

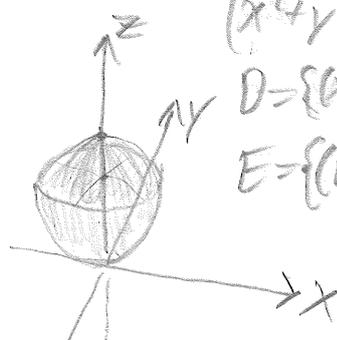
8.1

$$D = \{(x, y); x^2 + y^2 \leq 2\} \quad E = \{(r, \varphi); 0 \leq r \leq \sqrt{2}, 0 \leq \varphi \leq 2\pi\}$$

$$\begin{aligned} \iint_D (2 - (x^2 + y^2)) dx dy &= \iint_E (2 - r^2) r dr d\varphi = 2\pi \int_0^{\sqrt{2}} (2r - r^3) dr = \\ &= 2\pi \left[r^2 - \frac{r^4}{4} \right]_0^{\sqrt{2}} = 2\pi (2 - 1) = 2\pi \end{aligned}$$

$$\begin{aligned} \bullet V_{\text{rot}} &= 2 \cdot \pi r^2 = 2\pi \cdot 2 = 4\pi \\ V_1 &= V_2 = 2\pi \end{aligned} \quad \square$$

8.2



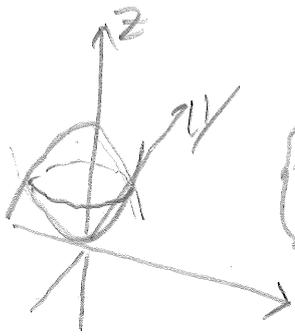
$$(x^2 + y^2)^2 + x^2 + y^2 - z = 0 \quad t^2 + t - 2 = 0$$

$$D = \{(x, y); x^2 + y^2 \leq 1\} \quad t = -\frac{1}{2} \pm \frac{3}{2} \quad t_1 = -2$$

$$E = \{(r, \varphi); 0 \leq r \leq 1, 0 \leq \varphi < 2\pi\} \quad t_2 = 1$$

$$\begin{aligned} V &= \iint_D (\sqrt{2 - (x^2 + y^2)} - (x^2 + y^2)) dx dy = \\ &= \iint_E (\sqrt{2 - r^2} - r^2) r dr d\varphi = 2\pi \int_0^1 (r\sqrt{2 - r^2} - r^3) dr = \\ &= 2\pi \left[-\frac{1}{3}(2 - r^2)^{3/2} - \frac{r^4}{4} \right]_0^1 = \\ &= 2\pi \left(-\frac{1}{3} - \frac{1}{4} + \frac{1}{3} 2^{3/2} \right) = 2\pi \left(\frac{4 \cdot 2^{3/2} - 3 - 4}{12} \right) = \\ &= \frac{\pi}{6} (8\sqrt{2} - 7) \end{aligned} \quad \square$$

8.3



$$D = \{(x, y) \mid x^2 + y^2 \leq 1\} \quad E = \{(r, \varphi) \mid 0 \leq r \leq 1, 0 \leq \varphi \leq 2\pi\}$$

$$\begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \end{cases} \Rightarrow \frac{d(x, y)}{d(r, \varphi)} = r$$

$$V = \iint_D (2 - (x^2 + y^2) - (x^2 + y^2)) dx dy = \iint_E (2 - 2r^2) r dr d\varphi =$$

$$= 2\pi \int_0^1 (2r - 2r^3) dr = 4\pi \left[\frac{r^2}{2} - \frac{r^4}{4} \right]_0^1 = 4\pi \left(\frac{1}{2} - \frac{1}{4} \right) = \pi$$

□

8.4 $z^2 = 49 - x^2 - y^2 = 1 + x^2 + y^2 \quad x^2 + y^2 = 24 = (2\sqrt{6})^2$

$$D = \{(x, y) \mid x^2 + y^2 \leq 24\}, \quad E = \{(r, \varphi) \mid 0 \leq r \leq 2\sqrt{6}, 0 \leq \varphi \leq 2\pi\}$$

$$V = \iiint_D (\sqrt{49 - x^2 - y^2} - \sqrt{1 + x^2 + y^2}) dx dy =$$

$$= \iint_E (\sqrt{49 - r^2} - \sqrt{1 + r^2}) r dr d\varphi = 2\pi \int_0^{2\sqrt{6}} (r\sqrt{49 - r^2} - r\sqrt{1 + r^2}) dr =$$

$$= 2\pi \left[-\frac{1}{3}(49 - r^2)^{3/2} - \frac{1}{3}(1 + r^2)^{3/2} \right]_0^{2\sqrt{6}} = -\frac{2\pi}{3} \left[(49 - r^2)^{3/2} + (1 + r^2)^{3/2} \right]_0^{2\sqrt{6}} =$$

$$= -\frac{2\pi}{3} (125 + 125 - 343 - 1) = \frac{2\pi}{3} (344 - 250) =$$

$$= \frac{2\pi}{3} \cdot 94 = \frac{188\pi}{3}$$

□

8.5

$$x^2 + y^2 = 2 - 3x - 2y \quad x^2 + 3x + y^2 + 2y = 2$$

$$\left(x + \frac{3}{2}\right)^2 + (y + 1)^2 = \frac{21}{4} = \left(\frac{\sqrt{21}}{2}\right)^2$$

$$\left(\frac{2x+3}{\sqrt{21}}\right)^2 + \left(\frac{2y+2}{\sqrt{21}}\right)^2 = 1$$

$$\begin{cases} \frac{2x+3}{\sqrt{21}} = r \cos \varphi \\ \frac{2y+2}{\sqrt{21}} = r \sin \varphi \end{cases}$$

$$\bullet \frac{d(x,y)}{d(r,\varphi)} = \frac{21}{4} r$$

$$\begin{cases} x = \frac{\sqrt{21}}{2} r \cos \varphi - \frac{3}{2} \\ y = \frac{\sqrt{21}}{2} r \sin \varphi - 1 \end{cases}$$

$$\bullet E = \{(r, \varphi) : 0 \leq r \leq 1, 0 \leq \varphi \leq 2\pi\}$$

$$V = \iint_D (2 - 3x - 2y - x^2 - y^2) dx dy = \iint_E \left(2 + \frac{9}{2} + 2 - \frac{21}{4} r \cos^2 \varphi - \frac{21}{4} r \sin^2 \varphi + \right.$$

$$\left. - \frac{9}{4} - 1\right) \frac{21}{4} r dr d\varphi = \frac{21}{4} \iint_E \left(\frac{21}{4} - \frac{21}{4} r\right) r dr d\varphi =$$

$$\bullet = \frac{21^2}{4^2} \cdot 2\pi \int_0^1 (r - r^2) dr = \frac{441}{8} \pi \cdot \left[\frac{r^2}{2} - \frac{r^3}{3}\right]_0^1 =$$

$$\bullet = \frac{441}{8} \pi \left(\frac{1}{2} - \frac{1}{3}\right) = \frac{441}{8} \pi \cdot \frac{1}{6} = \frac{441}{48} \pi$$

