{1}{25}\begin{pmatrix} 75 \\ -100 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$A^{-1} = A$!!

8.3 Från a)

$$\underbrace{\frac{1}{25}\begin{pmatrix} -7 & -24 \\ -24 & 7 \end{pmatrix}}_{A}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} \Leftrightarrow \begin{cases} -7x_1 - 24x_2 = -125 \\ -24x_1 + 7x_2 = 75 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{array}{r} \\ 7\text{II}-24\text{I} \end{array}\begin{cases} -7x_1 - 24x_2 = -125 \\ 625x_2 = 3525 \end{cases} \Leftrightarrow \begin{cases} \ldots \\ 25x_2 = 141 \end{cases}$$

$$\begin{pmatrix} 7 \cdot 75 = 525 \\ 24 \cdot 125 = 3000 \end{pmatrix} \qquad \Leftrightarrow \begin{cases} x_1 = \left(-125 + 24 \cdot \frac{141}{25}\right) \cdot -\frac{1}{7} = -\frac{37}{25} \\ x_2 = \frac{141}{25} \end{cases}$$

$$\left(-125 + 24 \cdot \frac{141}{25} = \frac{-3125 + 3384}{25} \cdot -\frac{1}{7} = \frac{259}{25} \cdot -\frac{1}{7} = -\frac{37}{25} \right)$$

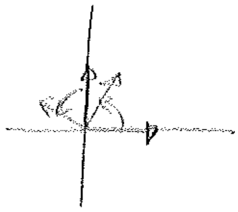
$$\begin{array}{r} 141 \\ 24 \\ \hline 564 \\ 282 \\ \hline 3384 \end{array}$$

Alltså $\begin{cases} x_1 = -\frac{37}{25} \\ x_2 = \frac{141}{25} \end{cases}$

INTE SÅ ROLIG UPPGIFT ^

JO, OM MAN NOTERAR ATT $A^{-1} = A$!!

8.5 a) $\frac{\pi}{3}$ moturs

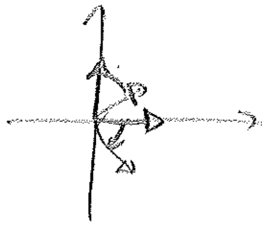


$(1,0) \mapsto \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$(0,1) \mapsto \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

så $A = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

b) $\frac{\pi}{3}$ medurs



$(1,0) \mapsto \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$(0,1) \mapsto \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

så $A = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$

8.6) Spegling i planet $x_1 + 2x_2 - 2x_3 = 0$ (origo i planet) $\bar{n} = (1, 2, -2)$

$$(y_1, y_2, y_3) = (x_1, x_2, x_3) - 2\,\frac{(x_1, x_2, x_3)\cdot(1,2,-2)}{1^2 + 2^2 + (-2)^2}\,(1,2,-2) =$$

$$= (x_1, x_2, x_3) - \frac{2x_1 + 4x_2 - 4x_3}{9}\cdot(1,2,-2) =$$

$$= \frac{1}{9}\left(7x_1 - 4x_2 + 4x_3,\ -4x_1 + x_2 + 8x_3,\ 4x_1 + 8x_2 + x_3\right)$$

Så

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \frac{1}{9}\begin{pmatrix} 7 & -4 & 4 \\ -4 & 1 & 8 \\ 4 & 8 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

↑
symmetrisk

8.9) $F(3u+2v) = \underbrace{F(3u)+F(2v)}_{F \text{ linjär}} = 3F(u)+2F(v) = 3\cdot(1,-1)+2(3,2) =$

$= (9, 1)$

8.10) $F(5u-2v) = 5F(u) - 2F(v)$ om F linjär

$F(5u-2v) = (4,3)$

$5F(u) - 2F(v) = 5\cdot(2,1) - 2(3,4) = (4,-3)$

Nej olika!

8.12) a) JA

b) NEJ, $F(2u) = 2u+w \overset{w \neq 0}{\neq} 2u+2w = 2F(u)$

c) NEJ, $F(2u) = |2u|\cdot 2u = 4|u|u \underset{u \neq 0}{\neq} 2|u|u = 2F(u)$

d) JA; $\tilde{F}(u) = \frac{u\cdot w}{|w|^2}\cdot w$ linjär ty proj på w.

Med $w = (1,2,3)$ är $|w|^2 = 14$ så

$F(u) = 14\hat{F}(u)$ så F linjär.

Kolla villkor för säkerhets skull.

