

Kapitel 6

6.2

$$e) (5-2i)^3 = 125 - 150i + 60i^2 + 8i = \boxed{65 - 142i}$$

$$f) (1-i)^4 = 1 - 4i - 6 + 4i + 1 = \boxed{-4}$$

6.5

$$b) \left| \frac{3+i}{4+3i} \right| = \left| \frac{(3+i)(4-3i)}{16-9i^2} \right| = \left| \frac{12-3i^2-5i}{25} \right| = \left| \frac{3-i}{5} \right| = \frac{\sqrt{9+1}}{5} = \boxed{\frac{\sqrt{10}}{5}}$$

6.6

$$\left| \frac{(1+2i)(7+\sqrt{3}i)^2}{(5+i)^2} \right| = \frac{|1+2i| \cdot |7+\sqrt{3}i|^2}{|5+i|^2} = \frac{\sqrt{5} \cdot 52}{26} = \boxed{2\sqrt{5}}$$

6.7

$$z + 2\bar{z} = 2 - i, z = a + bi$$

$$\Leftrightarrow a + bi + 2(a - bi) = 2 - i$$

$$\Leftrightarrow 3a - bi = 2 - i$$

$$\begin{cases} 3a = a \\ -b = -1 \end{cases} \Leftrightarrow \begin{cases} a = 2/3 \\ b = 1 \end{cases} \Rightarrow z = \boxed{\frac{2}{3} + i}$$

6.8

$$a) 3z - i\bar{z} = 7 - 5i, z = a + bi$$

$$\Leftrightarrow 3a + 3bi - i(a - bi) = 7 - 5i$$

$$\Leftrightarrow 3a - b + (3b - a)i = 7 - 5i$$

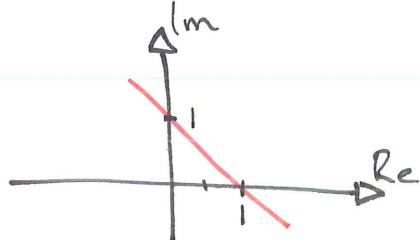
$$\Leftrightarrow \begin{cases} 3a - b = 7 \\ 3b - a = -5 \end{cases} \Leftrightarrow \begin{cases} 3a - b = 7 \\ 8b = -8 \end{cases} \Leftrightarrow \begin{cases} a = 2 \\ b = -1 \end{cases} \Rightarrow z = 2 - i$$

$$b) z \cdot 2\bar{z} = 1+i, z = a+bi$$

$$\Leftrightarrow 2(a^2 + b^2) = 1+i$$

$$\Rightarrow \begin{cases} 2a^2 + 2b^2 = 1 \\ 0 = 1 \end{cases} \Rightarrow \text{Lösning saknas}$$

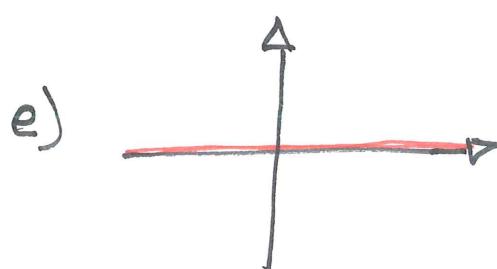
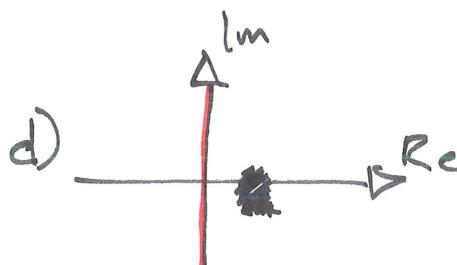
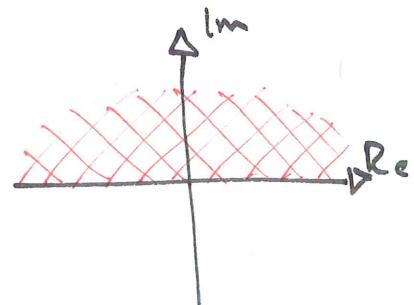
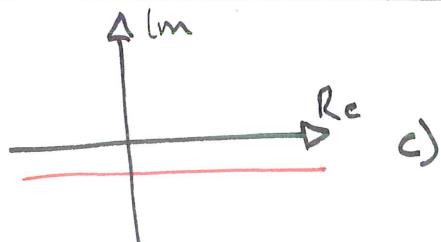
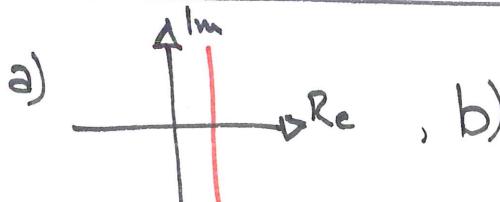
6.9



$$\operatorname{Re}(z) + \operatorname{Im}(z) = 1$$

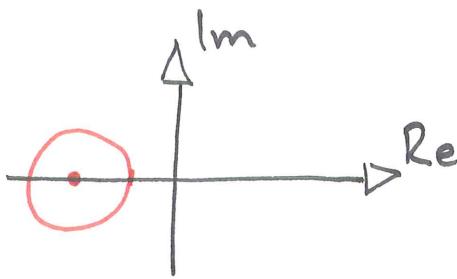
$$\operatorname{Im}(z) = 1 - \operatorname{Re}(z)$$