□

8.6

$$x^2 + y^2 \leq 4x$$

$$|z| \leq x^2 + y^2$$

$$(x-2)^2 + y^2 \leq 2^2 \quad \left(\frac{x-2}{2}\right)^2 + \left(\frac{y}{2}\right)^2 \leq 1$$

$$\begin{cases} x = 2 + 2r \cos \varphi \\ y = 2r \sin \varphi \end{cases} \Rightarrow \frac{d(x,y)}{d(r,\varphi)} = 4r$$

$$D = \{(x,y); \left(\frac{x-2}{2}\right)^2 + \left(\frac{y}{2}\right)^2 \leq 1\}$$

$$E = \{(r,\varphi); 0 \leq r \leq 1, 0 \leq \varphi \leq 2\pi\}$$

$$V = \iint_D ((x^2 + y^2) - (-x^2 - y^2)) dx dy = 2 \iint_D (x^2 + y^2) dx dy =$$

$$= 2 \iint_E (4 + 8r \cos \varphi + 4r^2 \cos^2 \varphi + 4r^2 \sin^2 \varphi) \cdot 4r dr d\varphi =$$

$$= 8 \iint_E (4r + 8r^2 \cos \varphi + 4r^3) dr d\varphi =$$

$$= 8 \int_0^{2\pi} \left[2r^2 + \frac{8}{3} r^3 \cos \varphi + r^4 \right]_0^1 d\varphi =$$

$$= 8 \int_0^{2\pi} \left(2 + \frac{8}{3} \cos \varphi + 1 \right) d\varphi = 8 \left[3\varphi + \frac{8}{3} \sin \varphi \right]_0^{2\pi} =$$

$$= 8 \cdot 3 \cdot 2\pi = 48\pi$$

□

8.7

$$0 \leq z \leq 10 - x^2 - y^2, \quad x + 1 - y^2 \geq 0, \quad x + y^2 - 1 \leq 0$$

$$(x=) y^2 - 1 = -y^2 + 1 \quad y^2 = 1 \quad y = \pm 1$$

$$\iint_D (10 - x^2 - y^2) dx dy = \int_{-1}^1 \left(\int_{y^2-1}^{1-y^2} (10 - x^2 - y^2) dx \right) dy = (y^2-1)^3.$$



$$= \int_{-1}^1 \left[10x - yx - \frac{x^2}{2} \right]_{y^2-1}^{1-y^2} dy =$$

$$= \int_{-1}^1 \left(10y^2 + 10 + y^4 - y^2 - \frac{(1-y^2)^2}{2} - 10y^2 + 10 + y^4 - y^2 + \frac{(y^2-1)^2}{2} \right) dy =$$

$$= 2 \int_{-1}^1 \left(\frac{(y^2-1)^2}{2} + y^4 - 11y^2 + 10 \right) dy =$$

$$= 2 \int_{-1}^1 \left(\frac{y^4 - 2y^2 + 1}{2} + y^4 - 11y^2 + 10 \right) dy =$$

$$= 2 \int_{-1}^1 \left(\frac{y^4}{2} - 10y^2 + \frac{29}{2} \right) dy = 2 \left[\frac{y^5}{10} - \frac{10}{3} y^3 + \frac{29}{2} y \right]_{-1}^1 =$$

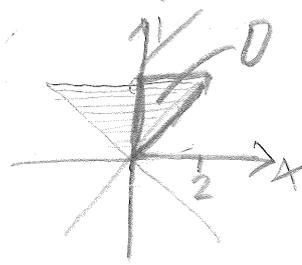
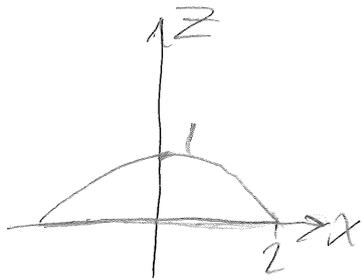
$$= 2 \left(\frac{1}{10} - \frac{10}{3} + \frac{29}{2} + \frac{1}{10} - \frac{10}{3} + \frac{29}{2} \right) = 4 \left(\frac{1 - 70 + 203}{10} \right) =$$

$$= \frac{4}{10} \cdot 134 = \frac{536}{10}$$

□

8.8

$$\begin{cases} x^2 = 4 - 4z \rightarrow z = 1 - \left(\frac{x}{2}\right)^2, z = 1 + \left(\frac{y}{2}\right)^2 \\ y^2 = 4 - 4z \\ z = 0 \end{cases}$$

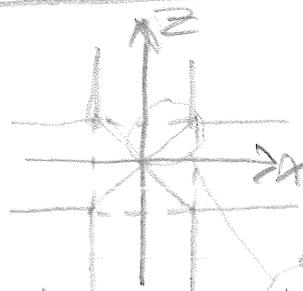


$$\begin{aligned} \frac{1}{8}V &= \iint_D \left(1 - \frac{x^2}{4}\right) dx dy = \int_0^2 \left(\int_{-x}^x \left(1 - \frac{x^2}{4}\right) dy \right) dx = \\ &= \int_0^2 \left[\left(y - \frac{x^3}{12}\right) \Big|_{-x}^x \right] dx = \int_0^2 \left(2 - \frac{8}{12}x + \frac{x^3}{12} \right) dx = \\ &= \left[\frac{4}{3}x - \frac{x^2}{2} + \frac{x^4}{48} \right]_0^2 = \frac{8}{3} - 2 + \frac{1}{3} = 1 \end{aligned}$$

$$\frac{1}{8}V = 1 \Leftrightarrow V = 8 \quad \square$$

8.9 $x^2 + y^2 = 1, y^2 + z^2 = 1$

$$y(z) = \sqrt{1 - z^2}$$



$$\begin{aligned} V &= 16 \cdot \iint_D \sqrt{1 - z^2} dx dz = 16 \int_0^1 \left(\int_{-x}^x \sqrt{1 - z^2} dz \right) dx = \\ &= 8 \int_0^1 \left[\left[z\sqrt{1 - z^2} + \arcsin z \right] \Big|_{-x}^x \right] dx = 8 \int_0^1 \left(\frac{2}{2} - x\sqrt{1 - x^2} - \arcsin x \right) dx = \\ &= 8 \left[\frac{2}{2}x + \frac{1}{3}(1 - x^2)^{3/2} - x\arcsin x - \sqrt{1 - x^2} \right]_0^1 \\ &= 8 \left(\frac{2}{2} - \frac{2}{2} - \frac{1}{3} + 1 \right) = 8 \cdot \frac{2}{3} = \frac{16}{3} \quad \square \end{aligned}$$

8.10

$$z = 2x^2 + 5y^2, \quad z^2 = 2x^2 + 5y^2 \quad \begin{cases} x = \frac{1}{\sqrt{2}} r \cos \varphi \\ y = \frac{1}{\sqrt{5}} r \sin \varphi \end{cases}$$

$$z^2 = z \Leftrightarrow \begin{cases} z_1 = 0 \\ z_2 = 1 \end{cases} \quad z = 1 \Rightarrow \left(\frac{x}{\sqrt{2}}\right)^2 + \left(\frac{y}{\sqrt{5}}\right)^2 = 1$$

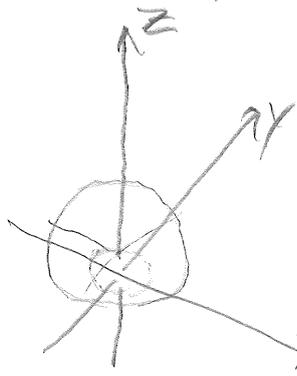
$$\frac{d(x,y)}{d(r,\varphi)} = \frac{1}{\sqrt{10}} r$$

$$\iint_D (\sqrt{2x^2 + 5y^2} - (2x^2 + 5y^2)) dx dy =$$

$$\frac{1}{\sqrt{10}} \iint_0^1 (r^2 - r^3) dr d\varphi = \frac{2\pi}{\sqrt{10}} \left[\frac{r^3}{3} - \frac{r^4}{4} \right]_0^1 = \frac{2\pi}{\sqrt{10}} \left(\frac{1}{3} - \frac{1}{4} \right) = \frac{2\pi\sqrt{10}}{60}$$