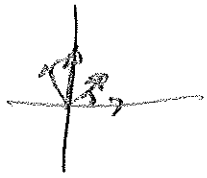
$F(\lambda u) = (\lambda u\cdot w)w = \lambda(u\cdot w)w = \lambda F(u)$

$F(u+v) = ((u+v)\cdot w)w = (u\cdot w+v\cdot w)w = (u\cdot w)w+(v\cdot w)w =$

$= F(u)+F(v)$

Ok!

8.14) Vridning $\frac{\pi}{6}$ moturs.



a) $(1,0) \mapsto \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$(0,1) \mapsto \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

b) $$A = \frac{1}{2}\begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix}$$

8.16 "$A: F \to \delta$"

a)
$$\begin{pmatrix}1\\0\end{pmatrix} \mapsto \begin{pmatrix}4\\1\end{pmatrix}, \quad \begin{pmatrix}0\\-1\end{pmatrix} \mapsto \begin{pmatrix}-1\\-3\end{pmatrix} \quad \text{så} \quad \begin{pmatrix}0\\1\end{pmatrix} \mapsto \begin{pmatrix}1\\3\end{pmatrix}$$

så
$$A = \begin{pmatrix}4 & 1\\1 & 3\end{pmatrix}$$

b) $\delta_x = 0$ ger
$$\begin{cases}4F_x + F_y = 0\\ F_x + 3F_y = \delta_y\end{cases} \quad \text{så} \quad F_y = -F_x$$

8.17) $F: \mathbb{R}^2 \to \mathbb{R}^2$
$$\begin{pmatrix}1\\2\end{pmatrix} \mapsto \begin{pmatrix}-3\\4\end{pmatrix}, \quad \begin{pmatrix}2\\5\end{pmatrix} \mapsto \begin{pmatrix}1\\1\end{pmatrix}$$

På matrisform
$$A \cdot \underbrace{\begin{pmatrix}1 & 2\\2 & 5\end{pmatrix}}_{B} = \begin{pmatrix}-3 & 1\\4 & 1\end{pmatrix}$$

$$B^{-1} = \begin{pmatrix}5 & -2\\-2 & 1\end{pmatrix} \quad \text{så} \quad A = \begin{pmatrix}-3 & 1\\4 & 1\end{pmatrix}\begin{pmatrix}5 & -2\\-2 & 1\end{pmatrix} = \begin{pmatrix}-17 & 7\\18 & -7\end{pmatrix}$$

8.18)

$(1,-2,1) \mapsto (0,0,0)$

$(1,0,-1) \mapsto (1,0,-1)$

$(0,1,0) \mapsto (0,1,0)$

$$A \cdot \begin{pmatrix} 1 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

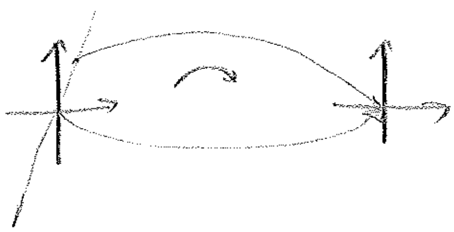
8.20) Vridning sedan spegling

$$\frac{1}{25}\begin{pmatrix} -7 & -24 \\ -24 & 7 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix} = \frac{1}{50}\begin{pmatrix} -7+24\sqrt{3} & -7\sqrt{3}-24 \\ -24-7\sqrt{3} & -24\sqrt{3}+7 \end{pmatrix}$$

8.22a)
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$-3\cdot(-1,2) = (3,-6)$ så värdemängden spanns av $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ och är 1-dim. Alltså ej bijektiv, t.ex. gäller

$$(0,0) \mapsto (0,0) \quad \text{och} \quad (1,3) \mapsto (0,0)$$



Ej bijektiv ⇒ invers saknas.

8.23 a)

$$\begin{cases} y_1 = x_1 - x_2 + x_3 \\ y_2 = 2x_1 + x_2 - x_3 \\ y_3 = 3x_1 + 3x_2 + x_3 \end{cases} \Leftrightarrow \begin{cases} y_1 = x_1 - x_2 + x_3 \\ -2y_1 + y_2 = 3x_2 - 3x_3 \\ -3y_1 + y_3 = 6x_2 - 2x_3 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y_1 = x_1 - x_2 + x_3 \\ -2y_1 + y_2 = 3x_2 - 3x_3 \\ y_1 - 2y_2 + y_3 = 4x_3 \end{cases}$$

Vi ser att y_1, y_2, y_3 kan väljas godtyckligt så värdemängden är $\mathbb{R}^3$, avbildningen är bijektiv och har invers. För invers räkna vidare (SEGT!!)