6.10



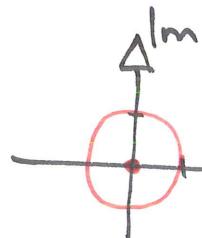
b.11

$$|z+2|=1$$

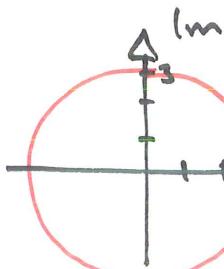


b.12

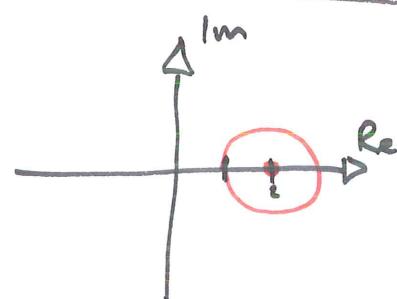
a)



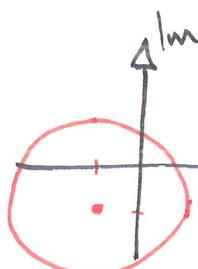
|b)



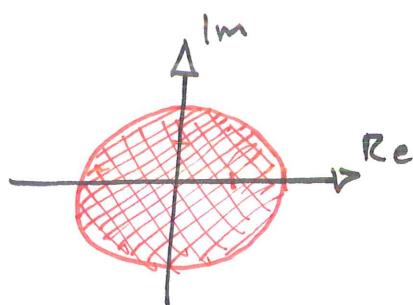
|c)



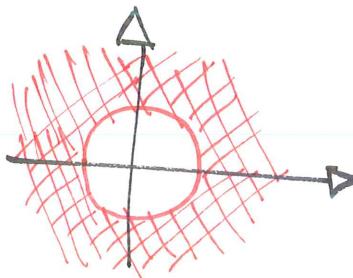
$$d) |z+1+i|=2 :$$



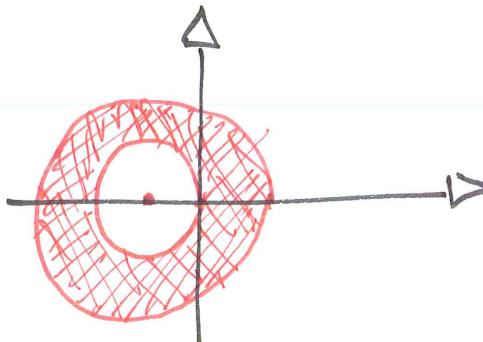
|e)



f)



g)



b.13

$$\begin{cases} |z-3i|=2 \\ z+\bar{z}=2 \end{cases}$$

$$, z = a + bi$$

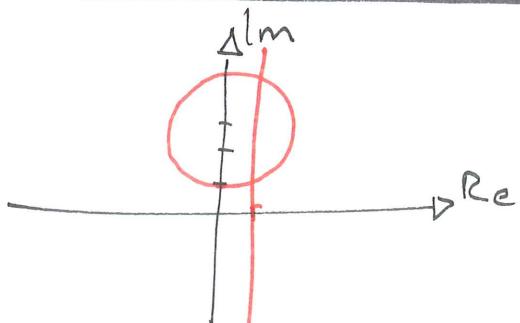
$$\hookrightarrow a = 1$$

$$|(1+(b-3)i)|=2$$

$$1+b^2-6b+9=4$$

$$b^2-6b+6=0$$

$$b = 3 \pm \sqrt{3}$$



$$\Rightarrow z = 1 + (3 \pm \sqrt{3})i$$

6.14

$$|z-1|=|z+1|, z=a+bi$$

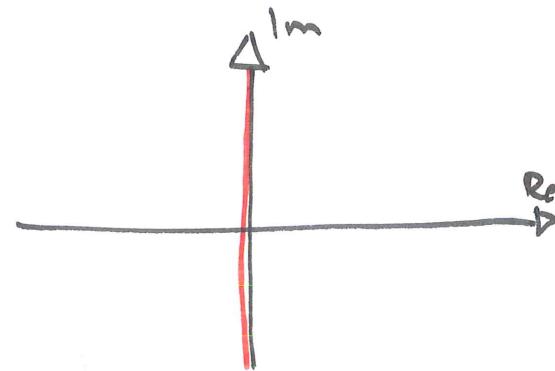
$$\Leftrightarrow |a-1+bi|=|a+1+bi|$$

$$\Leftrightarrow \sqrt{(a-1)^2 + (b)^2} = \sqrt{(a+1)^2 + (b)^2}$$

$$\Leftrightarrow a^2 - 2a + 1 - b^2 = a^2 + 2a + 1 - b^2$$

$$\Leftrightarrow 4a = 0 -$$

$\Leftrightarrow a = 0 \Rightarrow$ Alla z på imaginära axeln



6.15

$$|z-1|=2|z+1|, z=a+bi$$

$$\Leftrightarrow |a-1+bi|=2|a+1+bi|$$

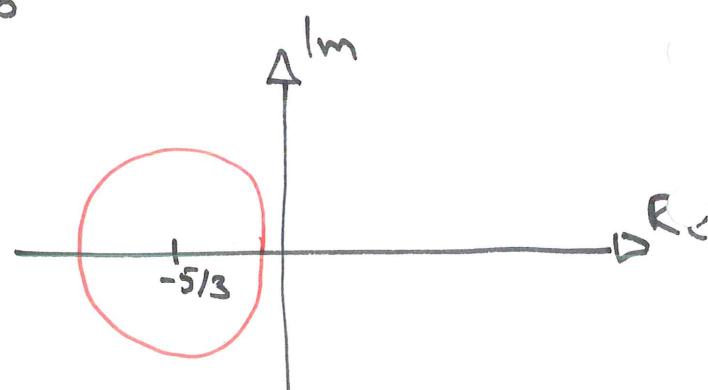
$$\Leftrightarrow \sqrt{(a-1)^2 + (b)^2} = 2\sqrt{(a+1)^2 + (b)^2}$$