□

8.11 $z \geq 2x^2 + y^2 - 1, \quad z \leq \sqrt{x^2 + y^2} + 1, \quad z \geq 0$



$$x^2 + y^2 - 1 = \sqrt{x^2 + y^2} + 1 \quad \begin{cases} D: 0 \leq r \leq 1 \\ E: 1 \leq r \leq 2 \end{cases}$$

$$x^2 + y^2 - \sqrt{x^2 + y^2} = 2$$

$$\left(\sqrt{x^2 + y^2} - \frac{1}{2}\right)^2 = \frac{9}{4} = \left(\frac{3}{2}\right)^2 \quad \begin{cases} A: \sqrt{x^2 + y^2} \leq 1 \\ B: 1 \leq \sqrt{x^2 + y^2} \leq 2 \end{cases}$$

$$\sqrt{x^2 + y^2} = 2 \quad x^2 + y^2 = 4 = 2^2$$

$$z = 0 \Rightarrow x^2 + y^2 = 1$$

$$V = \iint_A (\sqrt{x^2 + y^2} + 1) dx dy + \iint_B (\sqrt{x^2 + y^2} + 1 - (2x^2 + y^2 - 1)) dx dy =$$

$$= \iint_0^1 (r^2 + r) dr d\varphi + \iint_E (r^2 + 2r - r^3) dr d\varphi =$$

$$= 2\pi \left(\left[\frac{r^3}{3} + \frac{r^2}{2} \right]_0^1 + \left[\frac{r^3}{3} + r^2 - \frac{r^4}{4} \right]_1^2 \right) =$$

$$= 2\pi \left(\frac{1}{3} + \frac{1}{2} + \frac{8}{3} + 4 - 4 - \frac{1}{3} - 1 + \frac{1}{4} \right) = 2\pi \left(\frac{6 + 32 - 12 + 3}{12} \right) = \frac{29\pi}{6}$$

8.12

$$0 \leq z \leq \frac{1}{10}(x+y+100), \quad \frac{(x-5)^2}{9} + \frac{(y-7)^2}{4} \leq 1$$

$$\begin{cases} x = 5 + 3r\cos\varphi \\ y = 7 + 2r\sin\varphi \end{cases} \Rightarrow \frac{d(x,y)}{d(r,\varphi)} = 6r$$

$$D = \left\{ (x,y) : \frac{(x-5)^2}{9} + \frac{(y-7)^2}{4} \leq 1 \right\}, \quad E = \{ (r,\varphi) : 0 \leq r \leq 1, 0 \leq \varphi \leq 2\pi \}$$

$$\begin{aligned} V &= \iint_D \frac{1}{10}(x+y+100) \, dx \, dy = \frac{3}{5} \iint_E (112r + 3r^2\cos\varphi + 2r^2\sin\varphi) \, dr \, d\varphi = \\ &= \frac{3}{5} \int_0^{2\pi} \left[\left(112r\varphi + 3r^2\sin\varphi - 2r^2\cos\varphi \right) \Big|_0^{2\pi} \right] \, d\varphi = \\ &= \frac{3}{5} \int_0^{2\pi} (2240r - 2r^2 + 2r^2) \, d\varphi = \frac{3}{5} [1120r\varphi]_0^{2\pi} = \frac{3360\pi}{5} \end{aligned}$$

□

$$8.13 D: \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 \quad \begin{cases} x = ar\sin\theta\cos\varphi \\ y = br\sin\theta\sin\varphi \\ z = cr\cos\theta \end{cases} \frac{d(x,y,z)}{d(r,\theta,\varphi)} = abc r^2 \sin\theta$$

$$\begin{aligned} V &= \iiint_D dx \, dy \, dz = \iiint_E abc r^2 \sin\theta \, dr \, d\theta \, d\varphi = 2\pi abc \int_0^{\pi/2} r^2 \, dr \cdot \int_0^{\pi} \sin\theta \, d\theta = \\ &= 2\pi abc \left[\frac{r^3}{3} \right]_0^{\pi/2} \cdot \left[-\cos\theta \right]_0^{\pi} = \frac{4}{3}\pi abc \end{aligned}$$

□

8.14

$$y \geq 0, z \geq 0, y \leq x, z \leq \sqrt{4 - x^2 - y^2}$$

$$\rho(x, y, z) = x^2 + y^2$$

$$m = \iiint_K \rho(x, y, z) \, dx \, dy \, dz = \iiint_0^{\sqrt{4-x^2-y^2}} dz \cdot (x^2 + y^2) \, dx \, dy =$$

$$\bullet = \iint_D \sqrt{4-x^2-y^2} \cdot (x^2 + y^2) \, dx \, dy = \iint_D (r\sqrt{4-r^2}) \, dr \, d\phi =$$

$$\bullet = \int_0^{2\pi} d\phi \cdot \left(\left[-\frac{r^2}{3}(4-r^2)^{3/2} \right]_0^2 + 2 \int_0^2 \frac{r}{3}(4-r^2)^{3/2} \, dr \right) =$$

$$= \frac{\pi}{4} \left(-\frac{2^2}{3}(4-2^2)^{3/2} + \frac{2}{3} \left[-\frac{1}{5}(4-r^2)^{5/2} \right]_0^2 \right) =$$

$$= \frac{\pi}{4} \left(-\frac{4}{3}(4-4)^{3/2} - \frac{2}{15}(4-2^2)^{5/2} + \frac{64}{15} \right) =$$

$$= \frac{\pi}{4} \left(0 - 0 + \frac{64}{15} \right) = \frac{16\pi}{15}$$

□

$$8.15 \quad D: \sqrt{x^2+y^2} \leq \frac{1}{\sqrt{2}}$$

$$\begin{aligned} \iiint_K dx dy dz &= \iint_D (\sqrt{1-x^2-y^2} - \sqrt{x^2+y^2}) dx dy = \\ &= \iint_D (r\sqrt{1-r^2} - r^2) dr d\varphi = 2\pi \left[-\frac{1}{3}(1-r^2)^{3/2} - \frac{r^3}{3} \right]_0^{1/\sqrt{2}} = \\ &= 2\pi \left(-\frac{1}{3} \cdot \frac{1}{2\sqrt{2}} - \frac{1}{3} \cdot \frac{1}{2\sqrt{2}} + \frac{1}{3} \right) = \frac{2\pi}{3} \left(1 - \frac{1}{\sqrt{2}} \right) = \frac{\pi}{3} (2 - \sqrt{2}) \end{aligned}$$

$$\iiint_K z dx dy dz = \iint_D \left[\frac{z^2}{2} \right]_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} dx dy =$$

$$\begin{aligned} &= \frac{1}{2} \iint_D (1 - 2(x^2+y^2)) dx dy = \frac{1}{2} \iint_D (r - 2r^3) dr d\varphi = \\ &= \pi \left[\frac{r^2}{2} - \frac{r^4}{2} \right]_0^{1/\sqrt{2}} = \frac{\pi}{2} \left(\frac{1}{2} - \frac{1}{4} \right) = \frac{\pi}{8} \end{aligned}$$

$$\begin{aligned} Z_T &= \frac{\iiint_K z dx dy dz}{\iiint_K dx dy dz} = \frac{\pi/8}{\frac{\pi}{3}(2-\sqrt{2})} = \frac{3}{8(2-\sqrt{2})} = \frac{3(2+\sqrt{2})}{8(2-\sqrt{2})(2+\sqrt{2})} = \\ &= \frac{3(2+\sqrt{2})}{8(4-2)} = \frac{3}{16}(2+\sqrt{2}) \end{aligned}$$

□

8.16

$$x^2 + y^2 + z^2 \leq R^2, \quad x \geq 0, \quad z \geq 0$$

$$y_T = 0 \quad x_T = \frac{1}{m} \int_K x \, dm$$

$$m = \iiint_K dx \, dy \, dz = \iiint_L r^2 \sin \theta \, dr \, d\theta \, d\varphi =$$

$$= \pi \int_0^R r^2 \, dr \cdot \int_0^{\pi/2} \sin \theta \, d\theta = \frac{\pi}{3} R^3 \cdot \left[-\cos \theta \right]_0^{\pi/2} = \frac{\pi}{3} R^3$$

$$\iiint_K x \, dx \, dy \, dz = \iiint_L r^3 \sin^2 \theta \cdot \cos \varphi \, dr \, d\theta \, d\varphi = \left[\frac{r^4}{4} \right]_0^R \cdot \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/2} \cdot \left[\sin \varphi \right]_{-\pi/2}^{\pi/2} =$$

$$= \frac{\pi R^4}{4} \cdot \frac{\pi}{4} \cdot (1+1) = \frac{\pi}{8} R^4$$

$$\iiint_K z \, dx \, dy \, dz = \iiint_L r^3 \sin \theta \cdot \cos \theta \, dr \, d\theta \, d\varphi = \pi \left[\frac{r^4}{4} \right]_0^R \cdot \left[\frac{\sin^2 \theta}{2} \right]_0^{\pi/2} =$$

