KAPITEL 6

6.1

$$E_{medel}(\bar{n}) = \frac{1}{Z} \cdot \frac{\delta Z}{\delta \beta}$$

Detta ska deriveras map T.

$$U = N \cdot E_{medel}(\bar{n}) = N \cdot \frac{1}{Z} \cdot \frac{\delta Z}{\delta \beta} = -N\mu B \tanh\left(\frac{\mu B}{kT}\right)$$

Värmekaraciteten $C = \left(\frac{\delta U}{\delta T}\right)_V = N\mu B \cdot \frac{\frac{\mu B}{kT}}{\cosh^2\left(\frac{\mu B}{kT}\right)}$

$C \to 0$ då $T \to 0$ eller $T \to \infty$ vilket låter rimligt eftersom

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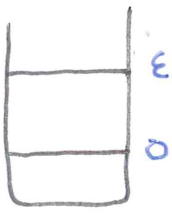
6.2



$E_1 = 0$, $E_2 = \varepsilon$

$P_n = \frac{1}{Z} \cdot e^{-\frac{E_n}{kT}}$, $P_1 = 4P_2$ $P_0 + P_1 = 1$

sätt in: $\frac{1}{2}e^0 = 4 \cdot \frac{1}{2} \cdot e^{-\varepsilon/kT} \Rightarrow -\frac{\varepsilon}{kT} = \ln\frac{1}{4} \Rightarrow$

$\Rightarrow \boxed{T = \frac{\varepsilon}{k \cdot \ln 4}}$, Medelenergi: $\bar{E} = \sum_n E_n \cdot P_n = 0 \cdot P_1 + \varepsilon P_2$

$= \varepsilon \cdot \frac{1}{2} e^{-\varepsilon/kT} = \varepsilon \frac{1}{2} \cdot e^{\ln\left(\frac{1}{4}\right)} = \boxed{\varepsilon \frac{1}{2} \cdot \frac{1}{4}}$

$$Z = \sum_n e^{-\beta E_n} = e^0 + e^{-\beta\varepsilon} = 1 + e^{-\beta\varepsilon} = 1 + e^{-\frac{\varepsilon}{kT}} = 1 + e^{\ln 1/4} = 1 + \frac{1}{4} = 1{,}25$$

$$\Rightarrow \boxed{E = \varepsilon \cdot \frac{1}{1{,}25} \cdot \frac{1}{4} = \frac{\varepsilon}{5}}$$

$$T \to \infty \Rightarrow P_n \to \frac{1}{Z}$$

$$Z = \sum e^{-\beta E_n} = \sum_N e^{-\frac{E_n}{kT}} \to N \text{ dá } T \to \infty$$

$$\Rightarrow P_n \to \frac{1}{N} \text{ dá } T \to \infty$$

$$N = 2 \Rightarrow \boxed{P_n \to \frac{1}{2} \text{ dá } T \to \infty}$$

6.3

$$Z_{rot} = \sum_{n=0}^{\infty} (2n+1) e^{-\frac{\varepsilon n(n+1)}{kT}}$$

Två termer:

$$Z_{rot} = e^{0} + 3e^{-\frac{2\varepsilon}{kT}} = 1 + 3e^{-2\varepsilon\beta}$$

Medelenergi: $\bar{E} = -\frac{1}{Z}\frac{\delta Z}{\delta \beta} = -\frac{3\cdot(-2\varepsilon)e^{-2\varepsilon\beta}}{Z} = \frac{+6\varepsilon e^{-2\varepsilon\beta}}{1+3e^{-2\varepsilon\beta}} =$

$$= \boxed{\frac{6\varepsilon}{e^{2\varepsilon\beta}+3}}, \quad U = \bar{E}\cdot N \Rightarrow U = \boxed{\frac{6N\varepsilon}{e^{2\varepsilon\beta}+3}}$$

Värmekapacitet: $C_V = \left(\frac{\delta U}{\delta T}\right)_V =$

$$= \frac{0(e^{2\varepsilon\beta}+3) - 6N\varepsilon\left(-\frac{2\varepsilon}{kT^2}\right)\cdot e^{\frac{2\varepsilon}{kT}}}{\left(e^{\frac{2\varepsilon}{kT}}+3\right)^2} =$$

$$= \boxed{\frac{12N\varepsilon^2 \cdot e^{\frac{2\varepsilon}{kT}}}{kT^2\left(e^{\frac{2\varepsilon}{kT}}+3\right)^2}} \to 0 \quad \text{då } T \to 0.$$

6.4

$$(\Delta E)^2 = \overline{E^2} - \overline{E}^2$$

$$\overline{E^2} = \sum_n E_n^2 \cdot P_n = \frac{1}{Z} \sum_n e^{-\frac{E_n^2}{kT}}$$

Vi deriverar $Z = \sum_n e^{-\beta \cdot E_n}$

$$\frac{\delta Z}{\delta \beta} = \sum_n -E_n e^{-\beta E_n} = Z \sum_n \frac{E_n \cdot e^{-\beta E_n}}{Z} = -Z\overline{E}$$

Vi deriverar igen:

$$\frac{\delta^2 Z}{\delta \beta^2} = \sum_n E_n^2 \cdot e^{-\beta E_n} = Z \sum_n \frac{E_n^2 \cdot e^{-\beta E_n}}{Z} = Z \cdot \overline{E^2}$$

$$\Rightarrow \overline{E^2} = \frac{1}{Z} \frac{\delta^2 Z}{\delta \beta^2}$$

Visa att $(\Delta E)^2 = \frac{1}{Z} \frac{\delta^2 Z}{\delta \beta^2} - \left(\frac{1}{Z} \cdot \frac{\delta Z}{\delta \beta} \right)^2 =$

HL: $(\Delta E)^2 = C k T^2 = \frac{\delta \overline{E}}{\delta T} k T^2 = \frac{\delta \left(-\frac{1}{Z} \frac{\delta Z}{\delta \beta} \right)}{\delta T} k T^2 =$

$$= \frac{\delta \overline{E}}{\delta T} k T^2$$