$$\Leftrightarrow a^2 - 2a + 1 + b^2 = 4a^2 + 8a + 4 + 4b^2$$

$$\Leftrightarrow 3a^2 + 10a + 3b^2 + 3 = 0$$

$$\Leftrightarrow a^2 + \frac{10}{3}a + b^2 + 1 = 0$$

$$\Leftrightarrow \left(a + \frac{5}{3}\right)^2 + b^2 = \left(\frac{4}{3}\right)^2$$

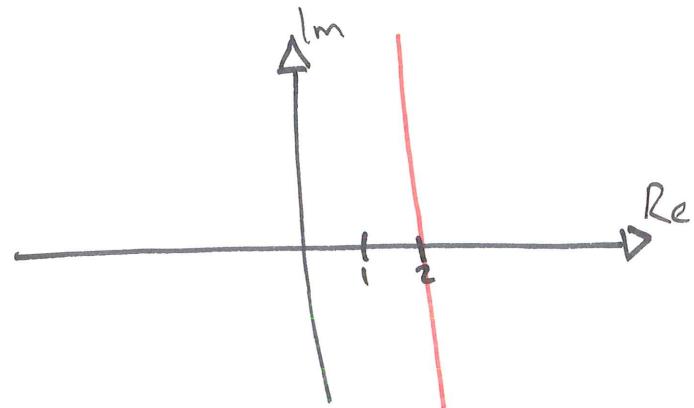


\Rightarrow cirkel med medelpunkt i $(-5/3, 0)$
och med radie $4/3$

6.16

$$\left| \frac{1}{z} - \frac{1}{4} \right| = \frac{1}{4}$$

$$\Leftrightarrow \left| \frac{4-z}{4z} \right| = \frac{1}{4}$$



$$\Leftrightarrow |4-z| = |z|, z = a+bi$$

$$\Leftrightarrow |4-a-bi| = |a+bi|$$

$$\Leftrightarrow \sqrt{(4-a)^2 + b^2} = \sqrt{a^2 + b^2}$$

$$\Leftrightarrow 16 - 8a + a^2 + b^2 = a^2 + b^2$$

$$\Leftrightarrow a = 2$$

$$\Rightarrow \boxed{\operatorname{Re}(z) = 2}$$

6.17

$$z + \frac{1}{z} = , z = a+bi, z \neq 0$$

$$= \frac{z^2 + 1}{z} = \frac{\bar{z}z^2 + \bar{z}}{|z|^2} = \frac{(a^2 + b^2)(a+bi) + (a-bi)}{|z|^2} =$$

$$= \frac{a^3 + a^2bi + ab^2 + b^3i + a - bi}{|z|^2} = \frac{a^3 + ab^2 + a + (a^2b + b^3 - b)i}{|z|^2}$$

$$\operatorname{Im}\left(z + \frac{1}{z}\right) = 0 \Rightarrow a^2b + b^3 - b = 0 \Leftrightarrow \begin{cases} b = 0 \\ a^2 + b^2 = 1 \Leftrightarrow |z| = 1 \end{cases}$$

svar: Alla tal på reella axeln utom origo samt enhetscirkeln.

6.18

a) $\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) = \boxed{1+i}$

b) $\cos \pi + i \sin \pi = \boxed{-1}$

c) $\sqrt{2} \left(\cos \frac{9\pi}{4} + i \sin \frac{9\pi}{4} \right) = \boxed{1+i}$

d) $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} = \boxed{i}$

e) $\cos 2\pi + i \sin 2\pi = \boxed{1}$

f) $\frac{1}{\sqrt{2}} \left(\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4}) \right) = \boxed{\frac{1}{2} - \frac{1}{2}i}$

