7.1 a)

$$3\cdot\begin{pmatrix}1&1\\2&3\\1&2\end{pmatrix} - 2\begin{pmatrix}6&-3\\2&9\\7&-5\end{pmatrix} = \begin{pmatrix}3&3\\6&9\\3&6\end{pmatrix} - \begin{pmatrix}12&-6\\4&18\\14&-10\end{pmatrix} = \begin{pmatrix}-9&-3\\2&-9\\-11&16\end{pmatrix}$$

b)

$$\frac{1}{2}\begin{pmatrix}1&-1\\1&1\end{pmatrix} - \frac{1}{2}\begin{pmatrix}1&1\\-1&1\end{pmatrix} = \begin{pmatrix}1/2&-1/2\\1/2&1/2\end{pmatrix} - \begin{pmatrix}1/2&1/2\\-1/2&1/2\end{pmatrix} = \begin{pmatrix}0&-1\\1&0\end{pmatrix}$$

7.2 a)

$$\begin{pmatrix}2&1\\-1&1\end{pmatrix}\cdot\begin{pmatrix}1&3\\2&4\end{pmatrix} = \begin{pmatrix}2\cdot1+1\cdot2 & 2\cdot3+1\cdot4\\-1\cdot1+1\cdot2 & -1\cdot3+1\cdot4\end{pmatrix} = \begin{pmatrix}4&10\\1&1\end{pmatrix}$$

b)

$$\begin{pmatrix}1&2&1\\-1&0&4\end{pmatrix}\cdot\begin{pmatrix}1&-1\\0&1\end{pmatrix}$$

$$2\times\boxed{3 \neq 2}\times 2$$

ej def

c)

$$\begin{pmatrix}1&2&3&4\\0&2&0&1\end{pmatrix}\cdot\begin{pmatrix}1&0&2\\0&-1&3\\0&0&-1\\1&0&1\end{pmatrix} = \begin{pmatrix}5&-2&9\\1&-2&7\end{pmatrix}$$

$2\times4 \qquad 4\times3 \qquad 2\times3$

d)

$$\begin{pmatrix}2&1&3\\0&5&0\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\-1&1&-1\\3&1&0\end{pmatrix} = \begin{pmatrix}10&4&-1\\-5&5&-5\end{pmatrix}$$

$2\times3 \qquad 3\times3 \qquad 2\times3$

7.3 a) $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \end{pmatrix}$

$1\times 3 \quad 3\times 1 \quad 1\times 1$

b) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 1 \\ 6 & 4 & 2 \\ 9 & 6 & 3 \end{pmatrix}$

$3\times 1 \quad 1\times 3 \quad 3\times 3$

7.4) a) $AB = \begin{pmatrix} 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ -3 & -2 \end{pmatrix}$

$2\times 3 \quad 3\times 2 \quad 2\times 2$

$BA = \begin{pmatrix} 2 & 1 \\ 0 & -1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$

$3\times 2 \quad 2\times 3 \quad 3\times 3$

b) $(AB)^T = \begin{pmatrix} 2 & -3 \\ 0 & -2 \end{pmatrix}$

$B^T A^T = \begin{pmatrix} 2 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ 0 & -2 \end{pmatrix}$

$2\times 3 \quad 3\times 2 \quad 2\times 2$

$(AB)^T = B^T A^T$!!

7.5 a) $(A+B)^2 = (A+B)\cdot(A+B) = A^2 + AB + BA + B^2$

Obs $AB \neq BA$ i allmänhet

b) $(A+B)(A-B) = A^2 - AB + BA - B^2$

c) $(A-B)(A+B) = A^2 + AB - BA - B^2$

7.6) $AB = \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} 8 & -16 \\ 4 & -8 \end{pmatrix}$

$BA = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Kommuterar ej (ty $AB \neq BA$)

7.7 a) $\begin{cases} x_1 + 2x_2 = y_1 \\ 2x_1 + 5x_2 = y_2 \end{cases} \iff \underbrace{\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}}_{Y}$

b) $\begin{array}{r} \\ \text{II}-2\text{I} \end{array} \begin{cases} x_1 + 2x_2 = y_1 \\ x_2 = -2y_1 + y_2 \end{cases} \iff \begin{array}{r} \text{I}-2\text{II} \\ \\ \end{array} \begin{cases} x_1 = 5y_1 - 2y_2 \\ x_2 = -2y_1 + y_2 \end{cases}$

$\iff \underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}}_{A^{-1}} \underbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}}_{Y}$

7.8 a)

$$\left(\begin{array}{cc|cc} 3 & 4 & 1 & 0 \\ 6 & 2 & 0 & 1 \end{array}\right) \sim \begin{array}{r} \\ \text{II}-2\text{I} \end{array} \left(\begin{array}{cc|cc} 3 & 4 & 1 & 0 \\ 0 & -6 & -2 & 1 \end{array}\right) \sim \begin{array}{r} 3\text{I}+2\text{II} \\ \\ \end{array} \left(\begin{array}{cc|cc} 9 & 0 & -1 & 2 \\ 0 & -6 & -2 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & -1/9 & 2/9 \\ 0 & 1 & 1/3 & -1/6 \end{array}\right)$$

$$\underbrace{}_{A^{-1}}$$

b)

$$\left(\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ -2 & 4 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{array}\right)$$

ej inv. bar.