$$= \pi \cdot \frac{R^4}{4} \cdot \frac{1}{2} = \frac{\pi}{8} R^4$$

$$x_T = z_T = \frac{3}{\pi R^3} \cdot \frac{\pi}{8} R^4 = \frac{3}{8} R$$

$$(x_T, y_T, z_T) = \left(\frac{3}{8} R, 0, \frac{3}{8} R \right)$$

□

8.17

$$D: x^2 + \frac{y^2}{a^2} \leq 1, y \geq 0$$

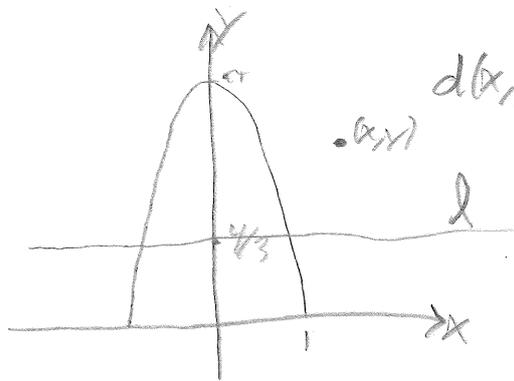
$$\begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \end{cases} \frac{d(x,y)}{d(r,\varphi)} = r$$

$$E: 0 \leq r \leq 1, 0 \leq \varphi \leq \pi$$

$$a) \iint_D y \, dx \, dy = a \iint_E r^2 \sin \varphi \, dr \, d\varphi = ar^2 \left[\frac{r^3}{3} \right]_0^1 \cdot [-\cos \varphi]_0^\pi = \frac{2}{3} a^3$$

$$\iint_D dx \, dy = a \iint_E r \, dr \, d\varphi = ar^2 \left[\frac{r^2}{2} \right]_0^1 = \frac{1}{2} a^2$$

$$y_{mc} = \frac{\iint_D y \, dx \, dy}{\iint_D dx \, dy} = \frac{\frac{2}{3} a^3}{\frac{1}{2} a^2} = \frac{4}{3}$$



$$d(x,y) = \left| y - \frac{4}{3} \right|$$

$$I = \iint_D (d(x,y))^2 \, dx \, dy = \iint_D \left(y - \frac{4}{3} \right)^2 \, dx \, dy =$$

$$= \iint_D \left(y^2 - \frac{8}{3}y + \frac{16}{9} \right) \, dx \, dy = a \iint_E \left(r^2 \sin^2 \varphi - \frac{8}{3} r \sin \varphi + \frac{16}{9} r \right) \, dr \, d\varphi =$$

$$= a \int_0^\pi \left[r^2 \sin^2 \varphi \cdot \frac{r^4}{4} - \frac{8r \sin \varphi \cdot r^3}{9} + \frac{8}{9} r^2 \right] d\varphi =$$

$$= a \int_0^\pi \left(\frac{a^2}{4} \sin^2 \varphi - \frac{8a}{9} \sin \varphi + \frac{8}{9} \right) d\varphi =$$

$$= a \left[\frac{a^2}{4} \left(\frac{\varphi}{2} - \frac{\sin 2\varphi}{4} \right) + \frac{8}{9} a \cos \varphi + \frac{8}{9} \varphi \right]_0^\pi =$$

$$= a \left(\frac{a^2}{8} - \frac{8}{9} a + \frac{8}{9} a - \frac{8}{9} a \right) = \frac{a^4}{8} - \frac{8a^2}{9}$$

□

8.20

forts...

$$\iint_{\Gamma} z \, dS = \iint_{\Gamma} \frac{(x^2+y^2)z}{\sqrt{1+4x^2+4y^2}} \, dx \, dy = \iint_{E'} \frac{r^5 \sin^2 \varphi}{\sqrt{1+4r^2}} \, dr \, d\varphi =$$

$$= \left(r^4 \cdot \frac{1}{4} \sqrt{1+4r^2} \right)' - \int_0^1 r^3 \sqrt{1+4r^2} \, dr \cdot \left[\frac{\varphi}{2} - \frac{\sin 2\varphi}{4} \right]_0^{2\pi} =$$

$$= \left(\frac{\sqrt{5}}{4} - \left[r^2 \cdot \frac{1}{12} (1+4r^2)^{3/2} \right]' + \frac{1}{8} \int_0^1 r (1+4r^2)^{3/2} \, dr \right) \cdot \pi =$$

$$= \pi \left(\frac{\sqrt{5}}{4} - \frac{5\sqrt{5}}{12} + \frac{1}{8} \left[\frac{1}{20} (1+4r^2)^{5/2} \right]' \right) =$$

$$= \pi \left(-\frac{\sqrt{5}}{6} + \frac{1}{120} (25\sqrt{5} - 1) \right) = \pi \left(-\frac{\sqrt{5}}{6} + \frac{5\sqrt{5}}{24} - \frac{1}{120} \right) =$$

$$= \pi \left(\frac{\sqrt{5}}{24} - \frac{1}{120} \right) = \frac{\pi}{24} \left(\sqrt{5} - \frac{1}{5} \right)$$

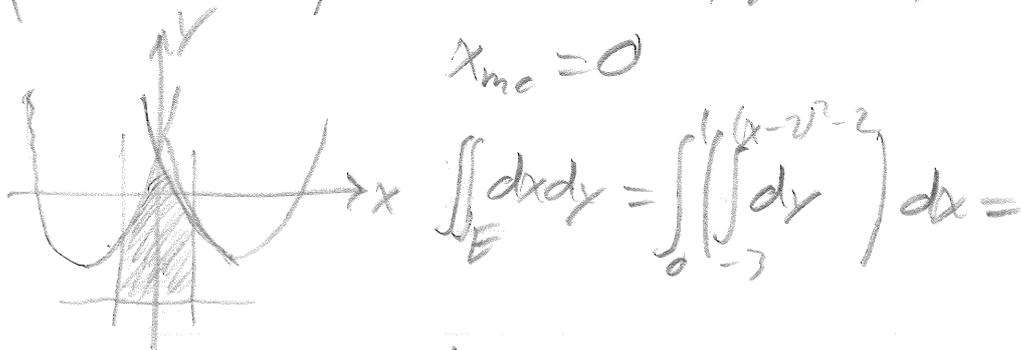
$$z_{mc} = \frac{\pi}{24} \left(\sqrt{5} - \frac{1}{5} \right) / \frac{\pi}{24} (\sqrt{5} + 1) = \frac{(\sqrt{5} - \frac{1}{5})(\sqrt{5} - 1)}{5 - 1} =$$

$$= \frac{5 - \sqrt{5} - \frac{\sqrt{5}}{5} + \frac{1}{5}}{4} = \frac{26 - 6\sqrt{5}}{20} = \frac{13 - 3\sqrt{5}}{10}$$

8.21

$$D: y \leq x^2 + 4x + 2, y \leq x^2 - 4x + 2, y \geq -3, -1 \leq x \leq 1$$

$$E: (y \leq (x+2)^2 - 2), y \leq (x-2)^2 - 2, y \geq -3, 0 \leq x \leq 1$$



$$x_{mc} = 0$$

$$\iint_E dx dy = \int_0^1 \left(\int_{-3}^{(x-2)^2-2} dy \right) dx =$$

$$= \int_0^1 ((x-2)^2 + 1) dx = \int_0^1 (x^2 - 4x + 5) dx = \left[\frac{x^3}{3} - 2x^2 + 5x \right]_0^1 =$$

$$= \frac{1}{3} - 2 + 5 = \frac{10}{3}$$

$$\iint_E y dx dy = \frac{1}{2} \int_0^1 \left(\left[y^2 \right]_{-3}^{(x-2)^2-2} \right) dx = \frac{1}{2} \int_0^1 (x^2 - 4x + 2)^2 - 9 dx =$$

$$= \frac{1}{2} \int_0^1 (x^4 - 8x^3 + 24x^2 - 32x + 16 - 4(x^2 - 4x + 4) + 4 - 9) dx =$$