g) $\cos(-100\pi) + i \sin(-100\pi) = \cos(0) + i \sin(0) = \boxed{1}$

6.22

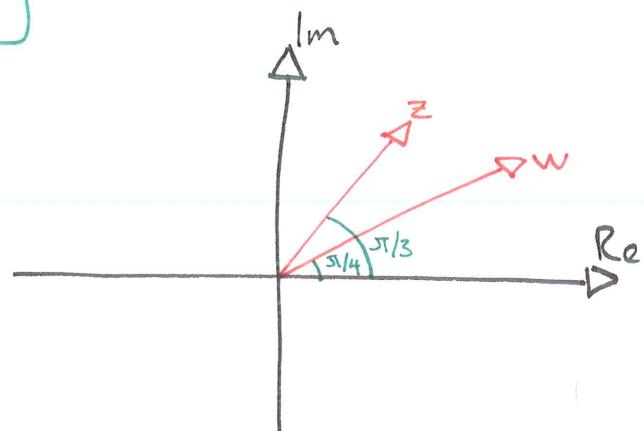
$\arg(z) = \pi/3$, $\arg(w) = \pi/4$

a) $\arg(zw) = \frac{\pi}{3} + \frac{\pi}{4} = \boxed{\frac{7\pi}{12}}$

b) $\arg(z/w) = \frac{\pi}{3} - \frac{\pi}{4} = \boxed{\frac{\pi}{12}}$

c) $\frac{\pi}{4} < \arg(z+w) < \frac{\pi}{3}$

$\frac{\pi}{3} < \arg(z-w) < \frac{3\pi}{4}$



6.23

$\arg(z) = \frac{\pi}{3}$, $\frac{2000}{3} = \frac{2\pi}{3} + 233 \cdot 2\pi$

svar: $\frac{2\pi}{3}$

6.24

$$z = \frac{1+i\sqrt{3}}{(2-2i)^3}, \quad \arg(1+i\sqrt{3}) = \frac{\pi}{3}$$

$$\arg(2-2i) = -\frac{\pi}{4}$$

$$\Rightarrow \arg\left(\frac{1+i\sqrt{3}}{(2-2i)^2}\right) = \arg(1+i\sqrt{3}) - 2\arg(2-2i) =$$

$$= \frac{\pi}{3} - 2\left(-\frac{\pi}{4}\right) = \frac{13\pi}{12}$$

svær: $\frac{13\pi}{12} + n \cdot 2\pi, n \in \mathbb{Z}$

6.25

$$\arg\left(\frac{(2+2i)(1+i\sqrt{3})}{3i(\sqrt{12}-2i)}\right) = \frac{\pi}{4} + \frac{\pi}{3} - \frac{\pi}{2} + \frac{\pi}{6} = \boxed{\frac{5\pi}{4}}$$

6.26

a) $\arg(1+2i\omega) = \boxed{\arctan(2\omega)}$

b) $\arg(-1+2i\omega) = \arctan(-2\omega) = \boxed{\pi - \arctan(2\omega)}$

c) $\arg\left(\frac{1}{1+2i\omega}\right) = -\arg(1+2i\omega) = \boxed{-\arctan(2\omega)}$

d) $\arg\left(\frac{1}{-1+2i\omega}\right) = -\arg(-1+2i\omega) = \boxed{\arctan(2\omega) - \pi}$

e) $\arg\left(\frac{e^{i\omega}}{(1+2i\omega)^2}\right) = \boxed{\omega - 2\arctan(2\omega)}$

6.27

$$\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{100} = \left(\cos \frac{\pi}{3} + i\sin \frac{\pi}{3}\right)^{100} = \\ = \cos \frac{100\pi}{3} + i\sin \frac{100\pi}{3} = \cos \frac{4\pi}{3} + i\sin \frac{4\pi}{3} =$$

$$= -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$

6.28

$$\cos 4\theta + i\sin 4\theta = e^{i4\theta} = (e^{i\theta})^4 = (\cos \theta + i\sin \theta)^4 =$$

$$= (\cos^2 \theta - \sin^2 \theta + 2i\sin \theta \cos \theta)^4 =$$

$$= \cos^4 \theta - \cos^2 \theta \sin^2 \theta + 2i\sin \theta \cdot \cos^3 \theta + \\ - \sin^2 \theta \cos^3 \theta + \sin^4 \theta - 2i\sin^3 \theta \cos \theta + \\ + 2i\sin \theta \cos^3 \theta - 2i\sin^3 \theta \cos \theta - 4\sin^2 \theta \cos^2 \theta =$$

$$= \cos^4 \theta + \sin^4 \theta - 6\sin^2 \theta \cos^2 \theta + \\ + i(4\sin \theta \cos^3 \theta - 4\sin^3 \theta \cos \theta)$$

$$\Rightarrow \boxed{\cos 4\theta = \cos^4 \theta + \sin^4 \theta - 6\sin^2 \theta \cos^2 \theta} \\ \boxed{\sin 4\theta = 4\sin \theta \cos^3 \theta - 4\sin^3 \theta \cos \theta}$$

6.29

$$\cos \alpha \cdot \sin \beta = \frac{e^{i\alpha} + e^{-i\alpha}}{2} \cdot \frac{e^{i\beta} - e^{-i\beta}}{2i} = \frac{1}{4i} \left(e^{i(\alpha+\beta)} - e^{i(\alpha-\beta)} + e^{i(\beta-\alpha)} - e^{i(-\alpha-\beta)} \right) =$$

$$= \frac{1}{4i} \left(\cancel{\cos(\alpha+\beta) + i\sin(\alpha+\beta)} - \cancel{\cos(\alpha-\beta) - i\sin(\alpha-\beta)} + \right.$$

$$\left. + \cancel{\cos(\beta-\alpha) + i\sin(\beta-\alpha)} - \cancel{\cos(-(α+β)) - i\sin(-(α+β))} \right) =$$

$$= \frac{1}{4i} (2i\sin(\alpha+\beta) + 2i\sin(\alpha-\beta)) =$$

$$= \frac{1}{2i} (\sin(\alpha+\beta) - \sin(\alpha-\beta)) =$$

$$= \boxed{\frac{1}{2} (\sin(\alpha+\beta) - \sin(\alpha-\beta))}$$

6.30

$$\sin^4 \theta = \frac{1}{(2i)^4} \cdot (e^{i\theta} - e^{-i\theta})^4 = \frac{1}{16} (e^{i\theta 2} - 2e^{\theta} + e^{-i\theta 2}) =$$

$$= \frac{1}{16} (e^{i4\theta} - 4e^{i2\theta} + 4 + 2(e^{\theta} - 2e^{-i\theta 2}) + e^{-i4\theta}) =$$

$$= \frac{1}{16} (e^{i4\theta} + e^{-i4\theta} - 4e^{i2\theta} - 4e^{-i2\theta} + 6) =$$

$$= \frac{1}{16} (\cos 4\theta + i\sin 4\theta + \cos(-4\theta) + i\sin(-4\theta) +$$

$$- 4(\cos 2\theta + i\sin 2\theta + \cos(-2\theta) + i\sin(-2\theta) + 6) =$$

$$= \frac{1}{16} (2\cos 4\theta - 8\cos 2\theta + 6) = \boxed{\frac{1}{8} (\cos 4\theta - 4\cos 2\theta + 3)}$$

