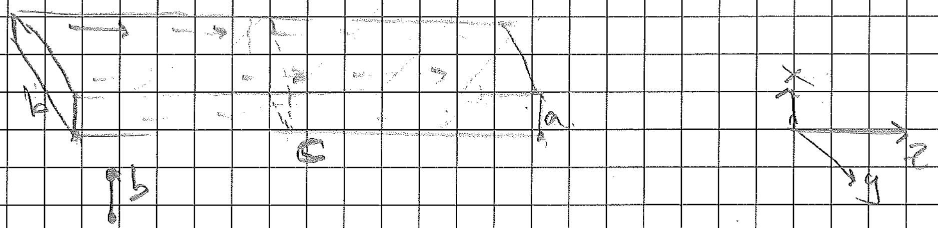
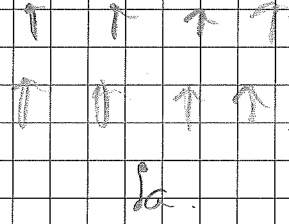
6.3

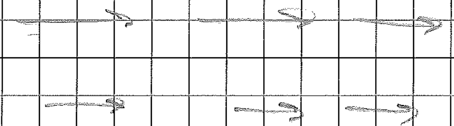


a)



$$R = \frac{l}{\sigma A} = \frac{a}{\sigma bc}$$

b)



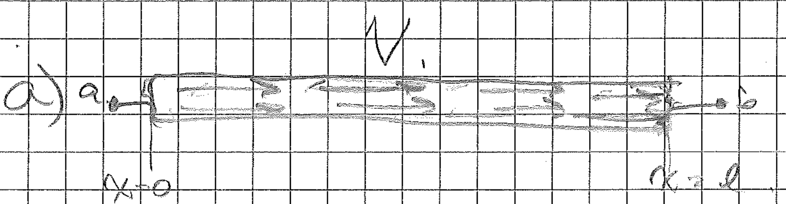
$$R = \frac{l}{\sigma A} = \frac{c}{\sigma ab}$$

$$\frac{c >> a}{bc >> ab} \Rightarrow R_{b)} \text{ är störst.}$$

A

6.4

a)



$$\sigma(x) = \sigma_0\left(1+\frac{x}{l}\right)$$

b) $i_{0l} = \int_A \vec{J}\cdot\vec{e}_n \, dS = J_0 A$

$$\vec{E}(x) = \frac{J_0 \vec{e}_x}{\sigma(x)}$$

Detta ger: $V_0 - V_l = \int_0^l \vec{E}\, d\vec{r} =$

$$= \int_0^l \left(\frac{J_0 \vec{e}_x}{\sigma(x)}\right) \vec{e}_x \, dx = \int_0^l \frac{J_0}{\sigma(x)} dx = \frac{J_0}{\sigma_0}\int_0^l \frac{1}{1+\frac{x}{l}} dx =$$

$$= \frac{lJ_0}{\sigma_0}\int_0^l \frac{1}{l+x} dx = \frac{lJ_0}{\sigma_0}\Big[\ln(l+x)\Big]_0^l =$$

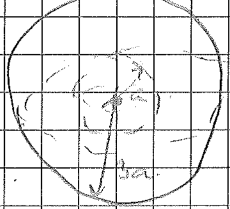
$$= \frac{lJ_0}{\sigma_0}\left(\ln(2l) - \ln(l)\right) = \frac{lJ_0}{\sigma_0}\ln 2.$$

Detta ger slutligen: $R = \dfrac{\frac{lJ_0}{\sigma_0}\ln 2}{J_0 A} = \dfrac{l \cancel{J_0} \ln 2}{\cancel{J_0} A \sigma_0} =$

$$= \frac{l}{\sigma_0 A}\ln 2.$$

c) $\dfrac{dP}{dV} = \vec{J}\cdot\vec{E} = \dfrac{i}{A}\vec{e}_x \cdot \dfrac{\frac{i}{A}\vec{e}_x}{\sigma(x)} = \dfrac{1}{\sigma_0\left(1+\frac{x}{l}\right)}\left(\dfrac{i}{A}\right)^2$

6.5



$$\sigma(r) = \sigma_0 \frac{r}{a} \qquad a<r<3a$$

$$\vec{J} = J(r)\vec{e}_r$$

$$i = \oint_S \vec{J}\cdot\vec{e}_n \, dS = \int_{r=b} J(r)\vec{e}_r\cdot\vec{e}_r \, dS = J(b)\, 4\pi b^2 \quad \text{där } a<b<3a$$

Alltså $J(r) = \dfrac{i}{4\pi r^2} \quad a<r<3a$

$$V_a - V_b = \int_a^{3a} \frac{1}{\sigma}\vec{J}\cdot d\vec{r} = \int_a^{3a} \frac{a}{\sigma_0 r}\,\frac{i}{4\pi r^2}\,\vec{e}_r\cdot\vec{e}_r \, dr =$$

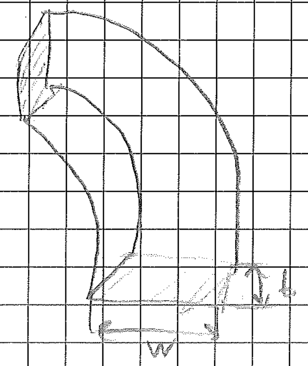
$$= \frac{i\cdot a}{\sigma_0 \cdot 4\pi} \int_a^{3a} \frac{1}{r^3}\, dr = \frac{ai}{4\pi\sigma_0}\left[-\frac{1}{2r^2}\right]_a^{3a} =$$