7.9 a)

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 4 & 1 & -1 & 0 & 1 & 0 \\ 2 & 3 & 0 & 0 & 0 & 1 \end{array}\right)$$

$\sim$

$$\begin{array}{r} \\ \text{II}-4\text{I} \\ \text{III}-2\text{I} \end{array}\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -7 & -1 & -4 & 1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 1 \end{array}\right)$$

$\sim$

$$\begin{array}{r} \\ \\ 7\text{III}-\text{II} \end{array}\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -7 & -1 & -4 & 1 & 0 \\ 0 & 0 & 1 & -10 & -1 & 7 \end{array}\right)$$

$\sim$

$$\begin{array}{r} \\ \text{II}+\text{III} \\ \\ \end{array}\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -7 & 0 & -14 & 0 & 7 \\ 0 & 0 & 1 & -10 & -1 & 7 \end{array}\right)$$

$\sim$

$$\begin{array}{r} 7\text{I}+2\text{II} \\ \\ \\ \end{array}\left(\begin{array}{ccc|ccc} 7 & 0 & 0 & -21 & 0 & 14 \\ 0 & -7 & 0 & -14 & 0 & 7 \\ 0 & 0 & 1 & -10 & -1 & 7 \end{array}\right)$$

$\sim$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{\begin{matrix} -3 & 0 & 2 \\ 2 & 0 & -1 \\ -10 & -1 & 7 \end{matrix}}_{A^{-1}} \\ 0 & 0 & 1 & \end{array}\right)$$

b)

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$\sim$

$$\begin{array}{r} \\ \text{II}-\text{I} \\ \text{III}-2\text{I} \end{array}\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & -1 & -2 & 0 & 1 \end{array}\right)$$

$\sim$

$$\begin{array}{r} \\ \\ \text{III}-\text{II} \end{array}\left(\begin{array}{c|ccc} \underbrace{\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}} & \begin{matrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 1 \end{matrix} \end{array}\right)$$

inv saknas.

7.10) $(AX+B)^{-1}=A \iff AX+B=A^{-1} \iff AX=A^{-1}-B \iff$

$\iff X= A^{-1}(A^{-1}-B).$

$$A^{-1}: \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \sim \begin{array}{c} \\ \\ \mathrm{III}-\mathrm{II} \end{array} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \sim$$

$$\begin{array}{c} \\ \\ \mathrm{III}+\mathrm{II} \end{array} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right) \sim \begin{array}{c} \mathrm{I}-\mathrm{III} \\ \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 2 & -1 & -1 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right) \sim$$

$$\begin{array}{c} \mathrm{I}+\mathrm{II} \\ \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right) \implies A^{-1}= \begin{pmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

$$A^{-1}-B= \begin{pmatrix} 0 & -2 & -4 \\ -3 & -6 & -6 \\ -8 & -7 & -8 \end{pmatrix}$$

$$A^{-1}(A^{-1}-B) = \begin{pmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ -1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & -2 & -4 \\ -3 & -6 & -6 \\ -8 & -7 & -8 \end{pmatrix} = \begin{pmatrix} 8 & 5 & 4 \\ 3 & 4 & 2 \\ -11 & -11 & -10 \end{pmatrix}$$

7.12) $AB = 0 \Leftrightarrow A^{-1}AB = A^{-1} \cdot 0 \Leftrightarrow B = 0.$

7.13a) $A^2 + A + I = 0$

$\Leftrightarrow$

$A(A+I) + I = 0$

$\Leftrightarrow$

$I = A(-A-I)$

så $A^{-1} = -A - I$

b) $A^2 + A + I = 0$

$\Leftrightarrow$

$A^3 + \underbrace{A^2 + A}_{-I} = 0$

$\Leftrightarrow$

$A^3 - I = 0$

$\Leftrightarrow$

$A^3 = I$

eller

$(A-I)(A^2+A+I) = 0$

$\Leftrightarrow$

$A^3 - I = 0$

7.14) a) $\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix} \neq I$ NIX!

b) $\begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}\begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ JA.

7.15) a) NEJ t.ex. kolonnvektor 1 ej längd 1

b) NEJ t.ex radvektor 1 och 2 ej ortogonala.

c) JA utför matrismult...