6.31 a) i , b) $z = -3+2i$, $|z| = \sqrt{13}$, $\arg(z) = -\arctan(\frac{2}{3}) + \pi$

$$z_{\perp} = -2-3i$$

6.32

a) $z = 1$, $\arg(z) = 0$, $|z| = 1$

$$\arg(z_c) = \frac{5\pi}{6} \quad |z_c| = 3$$

$$z_c = 3 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) = 3 \left(-\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2} \right) = \boxed{-\frac{3\sqrt{3}}{2} + i \frac{3}{2}}$$

b) $z = -1+i$, $\arg(z) = \frac{3\pi}{4}$, $|z| = \sqrt{2}$

$$\arg(z_c) = \frac{19\pi}{12} \quad |z_c| = 3\sqrt{2}$$

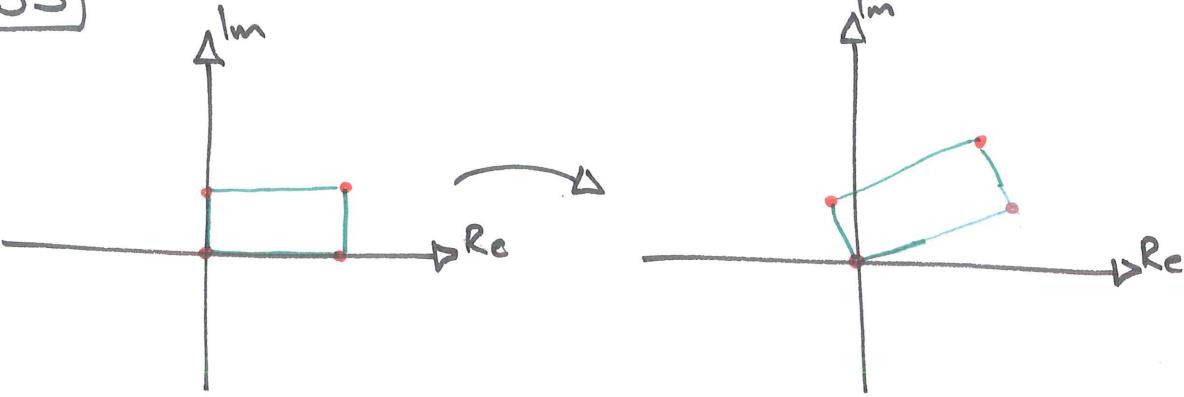
$$z_c = 3\sqrt{2} \left(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12} \right)$$

$$3e^{i\frac{5\pi}{6}} \cdot \sqrt{2} \cdot e^{i\frac{3\pi}{4}} = 3\sqrt{2} \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) =$$

$$= 3\sqrt{2} \left(-\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2} \right) \left(-\frac{1}{\sqrt{2}} + i \cdot \frac{1}{\sqrt{2}} \right) = 3 \left(-\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2} \right) \left(-1 + i \right) =$$

$$= 3 \left(\frac{\sqrt{3}}{2} - \frac{1}{2} + i \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} \right) \right) = \boxed{\frac{3}{2} (\sqrt{3}-1 - i(\sqrt{3}+1))}$$

6.33



$$\begin{aligned} 2 &\rightarrow 7+i \\ 0 &\rightarrow 0 \\ i &\rightarrow -\frac{1}{2} + \frac{\sqrt{3}}{2}i \\ 2+i &\rightarrow \frac{13}{2} + \frac{9}{2}i \end{aligned}$$

$$\arg(z_e) = \arctan \frac{1}{7}$$

$$k = \frac{\sqrt{50}}{2}$$

6.34

a) $e^0 = \boxed{1}$

b) $e^{i\frac{\pi}{2}} = \boxed{i}$

c) $e^{\frac{1}{2}\ln 2 + i\frac{\pi}{4}} = e^{\frac{1}{2}\ln 2} \cdot e^{i\frac{\pi}{4}} = e^{\frac{1}{2}\ln 2} \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = e^{\ln \sqrt{2}} \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = \boxed{1+i}$

d) $e^{i\pi} + 1 = 0 \Leftrightarrow \boxed{e^{i\pi} = -1}$

e) $e^{3-i} = e^3 \cdot e^{-i} = e^3 (\cos(-1) + i \sin(-1)) = \boxed{e^3 \cos 1 - e^3 i \sin 1}$

6.35

a) $f(x) = \frac{1+ix}{1-ix} = \frac{(1+ix)^2}{1+x^2} = \frac{1-x^2+2ix}{1+x^2}$

$$\boxed{\operatorname{Re}(f) = \frac{1-x^2}{1+x^2}, \quad \operatorname{Im}(f) = \frac{2x}{1+x^2}}$$

b) $g(x) = e^{(-1+i)x} = e^{-x+ix} = e^{-x} \cdot e^{ix} =$

$$= e^{-x}(\cos x + i \sin x) \Rightarrow \boxed{\operatorname{Re}(g) = e^{-x} \cos x, \quad \operatorname{Im}(g) = e^{-x} \sin x}$$