$$= \frac{1}{2} \int_0^1 (x^4 - 8x^3 + 20x^2 - 16x - 5) dx =$$

$$= \frac{1}{2} \left[\frac{x^5}{5} - 2x^4 + \frac{20}{3}x^3 - 8x^2 - 5x \right]_0^1 = \frac{1}{2} \left(\frac{1}{5} - 2 + \frac{20}{3} - 8 - 5 \right) =$$

$$= \frac{1}{2} \left(\frac{3 - 30 + 100 - 120 - 75}{15} \right) = \frac{1}{30} (-122) = -\frac{61}{15}$$

$$y_{mc} = \iint_0 y dx dy / \iint_0 dx dy = \iint_E y dx dy / \iint_E dx dy = -\frac{61}{15} / \frac{10}{3} =$$

$$= -\frac{61}{50} \quad (x_{mc}, y_{mc}) = \left(0, -\frac{61}{50} \right)$$

□

8.18

$$K: z = 2 - x^2 - y^2, \quad z = y^2 \quad \begin{cases} x = \sqrt{2} r \cos \varphi \\ y = r \sin \varphi \end{cases}$$

$$2 - x^2 - y^2 = y^2 \quad x^2 + 2y^2 = 2 \quad x^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = \sqrt{2}^2$$

$$\left(\frac{x}{\sqrt{2}}\right)^2 + y^2 = 1 : D \quad E: 0 \leq r \leq 1, 0 \leq \varphi \leq 2\pi$$

$$a) \quad V = \iiint_K dx dy dz = \iint_D (2 - x^2 - 2y^2) dx dy =$$

$$= \iint_D (2 - 2r^2 \cos^2 \varphi - 2r^2 \sin^2 \varphi) \sqrt{2} r dr d\varphi =$$

$$= \sqrt{2} \iint_D (2r - 2r^3) dr d\varphi = 4\sqrt{2}\pi \left[\frac{r^2}{2} - \frac{r^4}{4} \right]_0^1 =$$

$$= 4\sqrt{2}\pi \left(\frac{1}{2} - \frac{1}{4} \right) = \sqrt{2}\pi$$

$$b) \quad x_{mc} = 0, \quad y_{mc} = 0$$

$$\textcircled{1} \quad \iiint_K z dx dy dz = \frac{1}{2} \iint_D ((2 - x^2 - y^2)^2 - (y^2)^2) dx dy =$$

$$= \frac{1}{2} \iint_D (4 - 4x^2 - 4y^2 + x^4 + 2x^2y^2) dx dy =$$

$$= \frac{\sqrt{2}}{2} \iint_E (4r - 4r^3 + 2r^3 \cos^2 \varphi (2r^2 \cos^2 \varphi + 2r^2 \sin^2 \varphi)) dr d\varphi =$$

$$= \frac{\sqrt{2}}{2} \iint_E (4r - 4r^3 + 4r^5 \cos^2 \varphi) dr d\varphi =$$

$$= \frac{\sqrt{2}}{2} \int_0^{2\pi} \left[2r^2 - r^4 + \frac{2r^6}{3} \cos^2 \varphi \right]_0^1 d\varphi = \frac{\sqrt{2}}{2} \int_0^{2\pi} \left(1 + \frac{2}{3} \cos^2 \varphi \right) d\varphi =$$

$$= \frac{\sqrt{2}}{2} \left[\varphi + \frac{\varphi}{3} + \frac{\sin 2\varphi}{6} \right]_0^{2\pi} = \frac{\sqrt{2}}{2} \left(\frac{8\pi}{3} + 0 \right) = \frac{4\sqrt{2}}{3} \pi$$

$$z_{mc} = \frac{4\sqrt{2}\pi / \sqrt{2}\pi}{3} = \frac{4}{3}$$

8.19

$$K: x^2 + 4y^2 \leq z \leq 1 \quad D: x^2 + \left(\frac{y}{2}\right)^2 \leq 1 \quad \begin{cases} x = r \cos \varphi \\ y = \frac{1}{2} r \sin \varphi \end{cases}$$

$$\iiint_K dx dy dz = \iint_D (1 - (x^2 + 4y^2)) dx dy = \iint_E (1 - r^2) \frac{r}{2} dr d\varphi =$$

$$= \frac{1}{2} \iint_D (r - r^3) dr \cdot 2\pi = \pi \left[\frac{r^2}{2} - \frac{r^4}{4} \right]_0^1 = \frac{\pi}{4}$$

$$E = \iiint_K z dx dy dz = \frac{1}{2} \iint_D (1 - (x^2 + 4y^2)^2) dx dy =$$

$$= \frac{1}{4} \iint_E (r - r^5) dr d\varphi = \frac{\pi}{2} \left[\frac{r^2}{2} - \frac{r^6}{6} \right]_0^1 = \frac{\pi}{2} \left(\frac{1}{2} - \frac{1}{6} \right) = \frac{\pi}{6}$$

$$z_1 = \frac{\pi/6}{\pi/4} = \frac{4}{6} = \frac{2}{3} \quad (x_1, y_1, z_1) = (0, 0, 2/3)$$

□

8.20 $\Gamma: z = x^2 + y^2, x^2 + y^2 \leq 1, \sigma = \frac{y^2}{\sqrt{1 + 4x^2 + 4y^2}}$

$$x_{mc} = \frac{1}{m} \iint_{\Gamma} x \sigma dS$$

$$m = \iint_D \sigma dx dy = \iint_E \frac{r^3 \sin^2 \varphi}{\sqrt{1 + 4r^2}} dr d\varphi = \left(\left[r^2 \cdot \frac{1}{4} \sqrt{1 + 4r^2} \right]_0^1 - \frac{1}{2} \int_0^1 r \sqrt{1 + 4r^2} dr \right)$$

$$\left[\frac{\varphi}{2} - \frac{5 \sin 2\varphi}{4} \right]_0^{2\pi} = \pi \left(\frac{\sqrt{5}}{4} - \frac{1}{2} \left[\frac{1}{12} (1 + 4r^2)^{3/2} \right]_0^1 \right) =$$

$$= \pi \left(\frac{\sqrt{5}}{4} - \frac{1}{2} \left(\frac{5\sqrt{5}}{12} - \frac{1}{12} \right) \right) = \frac{\pi}{24} (1 + \sqrt{5})$$

forts...

8.22

$$K: \frac{2}{6+x^2+y^2} \leq z \leq \frac{1}{1+x^2+y^2}, D: x^2+y^2 \leq 4, E: 0 \leq r \leq 2, 0 \leq \theta \leq 2\pi$$

$$I = \iiint_K (x^2+y^2) dx dy dz = \iint_D (x^2+y^2) \left(\frac{1}{1+x^2+y^2} - \frac{2}{6+x^2+y^2} \right) dx dy =$$

$$= \iint_D r^3 \left(\frac{1}{1+r^2} - \frac{2}{6+r^2} \right) dr d\theta = 2\pi \int_0^2 \left(\frac{r^3}{1+r^2} - \frac{2r^3}{6+r^2} \right) dr =$$

$$= 2\pi \int_0^2 \left(r - \frac{r}{r^2+1} - 2r + \frac{12r}{6+r^2} \right) dr =$$

$$= 2\pi \left[-\frac{r^2}{2} - \frac{1}{2} \ln|r^2+1| + 6 \ln|r^2+6| \right]_0^2 =$$

$$= 2\pi \left(-2 - \frac{1}{2} \ln 5 + 6 \ln 10 - 6 \ln 6 \right) =$$

$$= 2\pi \left(-2 - \frac{1}{2} \ln 5 + 6(\ln 10 - \ln 6) \right) = 2\pi \left(-2 - \frac{1}{2} \ln 5 + 6 \ln \frac{10}{6} \right)$$

$$\stackrel{\oplus}{=} 2\pi \left(-2 - \frac{1}{2} \ln 5 + 6 \ln \frac{5}{3} \right) = 2\pi \left(-2 - \frac{1}{2} \ln 5 + 6(\ln 5 - \ln 3) \right) =$$

$$\stackrel{\ominus}{=} 2\pi (11 \ln 5 - 12 \ln 3 - 4) = \pi \cdot \ln \frac{5^{11}}{3^{12} \cdot e^4}$$