7.17) Vet $A^T = A^{-1}$, $B^T = B^{-1}$

$(AB)(AB)^T = ABB^TA^T = I$

$(A^TB)(A^TB)^T = A^TBB^TA = I$

DSV...

7.18) Koordbytesmatrisen är ortogonal (verifiera).

$X' = AX \Leftrightarrow A^{-1}X' = X \Leftrightarrow X = A^TX'$

så

$$\begin{cases} x_1 = \frac{1}{9}(x_1' + 8x_2' + 4x_3') \\ x_2 = \frac{1}{9}(4x_1' - 4x_2' + 7x_3') \\ x_3 = \frac{1}{9}(8x_1' + x_2' - 4x_3') \end{cases}$$

7.19) $e' = Ae \Leftrightarrow x = A^T x' \Leftrightarrow (A^T)^{-1} x = x' \Leftrightarrow x' = Ax$

om A ortogonal

Verifiera att koefficientmatrisen är ortogonal ...

$$\begin{cases} x_1' = \frac{2}{3}x_1 - \frac{2}{3}x_2 - \frac{1}{3}x_3 \\ x_2' = \frac{2}{3}x_1 + \frac{1}{3}x_2 + \frac{2}{3}x_3 \\ x_3' = \frac{1}{3}x_1 + \frac{2}{3}x_2 - \frac{2}{3}x_3 \end{cases}$$

7.23 Vi bestämmer (dimensionen på) nollrummet och använder sats, eller bestämmer värderummet och använder sats.

a) $\begin{cases} x - 2y + z = 0 \\ 2x - 6y + 6z = 0 \\ -3x + 5y - z = 0 \end{cases} \overset{1.11}{\Leftrightarrow} \begin{cases} x = 3t \\ y = 2t \\ z = t \end{cases}$

så $(3, 2, 1)$ bas i nollrum och rang 2.

c) $\begin{pmatrix} 2 & 1 & -3 & 4 & -3 \\ 3 & 2 & 1 & 2 & 2 \\ -4 & 3 & 2 & 1 & -4 \end{pmatrix} \sim \begin{array}{c} \\ 2II-3I \\ III+2I \end{array} \begin{pmatrix} 2 & 1 & -3 & 4 & -3 \\ 0 & 1 & 11 & -8 & 13 \\ 0 & 5 & -4 & 9 & -10 \end{pmatrix} \sim$

$\begin{array}{c} \\ \\ III-5II \end{array} \begin{pmatrix} 2 & 1 & -3 & 4 & -3 \\ 0 & 1 & 11 & -8 & 13 \\ 0 & 0 & -59 & 49 & -75 \end{pmatrix}$ rang 3, nolldim 2.

$\begin{cases} x_1 = \\ x_2 = \\ x_3 = -75s + 49t \\ x_4 = \quad 59t \\ x_5 = 59s \end{cases}$ osv "Superbråkigt"

$\longrightarrow$ 7.23 d)

7.23 d) Kolonnvektorerna uppfyller ($3k_1 + k_2 = k_3$)

Rang $= 2 \Rightarrow$ Nolldim $= 1$

och nollrummet spanns av $(3, 1, -1)$

7.26) $AXB = C \Leftrightarrow X = A^{-1}CB^{-1}$ om inverser existerar.

A^{-1}:
$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \sim \begin{array}{c} \\ II-I \\ \\ \end{array}\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \sim$$

$$\begin{array}{c} \\ \\ III-II \end{array}\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right) \sim \begin{array}{c} \\ II-III \\ \\ \end{array}\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 2 & -1 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right) \sim$$

$$\begin{array}{c} I-II \\ \\ \\ \end{array}\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & 1 \\ 0 & 1 & 0 & -2 & 2 & -1 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right) \quad \text{så} \quad A^{-1} = \begin{pmatrix} 3 & -2 & 1 \\ -2 & 2 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

B^{-1}:
$$\left(\begin{array}{cc|cc} 2 & 4 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right) \sim \begin{array}{c} \\ 2II-5I \end{array}\left(\begin{array}{cc|cc} 2 & 4 & 1 & 0 \\ 0 & -14 & -5 & 2 \end{array}\right) \sim \begin{array}{c} 7I+2II \\ \\ \end{array}\left(\begin{array}{cc|cc} 14 & 0 & -3 & 4 \\ 0 & -14 & -5 & 2 \end{array}\right)$$

så $B^{-1} = \frac{1}{14}\begin{pmatrix} -3 & 4 \\ 5 & -2 \end{pmatrix}$

$$A^{-1}C = \begin{pmatrix} 3 & -2 & 1 \\ -2 & 2 & -1 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$(A^{-1}C)B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{14}\begin{pmatrix} -3 & 4 \\ 5 & -2 \end{pmatrix} = \frac{1}{14}\begin{pmatrix} -3 & 4 \\ 0 & 0 \\ 5 & -2 \end{pmatrix}$$