6.36

$$z = x + iy$$

$$|e^z| = |e^{x+iy}| = e^x \cdot |e^{iy}| = e^x |\cos y + i \sin y| =$$

$$= e^x \sqrt{\cos^2 y + \sin^2 y} = e^x \cdot 1 = \boxed{e^x}$$

#

6.37

$$\text{a) } z^2 = 5 + 12i, \quad z = a + bi \Rightarrow a^2 - b^2 + 2abi = 5 + 12i$$

$$\begin{cases} a^2 - b^2 = 5 \\ 2ab = 12 \end{cases} \Leftrightarrow \begin{cases} b^2 = 4 \\ a^2 = 9 \end{cases} \Leftrightarrow \begin{cases} b = \pm 2 \\ a = \pm 3 \end{cases}$$

$$\Rightarrow \boxed{z = \pm(3+2i)}$$

$$\text{b) } z^2 - (2+2i)z - 5-10i = 0$$

Pq-Formeln

$$z = 1+i \pm \sqrt{(1+i)^2 + 5+10i} = 1+i \pm \sqrt{\underbrace{5+12i}_{(3+2i)^2}} = 1+i \pm \sqrt{(3+2i)^2}$$

$$\Leftrightarrow z = 1+i \pm 3+2i$$

(testa, det fungerar)

$$\Leftrightarrow \boxed{z_1 = 4+3i}$$

$$\boxed{z_2 = -2-i}$$

6.38

$$\text{a) } z^2 = -i \quad , \quad z = a + bi \quad , \quad z^2 = a^2 - b^2 + 2abi$$

$$\begin{array}{l} \text{Re}(z) \\ |z| \\ \text{Im}(z) \end{array} \left\{ \begin{array}{l} a^2 - b^2 = 0 \\ a^2 + b^2 = 1 \\ 2ab = -1 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} a^2 = 1/2 \\ b^2 = 1/2 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} z_1 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \\ z_2 = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \end{array} \right. \Rightarrow z = \pm \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right)$$

$$\text{b) } z^2 = 1 + i\sqrt{3}$$

$$\begin{array}{l} |z| \\ \text{Re}(z) \\ \text{Im}(z) \end{array} \left\{ \begin{array}{l} a^2 + b^2 = 2 \\ a^2 - b^2 = 1 \\ 2ab = \sqrt{3} \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} a^2 = 3/2 \\ b^2 = 1/2 \end{array} \right. \Rightarrow z = \pm \frac{1}{\sqrt{2}} (\sqrt{3} + i)$$

$$\text{c) } z^2 = 4i + 3$$

$$\left\{ \begin{array}{l} a^2 + b^2 = 5 \\ a^2 - b^2 = 3 \\ 2ab = 4 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} a^2 = 4 \\ b^2 = 1 \\ ab > 0 \end{array} \right. \Rightarrow z = \pm (2+i)$$

$$6.39 \quad z^2 + 2iz - 1 + 2i = 0 \quad \text{Pq: } z = -i \pm \sqrt{(-i)^2 + 1 - 2i} = -i \pm \sqrt{-2i}$$

$$w^2 = 2i - 2 \quad , \quad w = a + bi$$

$$\left\{ \begin{array}{l} a^2 + b^2 = 2\sqrt{2} \\ a^2 - b^2 = -2 \\ 2ab = 2 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} a^2 = \sqrt{2} - 1 \\ b^2 = \sqrt{2} + 1 \\ ab = 1 > 0 \end{array} \right. \Rightarrow w = \pm \left(\sqrt{\sqrt{2} - 1} + \sqrt{\sqrt{2} + 1} \right)$$

$$z = x + yi$$

$$x^2 + y^2 + 2xyi + 2xi - 2y - 1 + 2i = 0$$

$$\left\{ \begin{array}{l} x^2 - y^2 - 2y - 1 = 0 \\ xy + x + 2 = 0 \end{array} \right. \Rightarrow x = -\frac{2}{y+1}$$

$$\frac{4}{(y+1)^2} - y^2 - 2y - 1 = 0 \quad , \quad \boxed{y \neq -1}$$

$$4 - y^2(y^2 + 2y + 1) + 2y(y^2 + 2y + 1) - y^2 - 2y - 1 = 0, \quad y \neq -1$$

$$\Leftrightarrow 4 - y^2 - 2y^3 - y^2 - 2y^3 - 4y^2 - 2y - y^2 - 2y - 1 = 0$$

$$\Leftrightarrow 3 - y^4 - 4y^3 - 6y^2 - 4y = 0$$

6.39

$$b) z^2 + (2-2i)z - 6i - 3 = 0$$

$$\text{pq: } z = i - 1 \pm \sqrt{-1 - 2i + 1 + 6i + 3} = i - 1 \pm \sqrt{4i + 3} = i - 1 \pm (2+i)$$

$$\Rightarrow z_1 = 2i + 1$$

$$z_2 = -3$$

6.40

$$(2+i)z^2 + (1-7i)z - 5 = 0 \Leftrightarrow z^2 + \frac{1-7i}{2+i}z - \frac{5}{2+i} = 0$$

$$\frac{1-7i}{2+i} = \frac{(1-7i)(2-i)}{5} = \frac{2-i-14i+7}{5} = \frac{5+15i}{5} = -(1+3i)$$

$$\frac{5}{2+i} = \frac{5 \cdot (2-i)}{5} = 2-i$$

Vi sätter in

$$z^2 - (1+3i)z - (2-i) = 0$$

$$z = \frac{1+3i}{2} \pm \sqrt{\frac{1+6i-9}{4} + \frac{8-4i}{4}} = \frac{1+3i}{2} \pm \sqrt{\frac{2i}{2}} = \frac{1+3i \pm \sqrt{2i}}{2}$$