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 3 & -3 & -2 & 1 & 0 \\ 0 & 0 & 4 & 1 & -2 & 1 \end{array}\right) \sim \begin{array}{c} 4I - III \\ 4II + 3III \\ \end{array} \left(\begin{array}{ccc|ccc} 4 & -4 & 0 & 3 & 2 & -1 \\ 0 & 12 & 0 & -5 & -2 & 3 \\ 0 & 0 & 4 & 1 & -2 & 1 \end{array}\right) \sim$$

$$\begin{array}{c} 3I + II \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 12 & 0 & 0 & 4 & 4 & 0 \\ 0 & 12 & 0 & -5 & -2 & 3 \\ 0 & 0 & 4 & 1 & -2 & 1 \end{array}\right)$$

så

$$A^{-1} = \frac{1}{12}\begin{pmatrix} 4 & 4 & 0 \\ -5 & -2 & 3 \\ 3 & -6 & 3 \end{pmatrix}$$

b)

$$\begin{cases} y_1 = 3x_1 - 2x_2 + x_3 \\ y_2 = 2x_1 + 2x_2 - x_3 \\ y_3 = x_1 - 4x_2 + 2x_3 \end{cases} \Leftrightarrow \begin{cases} y_1 = 3x_1 - 2x_2 + x_3 \\ -2y_1 + 3y_2 = 10x_2 - 5x_3 \\ -y_1 + 3y_3 = -10x_2 + 5x_3 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \ldots \\ -3y_1 + 3y_2 + 3y_3 = 0 \end{cases}$$ ← Värdemängd, 2-dim, ej bijektiv, ingen invers.

8.25) Avbildningsmatris $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$

F^{-1} existerar $\Leftrightarrow$ A^{-1} existerar.

$$\left(\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 0 & -3 & 5 & -1 & 2 & 0 \\ 0 & -1 & 3 & -1 & 0 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 6 & 0 & 2 & 2 & 2 & 0 \\ 0 & -3 & 5 & -1 & 2 & 0 \\ 0 & 0 & 4 & -2 & -2 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 12 & 0 & 0 & 6 & 6 & -6 \\ 0 & -12 & 0 & 6 & 18 & -30 \\ 0 & 0 & 4 & -2 & -2 & 6 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 1 & -1 \\ 0 & -2 & 0 & 1 & 3 & -5 \\ 0 & 0 & 2 & -1 & -1 & 3 \end{array}\right)$$

Så

$$A^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 & -1 \\ -1 & -3 & 5 \\ -1 & -1 & 3 \end{pmatrix}$$

8.26) $F: \mathbb{R}^3 \to \mathbb{R}^3$ (samma rum)

$$Y = AX \quad , \quad X = SX' \; , \; Y = SY'$$

$$\Leftrightarrow$$

$$SY' = ASX'$$

$$\Leftrightarrow$$

$$Y' = \underbrace{S^{-1}AS}_{A'}X'$$

S^{-1}:

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 1 \end{array}\right) \sim \overset{I+III}{} \left(\begin{array}{ccc|ccc} -1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & -1 & 0 & 1 & 1 \end{array}\right)$$

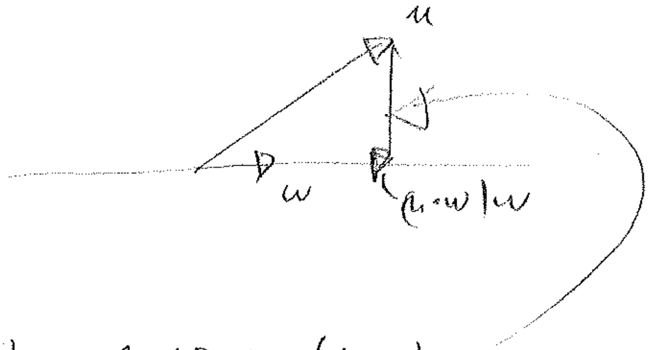
$$S^{-1} = \begin{pmatrix} -1 & 0 & -1 \\ 1 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix}$$

$$A' = \begin{pmatrix} -1 & 0 & -1 \\ 1 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix} S = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & -1 & -1 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

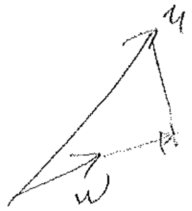
8.29) NEJ; eftersom planet ej går genom origo så gäller $F(0) \neq 0$

8.31) a) $u \mapsto (u \cdot w)w$ $|w| = 1$



b) $u \mapsto u - (u \cdot w)w$

c) $u \mapsto w \times (u - (u \cdot w)w) = w \times u$



d) ...