5.42 Homogen skiva $\Rightarrow$ samma densitet.

Massan halvcirkeln:

$$m_1 = \rho \cdot \frac{1}{2}\pi R^2$$

Massa triangeln:

$$m_2 = \rho \cdot \frac{2R \cdot 2R}{2} = \rho 2R^2$$

Masscentrum blir (X-axeln):

$$X_{h} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{-\frac{1}{2}\pi \cancel{R^2} \cdot \frac{4R}{3\pi} + 2\cancel{R^2} \cdot \frac{2R}{3}}{\frac{1}{2}\pi \cancel{R^2} + 2\cancel{R^2}}$$

$$= \frac{-\frac{2R}{3} + \frac{4R}{3}}{\frac{\pi}{2} + 2} = \frac{\frac{2R}{3}}{\frac{\pi}{2} + \frac{4}{2}} = \frac{\frac{2R}{3}}{\frac{\pi + 4}{2}} = \frac{2R}{3} \cdot \frac{2}{\pi + 4}$$

$$= \frac{4R}{3\pi + 12}$$

5.4.5 Pga symmetri ligger masscentrum i $y_G = z_G = 0$.

Bestämmer x_G:

$$\frac{R}{r} = \frac{H}{h} \Rightarrow r = \frac{Rh}{H}$$

Massa lilla kon:

$$m_1 = \rho \cdot \frac{\pi r^2 \cdot h}{3}, \quad x_{G1} = \frac{3h}{4}$$

Massa stora kon:

$$m_2 = \rho \cdot \frac{\pi R^2 \cdot H}{3}, \quad x_{G2} = \frac{3H}{4}$$

Massa vår kon:

$$m_3 = \rho \cdot \left(\frac{\pi R^2 \cdot H}{3} - \frac{\pi r^2 \cdot h}{3}\right), \quad x_{G3} = ?$$

$$\frac{3H}{4} = \frac{\frac{\cancel{\pi} r^2 \cdot h}{\cancel{3}} \cdot \frac{3h}{4} + \left(\frac{\cancel{\pi} R^2 \cdot H}{\cancel{3}} - \frac{\cancel{\pi} r^2 \cdot h}{\cancel{3}}\right) x_{G3}}{\frac{\cancel{\pi} r^2 \cdot h}{\cancel{3}} + \left(\frac{\cancel{\pi} R^2 \cdot H}{\cancel{3}} - \frac{\cancel{\pi} r^2 \cdot h}{\cancel{3}}\right)}$$

$$\frac{3H}{4} = \frac{\frac{r^2 \cdot h \cdot 3h}{4} + \frac{4(R^2 H - r^2 \cdot h)}{4} x_{G3}}{\cancel{r^2 \cdot h} + (R^2 \cdot H - \cancel{r^2 \cdot h})}$$

$$3H = \frac{r^2 \cdot h \cdot 3h + 4R^2 H x_{G3} - 4r^2 h \cdot x_{G3}}{R^2 H}$$

$$3H^2 R^2 = r^2 \cdot h \cdot 3h + 4R^2 H x_{G3} - 4r^2 h \cdot x_{G3}$$

$$3H^2 R^2 = \frac{R^2 h^2}{H^2} \cdot h \cdot 3h + 4R^2 H x_{G3} - 4\frac{R^2 h^2}{H^2} \cdot h \cdot x_{G3}$$

S.4.5 (Fortsättning)

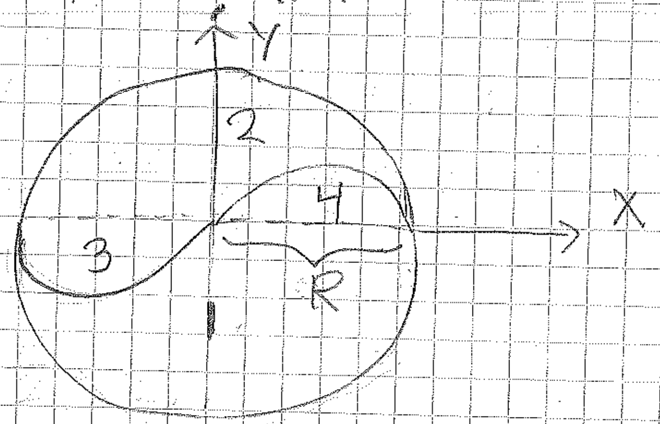
$$3H^4R^2 = R^2h^4 \cdot 3 + 4R^2H^3x_{43} - 4R^2h^3 \cdot x_{43}$$