Ansatz $\omega^2 = 2i$, $\omega = a+bi$

$$\begin{cases} a^2 - b^2 = 0 \\ a^2 + b^2 = 2 \\ 2ab = 2 \end{cases} \Rightarrow \begin{cases} a^2 = 1 \\ b^2 = 1 \\ ab = 1 > 0 \end{cases} \Rightarrow \omega = \begin{cases} 1+i \\ -(1+i) \end{cases} = \pm(1+i)$$

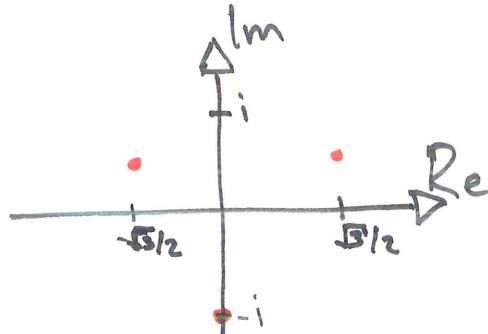
$$\Rightarrow z = \frac{1+3i \pm (1+i)}{2} = \begin{cases} 1+2i \\ i \end{cases}$$

6.41

$$a) z^3 = r^3 \cdot e^{i3\theta}, \quad i = e^{i(\frac{\pi}{2} + n \cdot 2\pi)}, \quad n \in \mathbb{Z}$$

$$r=1, 3\theta = \frac{\pi}{2} + n \cdot 2\pi \Leftrightarrow \theta = \frac{\pi}{6} + n \cdot \frac{2\pi}{3}$$

$$z = \cos\left(\frac{\pi}{6} + n \cdot \frac{2\pi}{3}\right) + i \sin\left(\frac{\pi}{6} + n \cdot \frac{2\pi}{3}\right), \quad n \in \mathbb{Z}$$



$$\begin{aligned} z_1 &= \frac{\sqrt{3}}{2} + \frac{1}{2}i \\ z_2 &= -\frac{\sqrt{3}}{2} + \frac{1}{2}i \\ z_3 &= -i \end{aligned}$$

$$b) z^3 = 1+i, \quad z = r e^{i\theta} \Rightarrow z^3 = r^3 e^{i3\theta}$$

$$1+i = \sqrt{2} \cdot e^{i(\frac{\pi}{4} + n \cdot 2\pi)}, \quad n \in \mathbb{Z}$$

$$z^3 = 1+i \Leftrightarrow r^3 e^{i3\theta} = \sqrt{2} e^{i(\frac{\pi}{4} + n \cdot 2\pi)}$$

$$\Rightarrow r^3 = \sqrt{2} \Leftrightarrow r = 2^{\frac{1}{6}}$$

$$3\theta = \frac{\pi}{4} + n \cdot 2\pi \Leftrightarrow \theta = \frac{\pi}{12} + n \cdot \frac{2\pi}{3}$$

$$\begin{cases} \theta_1 = \frac{5\pi}{12} \\ \theta_2 = \frac{9\pi}{12} = \frac{3\pi}{4} \\ \theta_3 = \frac{17\pi}{12} \end{cases} \Rightarrow$$

$$\begin{cases} z_1 = 2^{\frac{1}{6}} \cdot e^{i\frac{5\pi}{12}} \\ z_2 = 2^{\frac{1}{6}} \cdot e^{i\frac{3\pi}{4}} \\ z_3 = 2^{\frac{1}{6}} \cdot e^{i\frac{17\pi}{12}} \end{cases}$$

$$c) z^4 = 16 \quad , \quad z = r e^{i\theta} \Rightarrow z^4 = r^4 e^{i4\theta}$$

$$16 = 2^4 e^{i(n \cdot 2\pi)}$$

$$z^4 = 16 \Leftrightarrow r^4 \cdot e^{i4\theta} = 2^4 \cdot e^{i(n \cdot 2\pi)}$$

$$\Rightarrow r = 2 \quad \begin{cases} z_1 = 2 \\ z_2 = 2i \\ z_3 = -2 \\ z_4 = -2i \end{cases} \Rightarrow z = \begin{cases} \pm 2 \\ \pm 2i \end{cases}$$

$$d) z^3 = i\sqrt{3} - 1 \quad , \quad z = r e^{i\theta} \Rightarrow z^3 = r^3 e^{i3\theta}$$

$$i\sqrt{3} - 1 = 2 e^{i(\frac{2\pi}{3} + n \cdot 2\pi)}, \quad n \in \mathbb{Z}$$

