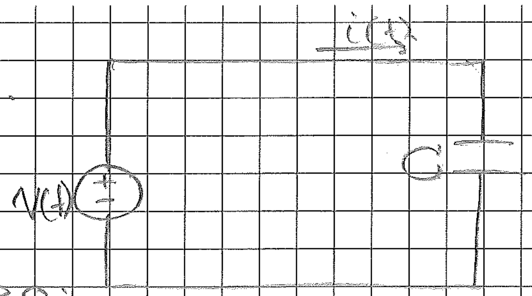
7.1



$$v(t) = \begin{cases} V_0 \sin(\omega t) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

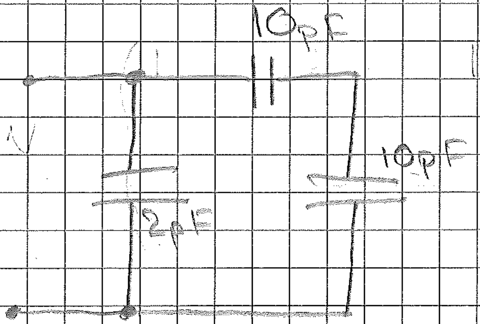
For $t \geq 0$:

$$i = C\frac{dv}{dt} = C\frac{d}{dt}\left(V_0 \sin(\omega t)\right) = C V_0 \omega \cos(\omega t)$$

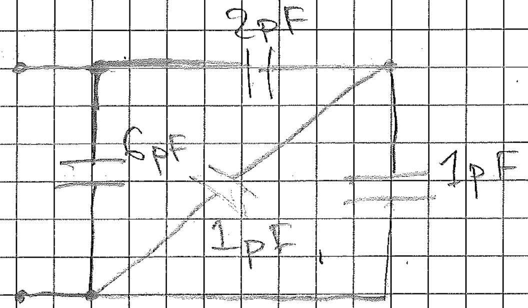
$$p = v \cdot i = C V_0^2 \omega \sin(\omega t)\cos(\omega t)$$

$$w(t) = \frac{1}{2} C v^2 = \frac{1}{2} C V_0^2 \sin^2(\omega t).$$

7.8 a)



b)



a) 1) $\frac{1}{C'} = \frac{1}{10} + \frac{1}{10} \Rightarrow \frac{1}{C'} = \frac{1}{5} \Rightarrow C' = 5\,pF$

$C_{eq} = (5+2)\,pF = 7\,pF.$

b) $C' = 1 + 1 = 2\,pF.$

$C'' = \frac{1}{2} + \frac{1}{2} = 1\,pF$

$C_{eq} = 6 + 1 = 7\,pF.$

7.4 $C = \frac{\varepsilon_0 A}{d}$

$$W = \frac{\varepsilon_0 A}{2d} V_0^2$$

$$q = C \cdot V_0 = \frac{\varepsilon_0 A}{d} V_0$$

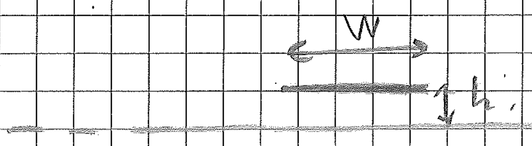
Ny kapacitans: $C' = \frac{\varepsilon_0 A}{2d}$

$$\Rightarrow V_{ny} = \frac{\varepsilon_0 A}{d} V_0 \cdot \frac{2d}{\varepsilon_0 A} = 2V_0$$

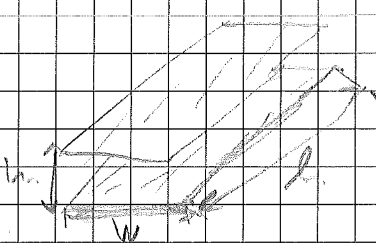
$$W_{ny} = \frac{1}{2} \frac{\varepsilon_0 A}{2d} 4V_0^2 = \frac{\varepsilon_0 A}{d} V_0^2 = 2W_0$$

"Mekaniskt arbete för att separera plattorna"

7.5



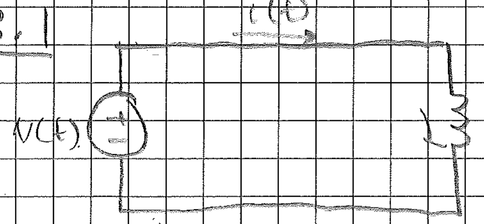
Vi antar längden l:



Detta ger: $C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 w l}{h}$

$$\frac{C}{l} = \frac{\varepsilon_0 w}{h}$$

8.1



$V(t) = V_0 \sin(\omega t)$ $\quad t \geq 0$

$i(t) = 0$ $\quad t \leq 0$

$$i(t) = \int V \frac{1}{L} dt = \frac{1}{L} V_0 \int \sin(\omega t) dt = -\frac{1}{L}\frac{V_0}{\omega}\cos(\omega t)$$

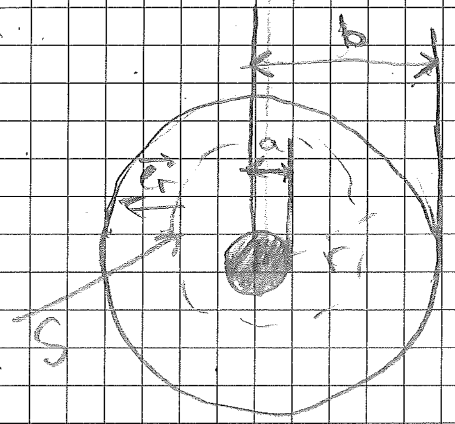
$$i(t) = \begin{cases} -\frac{1 V_0}{L\omega}\cos(\omega t) & t > 0 \\ 0 & t = 0 \end{cases}$$

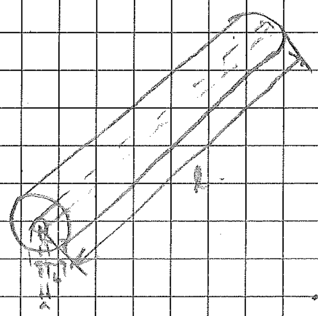
$$P = V \cdot i = V_0 \sin(\omega t) \cdot \frac{1 V_0}{\omega}\cos(\omega t) = -\frac{1 V_0^2}{\omega}\sin(\omega t)\cos(\omega t)$$

$$P = \begin{cases} -\frac{V_0^2}{L\omega}\sin(\omega t)\cos(\omega t) & t > 0 \\ 0 & t = 0 \end{cases}$$

$$w(t) = \frac{1}{2} L i^2 = \frac{1}{2} L \cdot \frac{1}{L^2}\frac{V_0^2}{\omega^2}\cos^2(\omega t) \Rightarrow w(t) = \begin{cases} \frac{1}{2\omega^2 L} V_0^2 \cos(\omega t) & t > 0 \\ 0 & t = 0 \end{cases}$$

7.7

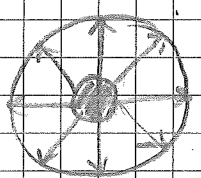




Laddning på innerledare: q

Skapar en cylinderyta S med radie r så att $a<r<b$

Symmetrin ger följande fältbild:



$\vec{E} = E(r)\vec{e}_r$. Gauss lag ger $\varepsilon_0 \oint_S \vec{E}\cdot\vec{e}_n \, dS = q$ $\Rightarrow$

$$\frac{q}{\varepsilon_0} = \oint_S \vec{E}\cdot\vec{e}_n \, dS = \oint_S E(r)\,\vec{e}_r\cdot\vec{e}_r \, dS = 2\pi r l E(r)$$