□

8.23

$$K: \begin{cases} x^2 + y^2 \leq 1 \\ 0 \leq z \leq 1 + \sqrt{1 - x^2 - y^2} \end{cases}$$

$$\iiint_K (x^2 + y^2) dx dy dz = \iint_D (x^2 + y^2)(1 + \sqrt{1 - x^2 - y^2}) dx dy =$$

$$= \iint_E r^2(1 + \sqrt{1 - r^2}) \cdot r dr d\phi = 2\pi \int_0^1 (r^3 + r^3 \sqrt{1 - r^2}) dr =$$

$$= 2\pi \left(\left[\frac{r^4}{4} \right]_0^1 - \frac{1}{3} \left[r^2(1 - r^2)^{3/2} \right]_0^1 + \frac{2}{3} \int_0^1 r(1 - r^2)^{3/2} dr \right) =$$

$$= 2\pi \left(\frac{1}{4} + \frac{2}{3} \left[-\frac{1}{5}(1 - r^2)^{5/2} \right]_0^1 \right) = 2\pi \left(\frac{1}{4} + \frac{2}{15} \right) =$$

$$= 2\pi \left(\frac{15 + 8}{60} \right) = \frac{23\pi}{30}$$

□

$$8.24 \quad \iiint_K (x^2 + y^2) dx dy dz = \iint_D (x^2 + y^2)(1 - (x^2 + y^2)) dx dy =$$

$$= \iint_E r^2(1 - r^2) dr d\phi = 2\pi \int_{1/2}^1 r^2(1 - r^2) dr = 2\pi \left[\frac{r^4}{4} - \frac{r^6}{6} \right]_{1/2}^1 =$$

$$= 2\pi \left(\frac{1}{4} - \frac{1}{6} - \frac{1}{64} + \frac{1}{384} \right) = 2\pi \left(\frac{96 - 64 - 6 + 1}{384} \right) = \frac{23\pi}{192} = \frac{9\pi}{64}$$

□

$$8.25 \quad 2x^2 + 3y^2 \leq 6 \quad D: \left(\frac{x}{\sqrt{3}}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 \leq 1 \quad \begin{cases} x = \sqrt{3}r\cos\varphi \\ y = \sqrt{2}r\sin\varphi \end{cases}$$

$$K: x^2 + y^2 \leq z \leq 6 - x^2 - 2y^2 \quad E: 0 \leq r \leq 1, 0 \leq \varphi \leq 2\pi$$

$$\iiint_K (x^2 + y^2) dx dy dz = \iint_D (x^2 + y^2) (6 - 2x^2 - 3y^2) dx dy =$$

$$= \sqrt{6} \iint_E (3r^2 \cos^2\varphi + 2r^2 \sin^2\varphi) (6 - 6r^2 \cos^2\varphi - 6r^2 \sin^2\varphi) r dr d\varphi =$$

$$= \sqrt{6} \iint_E (2r^3 + r^3 \cos^2\varphi) (6 - 6r^2) dr d\varphi =$$

$$= 6\sqrt{6} \iint_E (2r^3 - 2r^5 + r^3 - r^5) \cos^2\varphi dr d\varphi =$$

$$= 6\sqrt{6} \int_0^{2\pi} \left[(2r^3 - 2r^5)\varphi + (r^3 - r^5) \left(\frac{\varphi}{2} + \frac{\sin 2\varphi}{4} \right) \right]_0^{2\pi} d\varphi =$$

$$= 6\sqrt{6} \int_0^1 (2\pi(2r^3 - 2r^5) + \pi(r^3 - r^5)) dr =$$

$$= 6\sqrt{6}\pi \int_0^1 (5r^3 - 5r^5) dr = 30\sqrt{6}\pi \left[\frac{r^4}{4} - \frac{r^6}{6} \right]_0^1 =$$

$$= 30\sqrt{6}\pi \left(\frac{1}{4} - \frac{1}{6} \right) = 30\sqrt{6}\pi \cdot \frac{1}{12} = \frac{5\pi\sqrt{6}}{2}$$

□

8.26

$$K: 2x^2 + y^2 + z^2 + 2y + 4z \leq 0$$

$$2x^2 + (y+1)^2 + (z+2)^2 \leq 5$$

$$\left(\frac{x}{\sqrt{5/2}}\right)^2 + \left(\frac{y+1}{\sqrt{5}}\right)^2 + \left(\frac{z+2}{\sqrt{5}}\right)^2 \leq 1$$

$$L: 0 \leq r \leq 1, 0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi$$

$$\begin{cases} x = \sqrt{\frac{5}{2}} r \sin \theta \cos \phi \\ y = \sqrt{5} r \sin \theta \sin \phi - 1 \\ z = \sqrt{5} r \cos \theta - 2 \end{cases}$$

$$\frac{dx, y, z}{d(r, \theta, \phi)} = 5\sqrt{\frac{5}{2}} r^2 \sin \theta$$

$$\iiint_K (x^2 + y^2) dx dy dz = 5\sqrt{\frac{5}{2}} \iiint_L \left(\frac{5}{2} r^2 \sin^2 \theta \cos^2 \phi + 5 r^2 \sin^2 \theta \sin^2 \phi + \right.$$

$$\left. - 2\sqrt{5} r \sin \theta \cos \phi + 1 \right) r^2 \sin \theta dr d\theta d\phi =$$

$$= 5\sqrt{\frac{5}{2}} \iiint_L \left(\frac{5}{2} r^4 \sin^3 \theta + \frac{5}{2} r^4 \sin^3 \theta \cdot \sin^2 \phi - 2\sqrt{5} r^3 \sin^2 \theta \cos \phi + r^2 \sin \theta \right) d\theta d\phi dr =$$

$$= 5\sqrt{\frac{5}{2}} \int_0^\pi \left[\left(\frac{5}{2} r^4 \sin^3 \theta \cdot \phi + \frac{5}{2} r^4 \sin^3 \theta \left(\frac{\phi}{2} - \frac{\sin 2\phi}{4} \right) - 2\sqrt{5} r^3 \sin^2 \theta \sin \phi + r^2 \sin \theta \cdot \phi \right) \right]_0^{2\pi} d\theta =$$

$$= 5\sqrt{\frac{5}{2}} \int_0^\pi \left(5\pi r^4 \sin^3 \theta (1 - \cos^2 \theta) + \frac{5}{2} \pi r^4 \sin^3 \theta (1 - \cos^2 \theta) + 2\pi r^2 \sin \theta \right) d\theta =$$

$$= 5\sqrt{\frac{5}{2}} \int_0^\pi \left(\frac{15}{2} \pi r^4 \sin^3 \theta (1 - \cos^2 \theta) + 2\pi r^2 \sin \theta \right) d\theta =$$

$$= 5\sqrt{\frac{5}{2}} \pi \int_0^\pi \left(\frac{15}{2} r^4 (-\cos \theta + \frac{1}{3} \cos^3 \theta) - 2r^2 \cos \theta \right) d\theta =$$

$$= 5\sqrt{\frac{5}{2}} \pi \int_0^\pi (10r^4 + 4r^2) d\theta = 5\sqrt{\frac{5}{2}} \pi \left[2r^5 + \frac{4}{3} r^3 \right]_0^\pi =$$

$$= 5\sqrt{\frac{5}{2}} \pi \left(2 + \frac{4}{3} \right) = \frac{5\sqrt{10}}{2} \left(\frac{6+4}{3} \right) \pi = \frac{50\sqrt{10}}{6} \pi = \frac{25\sqrt{10}}{3} \pi$$