$$r^3 e^{i3\theta} = 2 e^{i(\frac{2\pi}{3} + n \cdot 2\pi)}$$

$$r = 2^{\frac{1}{3}}$$

$$\Theta = \frac{2\pi}{9} + n \cdot \frac{2\pi}{3}$$

$$\begin{cases} \Theta_1 = \frac{2\pi}{9} \\ \Theta_2 = \frac{8\pi}{9} \\ \Theta_3 = \frac{14\pi}{9} \end{cases} \Rightarrow \begin{cases} z_1 = 2^{\frac{1}{3}} \cdot e^{i\frac{2\pi}{9}} \\ z_2 = 2^{\frac{1}{3}} \cdot e^{i\frac{8\pi}{9}} \\ z_3 = 2^{\frac{1}{3}} \cdot e^{i\frac{14\pi}{9}} \end{cases}$$

$$e) z^5 = 4i \quad , \quad z = r e^{i\theta} \quad , \quad 4i = 4e^{i(\frac{\pi}{2} + n \cdot 2\pi)}, \quad n \in \mathbb{Z}$$

$$r^5 \cdot e^{i5\theta} = 4e^{i(\frac{\pi}{2} + n \cdot 2\pi)}$$

$$\Rightarrow r = 4^{1/5}, \quad \theta = \frac{\pi}{10} + n \cdot \frac{2\pi}{5}$$

$$\Rightarrow z = 4^{1/5} \cdot \begin{cases} e^{i\pi/10} \\ e^{i\pi/2} = i \\ e^{-i9\pi/10} \\ e^{i13\pi/10} \\ e^{i17\pi/10} \end{cases}$$

$$f) z^4 = -1 \quad , \quad z = r e^{i\theta} \quad , \quad -1 = 1 \cdot e^{i(\pi + n \cdot 2\pi)}, \quad n \in \mathbb{Z}$$

$$r^4 e^{i4\theta} = e^{i(\pi + n \cdot 2\pi)}, \quad r = 1, \quad \theta = \frac{\pi}{4} + n \cdot \frac{\pi}{2}$$

$$\Rightarrow z = \frac{1}{\sqrt[4]{2}} \begin{cases} 1+i \\ -1+i \\ -1-i \\ 1-i \end{cases}$$

6.42

$$z^6 - 2z^3 + 2 = 0, w = z^3 \Rightarrow w^2 - 2w + 2 = 0$$

$$\text{Pq: } w = 1 \pm i, z^3 = w = 1 \pm i$$

$$r^3 \cdot e^{i\frac{3\pi}{2}} = \sqrt{2} e^{i(\frac{\pi}{4} + n \cdot 2\pi)}, n \in \mathbb{Z}$$

$$r = 2^{\frac{1}{6}}$$

$$\Theta_1 = \frac{\pi}{12} + n \cdot \frac{2\pi}{3}$$

$$\Theta_2 = -\frac{\pi}{12} + n \cdot \frac{2\pi}{3}$$

$$\Rightarrow z = 2^{\frac{1}{6}} \left\{ \begin{array}{l} e^{i\frac{5\pi}{12}} \\ e^{i\frac{9\pi}{12}} = e^{i\frac{3\pi}{4}} \\ e^{i\frac{13\pi}{12}} \\ e^{-i\frac{5\pi}{12}} \\ e^{i\frac{7\pi}{12}} \\ e^{i\frac{5\pi}{12}} \end{array} \right.$$

6.43

$$(1+z^2)^3 = -8, w = 1+z^2, w^3 = -8$$

$$w = r \cdot e^{i\theta}, r^3 \cdot e^{i\frac{3\pi}{2}} = 2 \cdot e^{i(\pi + n \cdot 2\pi)}, n \in \mathbb{Z}$$

$$\left\{ \begin{array}{l} w_1 = 2 e^{i\frac{\pi}{3}} = 1 + \sqrt{3}i \\ w_2 = 2 e^{i\pi} = -2 \\ w_3 = 2 e^{i\frac{5\pi}{3}} = 1 - \sqrt{3}i \end{array} \right.$$

$$w = 1+z^2 \Leftrightarrow z^2 = w-1$$

$$\left\{ \begin{array}{l} z_1^2 = \sqrt{3}i \\ z_2^2 = -3 \Rightarrow z_2 = \pm i\sqrt{3} \\ z_3^2 = -\sqrt{3}i \end{array} \right.$$

$$z_1 = a+bi \quad , \quad a^2 - b^2 + 2abi = \sqrt{3}i$$

$$\begin{cases} a^2 - b^2 = 0 \\ a^2 + b^2 = \sqrt{3} \\ 2ab = \sqrt{3} \end{cases} \Leftrightarrow \begin{cases} a^2 = \frac{\sqrt{3}}{2} \\ b^2 = \frac{\sqrt{3}}{2} \\ ab = \frac{\sqrt{3}}{2} > 0 \end{cases} \Rightarrow z_1 = \pm \sqrt{\frac{\sqrt{3}}{2}} \cdot (1+i)$$



$$z_3 = x+iy, \quad x^2 - y^2 + 2xyi = -\sqrt{3}i$$

$$\begin{cases} x^2 - y^2 = 0 \\ x^2 + y^2 = \sqrt{3} \\ 2xy = -\sqrt{3} \end{cases} \Leftrightarrow \begin{cases} x^2 = \frac{\sqrt{3}}{2} \\ y^2 = \frac{\sqrt{3}}{2} \\ xy = -\frac{\sqrt{3}}{2} < 0 \end{cases} \Rightarrow z = \pm \sqrt{\frac{\sqrt{3}}{2}} (1-i)$$

$$\Rightarrow z = \left(\frac{3}{4}\right)^{1/4} (\pm 1 \pm i) \text{ eller } z = \pm \sqrt{3}i$$