Detta ger $E(r) = \dfrac{q}{2\pi r l \varepsilon_0}$

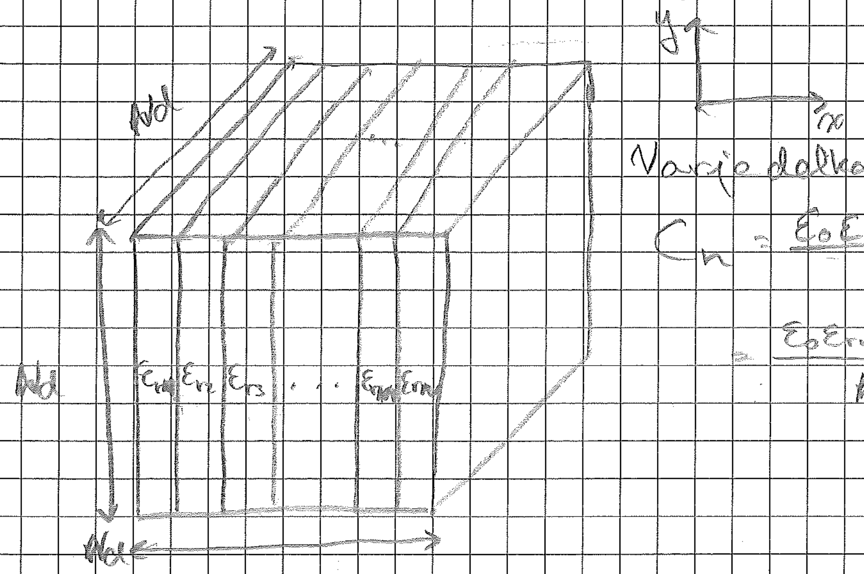
$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{r} = \int_a^b \frac{q}{2\pi r l \varepsilon_0}\, d\vec{r} = \frac{q}{2\pi l \varepsilon_0}\int_a^b \frac{1}{r}\, dr =$$

$= \dfrac{q}{2\pi l \varepsilon_0} \ln\dfrac{b}{a}$. Detta ger $C = \dfrac{2\pi l \varepsilon_0}{\ln\frac{b}{a}}$

Kapacitansen/längdenhet blir då

$$\boxed{\frac{C}{l} = \frac{2\pi\varepsilon_0}{\ln(b/a)}}$$

7.12



Varje delkapacitans:

$$C_n = \frac{\varepsilon_0 \varepsilon_{rn} A_n}{d_n} = \frac{\varepsilon_0 \varepsilon_{rn} (Nd \cdot Nd)}{N \cdot d}$$

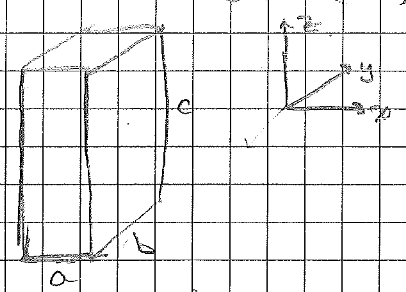
a) Vinkelrätt blir en summa av parallellkopplade kapacitanser.

$$C = \sum_{n=1}^{N} C_n = \sum_{n=1}^{N} \frac{\varepsilon_0 \varepsilon_{rn} (Nd)^2}{N \cdot d} = \varepsilon_0 N d \sum_{n=1}^{\infty} \varepsilon_{rn}$$

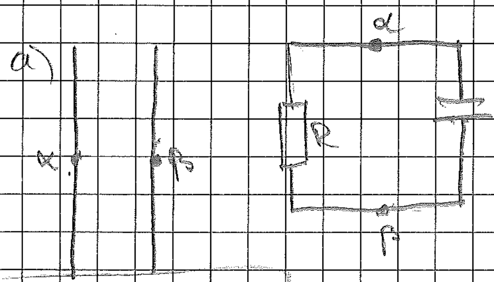
b) Parallellt blir en summa av seriekopplade kapacitanser:

$$C' = \sum_{n=1}^{\infty} \frac{1}{C_n} = \sum_{n=1}^{N} \frac{Nd}{\varepsilon_0 \varepsilon_{rn} (Nd)^2} = \frac{1}{\varepsilon_0 N d} \sum_{n=1}^{N} \frac{1}{\varepsilon_{rn}}$$

7.13



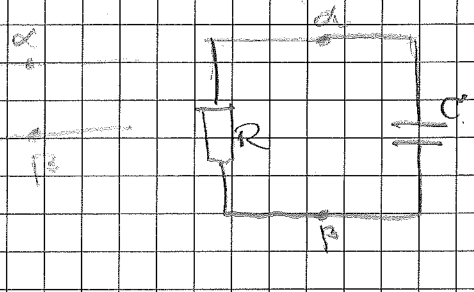
a)



$$R = \frac{a}{\sigma bc}$$

$$C = \frac{\varepsilon_0 \varepsilon_r bc}{a}$$

b)



$$R = \frac{c}{\sigma ab}$$

$$C = \frac{\varepsilon_0 \varepsilon_r ab}{c}$$

$a < b < c \Rightarrow$ Resistansen större i b; Kapacitansen större i a