□

8.27

$$L: x^2 + z^2 = 9, 0 \leq x \leq 2, D: 0 \leq s \leq 2, -3 \leq t \leq 3$$

$$r(s, t) = (s, t, \pm\sqrt{9-t^2})$$

$$r_s = (1, 0, 0)$$

$$r_t = (0, 1, \pm\frac{t}{\sqrt{9-t^2}})$$

$$\|r_s \times r_t\| = \left\| \left(0, \pm\frac{t}{\sqrt{9-t^2}}, 1 \right) \right\| = \sqrt{\frac{t^2}{9-t^2} + 1}$$

$$\bullet \iint_L (x^2 + z^2) \sqrt{x^2 + z^2} \, dS = \iint_D (t^2 + 9 - t^2) \cdot s t^2 \sqrt{\frac{t^2}{9-t^2} + 1} \, ds dt =$$

$$\bullet = 9 \iint_D s t^2 \sqrt{\frac{9}{9-t^2}} \, ds dt = 9 \int_0^2 s \, ds \cdot \int_{-3}^3 t^2 \sqrt{\frac{9}{9-t^2}} \, dt =$$

$$= 9 \left[\frac{s^2}{2} \right]_0^2 \cdot 3 \int_{-3}^3 \frac{t^2}{\sqrt{9-t^2}} \, dt = 54 \cdot \left(\int_{-3}^3 \left(\frac{9}{\sqrt{9-t^2}} - \sqrt{9-t^2} \right) dt \right) =$$

$$= 54 \cdot \int_{-3}^3 \left(\sqrt{1 - \left(\frac{t}{3}\right)^2} - \sqrt{9-t^2} \right) dt = 54 \cdot \left[\frac{9}{2} \arcsin\left(\frac{t}{3}\right) - \frac{t}{2} \sqrt{9-t^2} \right]_{-3}^3 =$$

$$= 54 \cdot \left(\frac{9\pi}{4} + \frac{9\pi}{4} \right) = 54 \cdot \frac{9\pi}{2} = 27 \cdot 9\pi = 243\pi$$

8.28

$$a) \Gamma: z = x^2 + y^2, x^2 + y^2 \leq 1$$

$$r(s, t) = (s, t, s^2 + t^2), s^2 + t^2 \leq 1$$

$$r'_s = (1, 0, 2s)$$

$$r'_t = (0, 1, 2t)$$

$$r'_s \times r'_t = (-2s, -2t, 1) = -2(s, t, -\frac{1}{2})$$

$$b) |r'_s \times r'_t| = \sqrt{1 + 4s^2 + 4t^2}$$

$$A = \iint_D \sqrt{1 + 4s^2 + 4t^2} ds dt = \iint_E \sqrt{1 + 4r^2} r dr d\varphi = 2\pi \left[\frac{1}{12} (1 + 4r^2)^{3/2} \right]_0^1 =$$

$$= 2\pi \left(\frac{5\sqrt{5}}{12} - \frac{1}{12} \right) = \frac{\pi}{6} (5\sqrt{5} - 1)$$

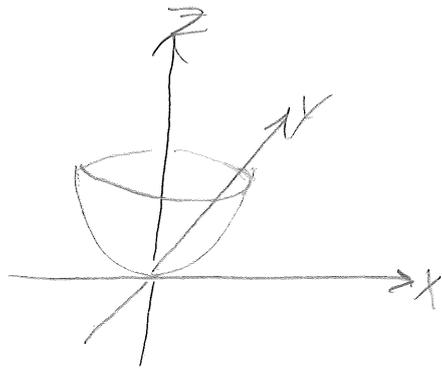
$$c) m = \iint_E s^2 \sqrt{1 + 4s^2 + 4t^2} ds dt = \iint_E r^3 \cos^2 \varphi \sqrt{1 + 4r^2} dr d\varphi =$$

$$= \left[r^2 \cdot \frac{1}{12} (1 + 4r^2)^{3/2} \right]_0^1 - \frac{1}{6} \int r (1 + 4r^2)^{3/2} dr \cdot \left[\frac{\varphi}{2} + \frac{\sin 2\varphi}{4} \right]_0^{2\pi} =$$

$$= \left(\frac{5\sqrt{5}}{12} - \left[\frac{1}{120} (1 + 4r^2)^{3/2} \right]_0^1 \right) \cdot \pi = \pi \left(\frac{5\sqrt{5}}{12} - \frac{25\sqrt{5}}{120} + \frac{1}{120} \right) =$$

$$= \frac{\pi}{120} (25\sqrt{5} + 1)$$

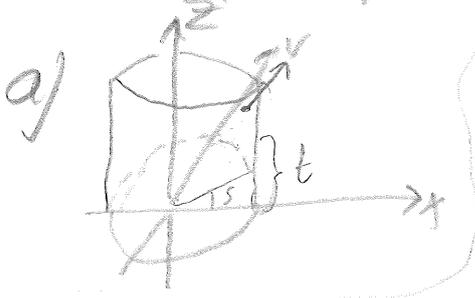
□



8.29

$$x^2 + y^2 = 4, 0 \leq z \leq 3$$

$$r(s, t) = (2\cos s, 2\sin s, t), 0 \leq s \leq 2\pi, 0 \leq t \leq 3$$



b) $r'_s = (-2\sin s, 2\cos s, 0)$

$$r'_t = (0, 0, 1)$$

$$r'_s\left(\frac{\pi}{3}, 1\right) = (-\sqrt{3}, 1, 0), r'_t\left(\frac{\pi}{3}, 1\right) = (0, 0, 1)$$

• $(1, \sqrt{3}, 2) = (2\cos s, 2\sin s, t) \Leftrightarrow \begin{cases} s = \pi/3 \\ t = 1 \end{cases}$

• $r'_s\left(\frac{\pi}{3}, 1\right) \times r'_t\left(\frac{\pi}{3}, 1\right) = (-\sqrt{3}, 1, 0) \times (0, 0, 1) = (1, \sqrt{3}, 0)$

c) $A = \iint_D |r'_s \times r'_t| ds dt = \iint_D \sqrt{4\cos^2 s + 4\sin^2 s} ds dt = 2 \iint_D ds dt =$

$$= 2 \cdot 2\pi \cdot 3 = 12\pi$$

d) $r(x, z) = (x, \sqrt{4-x^2}, z), -2 \leq x \leq 2, 0 \leq z \leq 3$

• $r'_x = \left(1, -\frac{x}{\sqrt{4-x^2}}, 0\right)$ $(x, \sqrt{4-x^2}, z) = (1, \sqrt{3}, 2)$

$r'_z = (0, 0, 1)$ $\Leftrightarrow (x, z) = (1, 2)$

• $r'_x(1, 2) = \left(1, -\frac{1}{\sqrt{3}}, 0\right)$

$r'_x(1, 2) \times r'_z(1, 2) = \left(-\frac{1}{\sqrt{3}}, -1, 0\right)$

□

8.30

$$\therefore x^2 + y^2 + z^2 = 4, z \geq 1$$

$$a) r(x, y) = (x, y, \sqrt{4 - x^2 - y^2}), x^2 + y^2 \leq 3$$

$$b) r_x = \left(1, 0, -\frac{x}{\sqrt{4 - x^2 - y^2}}\right)$$

$$r_y = \left(0, 1, -\frac{y}{\sqrt{4 - x^2 - y^2}}\right)$$

$$dx \times r_y = \left(\frac{x}{\sqrt{4 - x^2 - y^2}}, \frac{y}{\sqrt{4 - x^2 - y^2}}, 1\right)$$

$$c) |dx \times r_y| = \frac{1}{\sqrt{4 - x^2 - y^2} \sqrt{x^2 + y^2 + 4 - x^2 - y^2}} = \frac{2}{\sqrt{4 - x^2 - y^2}}$$

$$E: 0 \leq r \leq \sqrt{3}, 0 \leq \varphi \leq 2\pi$$

$$A = \iint_D \frac{2}{\sqrt{4 - x^2 - y^2}} dx dy = \iint_E \frac{2r}{\sqrt{4 - r^2}} dr d\varphi = 2\pi \left[-2\sqrt{4 - r^2} \right]_0^{\sqrt{3}} =$$

$$= 2\pi(-2 + 4) = 4\pi$$

$$d) r(\theta, \varphi) = (R \sin \theta \cos \varphi, R \sin \theta \sin \varphi, R \cos \theta)$$

$$R \cos \theta \geq h \Leftrightarrow 0 \leq \theta \leq \arccos \frac{h}{R}$$

$$r'_\theta = (R \cos \theta \cos \varphi, R \cos \theta \sin \varphi, -R \sin \theta)$$

$$r'_\varphi = (-R \sin \theta \sin \varphi, R \sin \theta \cos \varphi, 0)$$

$$|r'_\theta \times r'_\varphi| = \sqrt{(R^2 \sin^2 \theta \cos^2 \varphi)^2 + (R^2 \sin^2 \theta \sin^2 \varphi)^2 + (R^2 \sin \theta \cos \theta \cos^2 \varphi + R^2 \cos \theta \sin \theta \sin^2 \varphi)^2} =$$

$$= R^2 \sqrt{\sin^4 \theta \cos^2 \varphi + \sin^4 \theta \sin^2 \varphi + \sin^2 \theta \cos^2 \theta} =$$

$$= R^2 \sqrt{\sin^2 \theta} = R^2 \sin \theta$$

forts...