$$3H^4R^2 - R^2h^4 \cdot 3 = x_{43}(4R^2H^3 - 4R^2h^3)$$

$$x_{43} = \frac{3H^4\cancel{R^2} - \cancel{R^2}h^4 \cdot 3}{4\cancel{R^2}H^3 - 4\cancel{R^2}h^3} = \frac{3H^4 - 3h^4}{4H^3 - 4h^3}$$

$$x_{43} = \frac{3}{4}\left(\frac{H^4 - h^4}{H^3 - h^3}\right)$$

S.4.10



1: $x_1 = 0$ $m_1 = \rho \cdot \frac{\pi R^2}{2}$

$y_1 = -\frac{4R}{3\pi}$

2: $x_2 = 0$ $m_2 = -\rho \cdot \frac{\pi R^2}{2}$

$y_2 = \frac{4R}{3\pi}$

3: $x_3 = -\frac{R}{2}$ $m_3 = -\rho \frac{\pi}{2}\left(\frac{R}{2}\right)^2 = -\frac{\rho\pi}{8}R^2$

$y_3 = -\frac{4R}{6\pi} = -\frac{2R}{3\pi}$

4: $x_4 = \frac{R}{2}$ $m_4 = \rho \frac{\pi}{2}\left(\frac{R}{2}\right)^2 = \frac{\rho\pi}{8}R^2$

$y_4 = \frac{4R}{6\pi} = \frac{2R}{3\pi}$

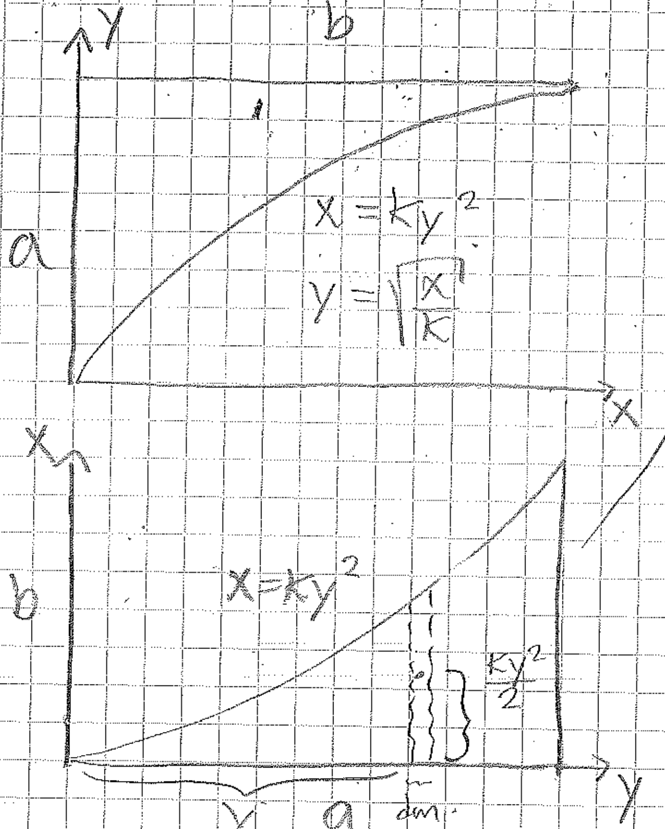
5.4.10 (Fortsättning)

$$X_4 = \frac{X_1 m_1 + x_2 m_2 + x_3 m_3 + x_4 m_4}{m_1 + m_2 + m_3 + m_4}$$

Tyngdpunkt 1–3:

$$m_1 x_1 = \frac{m_3 X_3 + m_{1-3} \cdot X_{1-3}}{m_3 + m_{1-3}}$$

5.4.12



Masscentrum för dm är i x-led $= \frac{ky^2}{2}$ och y-led $= y$.

Observera att axlarna bytt plats!

$b = ka^2$ (massa i y-led ökar i den proportionen med a)

$$m = \int_0^a ky^2\,dy = \left[\frac{ky^3}{3}\right]_0^a = \frac{ka^3}{3}$$

$$y_G = \frac{1}{m}\int_0^a ky^2 \cdot y\,dy = \frac{1}{m}\left[\frac{ky^4}{4}\right]_0^a = \frac{1}{m}\frac{ka^4}{4}$$

$$= \frac{3}{ka^3}\cdot\frac{ka^4}{4} = \underline{\underline{\frac{3a}{4}}}$$

$$x_G = \frac{1}{m}\int_0^a \frac{ky^2}{2}\cdot ky^2\,dy = \frac{1}{m}\left[\frac{k^2y^5}{10}\right]_0^a = \frac{3}{ka^3}\frac{k^2a^5}{10}$$

Små rektangulära delar där masscentrum är $\frac{x}{2}$

$$= \frac{3}{10}ka^2 = \underline{\underline{\frac{3}{10}b}}$$