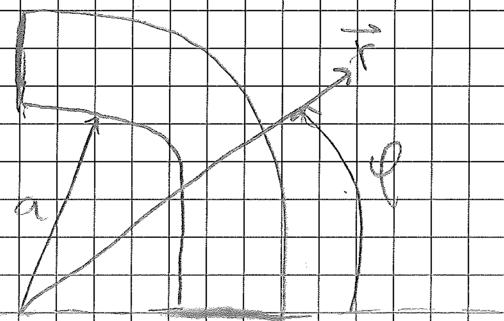
$$= \frac{ai}{4\pi\sigma_0}\left(-\frac{1}{18a^2} + \frac{1}{2a^2}\right) =$$

$$= \frac{\cancel{a}i}{\cancel{4}\pi\sigma_0}\,\frac{\cancel{8}}{18a^{\cancel{2}}} = \boxed{\frac{i}{9\pi\sigma_0 a}}$$

Detta ger: $R = \dfrac{V_a - V_{3a}}{i} = \dfrac{1}{9\pi\sigma_0 a}$.

6.6



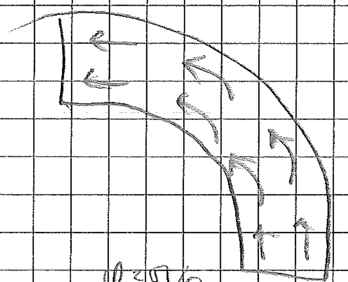


$A = Wt$ Ledningsförmåga: σ

$a < r < a+W$

$\vec{E} = E_0 \frac{a}{r} \vec{e}_\varphi \quad a<r<a+W$

a)



b) $V_a - V_b = \int_{\varphi=0}^{\varphi=\pi/2} E_0 \frac{a}{\cancel{r_0}} \vec{e}_\varphi \cdot \cancel{r_0} \vec{e}_\varphi \, d\varphi =$ $\quad d\vec{r} = r_0 \vec{e}_\varphi$

$= E_0 \cdot a \int_0^{\pi/2} d\varphi = E_0 \cdot a \left[\varphi \right]_0^{\pi/2} = \frac{\pi E_0 a}{2}$

$I_{ab} = \int_a^{a+W} \sigma E_0 \frac{a}{r} \vec{e}_\varphi \cdot \vec{e}_\varphi \cdot t\,dr = \sigma E_0 a t \int_a^{a+W} \frac{1}{r} dr =$

$= \sigma E_0 a t \left[\ln(r) \right]_a^{a+W} = \sigma E_0 a t \left(\ln(a+W) - \ln(a) \right) =$

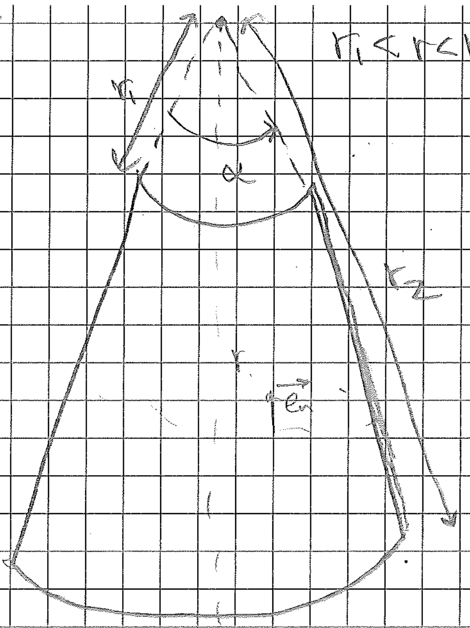
$= \sigma E_0 a t \ln \frac{a+W}{a}$

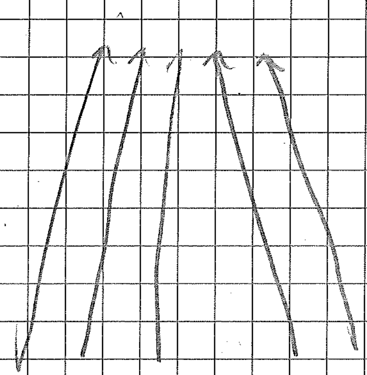
Detta ger resistansen:

$R = \frac{V_a - V_b}{I_{ab}} = \frac{\pi \cancel{E_0 a}}{2\sigma \cancel{E_0 a} t \ln \frac{a+W}{a}} = \frac{\pi}{2\sigma t \ln \frac{a+W}{a}}$

6.8

$r_1<r<r_2$ $\qquad$ $\sigma(\alpha)=\sigma$





$\vec{e}_n=-\vec{e}_r$

$\vec{J}=J(r)\vec{e}_r$

$\vec{E}=\frac{1}{\sigma}J(r)\vec{e}_r$

$i=\int_S \vec{J}\,\vec{e}_n\,dS=-t\int_0^{\alpha} J(r)\vec{e}_r\cdot\vec{e}_r\,d\varphi=-t\,r\,J(r)\int d\varphi=$

$=-t\,r\,J(r)\,\alpha \Rightarrow J(r)=-\frac{i}{t\,r\,\alpha}$

$\Rightarrow \vec{E}=-\frac{i}{\sigma\,t\,r\,\alpha}\vec{e}_r$

$V_a-V_b=\int_{r_1}^{r_2}\vec{E}\cdot d\vec{r}=\int_{r_1}^{r_2}-\frac{i}{\sigma\,t\,r\,\alpha}\cancel{\vec{e}_r}\cdot\cancel{\vec{e}_r}\,dr=$

$=\frac{i}{\sigma\,t\,\alpha}\int_{r_1}^{r_2}\frac{1}{r}\,dr=\frac{i}{\sigma\,t\,\alpha}\ln\frac{r_2}{r_1}$

$R=\frac{V_a-V_b}{i}=\frac{1}{\sigma\,t\,\alpha}\ln\frac{r_2}{r_1}$