8.30

fets...

$$A = \int_F R^2 \sin \theta d\theta d\phi = 2\pi R^2 \left[-\cos \theta \right]_0^{\arccos \frac{h}{R}} =$$

$$= 2\pi R^2 \left(-\frac{h}{R} + 1 \right) = 2\pi R^2 \left(-\frac{h}{R} + \frac{R}{R} \right) = 2\pi R(R-h)$$

□

8.31

$$\begin{cases} x = (2 - \cos t) \cos s \\ y = (2 - \cos t) \sin s \\ z = \sin t \end{cases}, -\pi \leq s \leq \pi, -\pi \leq t \leq \pi$$

$$r(s, t) = (x, y, z) \quad r_s = (\cos t - 2) \sin s, (2 - \cos t) \cos s, 0$$

$$r_t = (\sin t \cos s, \sin t \sin s, \cos t)$$

$$r_s \times r_t = ((2 - \cos t) \cos s \cos t, (2 - \cos t) \sin s \cos t, (\cos t - 2) \sin s \sin t +$$

$$-(2 - \cos t) \cos s \sin t) = (2 - \cos t) (\cos s \cos t, \sin s \cos t, -\sin t)$$

$$|r_s \times r_t| = (2 - \cos t)$$

$$A = \int_0^{2\pi} \int_{-\pi}^{\pi} (2 - \cos t) ds dt = 2\pi \left[2t - \sin t \right]_{-\pi}^{\pi} = 2\pi(2\pi + 2\pi) = 8\pi^2$$

□

8.32

$$\Gamma_1: |x| \leq z \leq 4, -4 \leq x \leq 4, -4 \leq y \leq 4, y^2 + z^2 = 16$$

$$\Gamma_2: x \leq z \leq 4, 0 \leq x \leq 4, 0 \leq y \leq 4, y^2 + z^2 = 16 \quad (\Gamma_1 = 4 \cdot \Gamma_2)$$

$$r(x, y) = (x, y, \sqrt{16 - y^2}), x^2 + y^2 \leq 16$$

$$r_x = (1, 0, 0)$$

$$r_y = (0, 1, -\frac{y}{\sqrt{16 - y^2}})$$

$$r_x \times r_y = (0, \frac{y}{\sqrt{16 - y^2}}, 1)$$

$$|r_x \times r_y| = \sqrt{\frac{y^2}{16 - y^2} + 1} = \frac{4}{\sqrt{16 - y^2}}$$

$$\frac{A}{4} = \iint_{\Gamma_2} ds = \iint_D \frac{4}{\sqrt{16 - y^2}} dx dy = \int_0^4 \left(\frac{4}{\sqrt{16 - y^2}} \cdot \sqrt{16 - y^2} \right) dy =$$

$$= 4 \int_0^4 dy = 16 \Rightarrow A = 4 \cdot 16 = 64 \text{ cm}^2$$

□

8.33

$$x^2 + y^2 \leq 4, 0 \leq y \leq 7, x \geq 0, z \geq 0$$

$$T(x, y, z) = y^2 \sqrt{2-x}$$

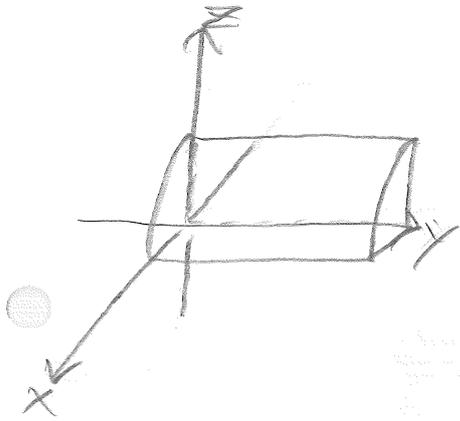
$$r(s, t) = (2\cos s, t, 2\sin s), 0 \leq s \leq \frac{\pi}{2}, 0 \leq t \leq 7$$

$$r'_s = (-2\sin s, 0, 2\cos s)$$

$$r'_t = (0, 1, 0)$$

$$r'_s \times r'_t = (-2\cos s, 0, -2\sin s)$$

$$|r'_s \times r'_t| = 2$$



$$\iint_M k t^4 dS = k \iint_M y^8 (2-x)^2 dx dy = k \int_0^{\pi/2} \int_0^7 t^8 (2-2\cos s)^2 \cdot 2 ds dt =$$

$$= 8k \int_0^{\pi/2} dt \cdot \int_0^7 (1-2\cos s + \cos^2 s) ds =$$

$$= 8k \left[\frac{t^9}{9} \right]_0^7 \cdot \left[s - 2\sin s + \frac{s}{2} + \frac{\sin 2s}{4} \right]_0^{\pi/2} =$$

$$= 8k \cdot \frac{7^9}{9} \cdot \left(\frac{\pi}{2} - 2 + \frac{\pi}{4} \right)$$

$$= \frac{8 \cdot 7^9}{9} \cdot k \left(\frac{3\pi}{4} - 2 \right) =$$

$$= k \cdot \frac{2 \cdot 7^9}{9} (3\pi - 8)$$

□

8.34

$$z = \sqrt{1-x^2-y^2}, x \geq 0, y \geq 0, x^2+y^2 \leq 1, |r_\theta \times r_\varphi| = \sin \theta$$

$$\sigma = k z \sqrt{x^2+y^2} \quad r(\theta, \varphi) = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$$

$$D: 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq \varphi \leq \frac{\pi}{2}$$

$$M = \iint_H \sigma \, ds = k \iint_H z \sqrt{x^2+y^2} \, dx \, dy = k \iint_D \cos \theta \cdot \sin^2 \theta \, d\theta \, d\varphi =$$

$$= k \cdot \frac{\pi}{2} \left[\frac{1}{3} \sin^3 \theta \right]_0^{\pi/2} = k \cdot \frac{\pi}{2} \cdot \frac{1}{3} \cdot 1 = k \cdot \frac{\pi}{6}$$

□

$$8.35 \Gamma: x^2+y^2+z^2=R^2, \rho(x,y,z) = \sqrt{R^2-x^2-y^2}$$

$$r(\theta, \varphi) = (R \sin \theta \cos \varphi, R \sin \theta \sin \varphi, R \cos \theta), D: 0 \leq \theta \leq \pi, 0 \leq \varphi \leq 2\pi$$

$$|r_\theta \times r_\varphi| = R^2 \sin \theta$$

$$D: 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq \varphi \leq 2\pi$$

$$\iint_H \rho \, ds = \iint_D \sqrt{R^2-R^2 \sin^2 \theta} \cdot R^2 \sin \theta \, d\theta \, d\varphi = R^3 \iint_D \cos \theta \cdot \sin \theta \, d\theta \, d\varphi = *$$

$$= 2R^3 \iint_{D_1} \cos \theta \sin \theta \, d\theta \, d\varphi = 4\pi R^3 \left[\frac{1}{2} \sin^2 \theta \right]_0^{\pi/2} = 4\pi R^3 \left(\frac{1}{2} \cdot 1 - 0 \right) =$$

$$= 2\pi R^3 \text{ [C]}$$

* ytladdningen är oberoende av z-koordinaten \Rightarrow

ytladdningen är lika stor på nedre halvan som övre \Rightarrow

$$p \text{ på } D = 2 \cdot (p \text{ på } D_1)$$

□