2.1) Fråga!

2.2) Fråga!

2.3) $\begin{cases} \hat{u} + \hat{v} = u \\ 2\hat{u} + 3\hat{v} = v \end{cases}$ Lös ut $\hat{u}$ och $\hat{v}$
(Vanlig hederlig gausning)

$\Leftrightarrow$

$\begin{cases} \hat{u} + \hat{v} = u \\ \hat{v} = -2u + v \end{cases}$ $\text{II} - 2\text{I}$

$\Leftrightarrow$ (ä.s)

$\begin{cases} \hat{u} = u - (-2u+v) = 3u - v \\ \hat{v} = -2u + v \end{cases}$

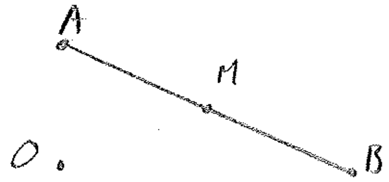
2.4) Vet $3\overline{AC} = \overline{CB}$ (eller $\overline{AC} = \frac{1}{4}\overline{AB}$). Visa $\overline{OC} = \frac{3}{4}\overline{OA} + \frac{1}{4}\overline{OB}$

Bevis:

$\overline{OC} = \overline{OA} + \overline{AC} = \overline{OA} + \frac{1}{4}\overline{AB} = \frac{3}{4}\overline{OA} + \frac{1}{4}\overline{OA} + \frac{1}{4}\overline{AB} =$

$= \frac{3}{4}\overline{OA} + \frac{1}{4}(\overline{OA} + \overline{AB}) = \frac{3}{4}\overline{OA} + \frac{1}{4}\overline{OB}$

2.5)



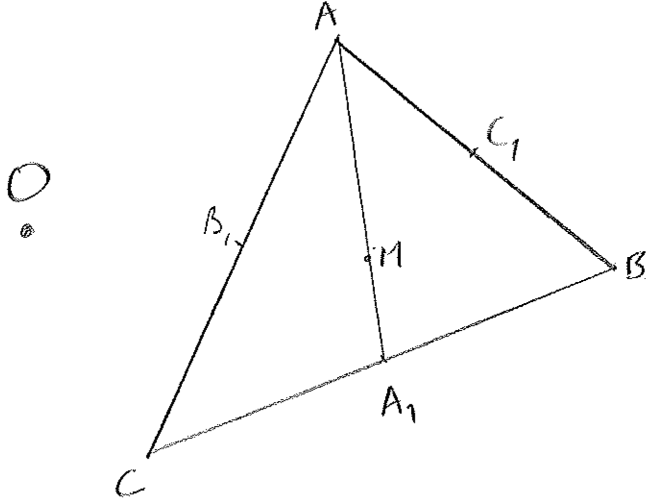
Vet $\overline{AM} = \frac{1}{2}\overline{AB}$.

Visa $\overline{OM} = \frac{1}{2}(\overline{OA} + \overline{OB})$

Bevis: $\overline{OM} = \overline{OA} + \overline{AM} = \overline{OA} + \frac{1}{2}\overline{AB} = \frac{1}{2}\overline{OA} + \frac{1}{2}\overline{OA} + \frac{1}{2}\overline{AB} =$

$= \frac{1}{2}\overline{OA} + \frac{1}{2}(\overline{OA} + \overline{AB}) = \frac{1}{2}\overline{OA} + \frac{1}{2}\overline{OB}$

2.6)



Vet: $\overline{CA_1} = \frac{1}{2}\overline{CB}$

$AM = \frac{2}{3}\overline{AA_1}$

Visa $\overline{OM} = \frac{1}{3}(\overline{OA} + \overline{OB} + \overline{OC})$

Bevis: $$\overline{OM} = \overline{OA} + \overline{AM} = \overline{OA} + \frac{2}{3}\overline{AA_1} = \overline{OA} + \frac{2}{3}\overbrace{\frac{1}{2}(\overline{AB} + \overline{AC})}^{\overline{AA_1}\ (\text{se } 2.5)} =$$

$$= \overline{OA} + \frac{1}{3}(\overline{AO} + \overline{OB} + \overline{AO} + \overline{OC}) = \overline{OA} + \frac{2}{3}\overline{AO} + \frac{1}{3}\overline{OB} + \frac{1}{3}\overline{OC} =$$

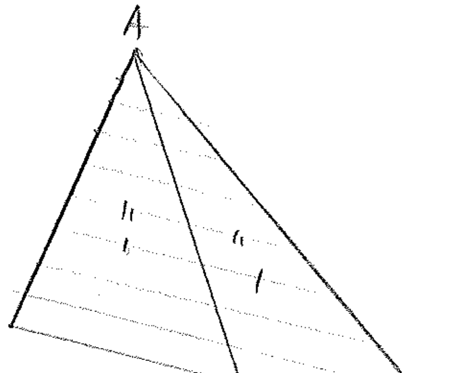
$$= \frac{1}{3}\overline{OA} + \frac{1}{3}\overline{OB} + \frac{1}{3}\overline{OC} = \frac{1}{3}(\overline{OA} + \overline{OB} + \overline{OC})$$

Om $\hat{M}$ är punkten som ligger på $\overline{BB_1}$ i förhållande 2:1 så ger samma räkningar

$$\overline{O\hat{M}} = \frac{1}{3}(\overline{OA} + \overline{OB} + \overline{OC})$$

Alltså är punkten i spetsen på $\frac{1}{3}(\overline{OA} + \overline{OB} + \overline{OC})$ på alla medianer. Alltså skär de varandra just här.

Tyngdpunkt.



2.7) $\overline{OM} = \overline{OA} + \overline{AM} = \overline{OA} + \frac{3}{4}\overline{AA_1} = \overline{OA} + \frac{3}{4}\cdot(\overline{AO} + \overline{OM'}) =$

$= \overline{OA} + \frac{3}{4}\overline{AO} + \frac{3}{4}\cdot\underbrace{\frac{1}{3}(\overline{OB} + \overline{OC} + \overline{OD})}_{\overline{OM} \text{ enligt } 2.6} = \frac{1}{4}\overline{OA} + \frac{1}{4}(\overline{OB} + \overline{OC} + \overline{OD}) =$

$= \frac{1}{4}(\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD})$

2.8) O är godtycklig punkt i 2.6. Välj $O = M$

så

$\overline{MM} = \frac{1}{3}(\overline{MA} + \overline{MB} + \overline{MC}) \Rightarrow \overline{MA} + \overline{MB} + \overline{MC} = 0.$

$= \overline{0}$ såklart.

2.12 VÄND →

2.13) Fråga!

2.14) Vi vet att (se. 2.3)

$$\begin{cases} \hat{u} = 3u - v \\ \hat{v} = -2u + v \end{cases} \quad \text{och} \quad \begin{cases} u = \hat{u} + \hat{v} \\ v = 2\hat{u} + 3\hat{v} \end{cases}$$

så

a) $u = (1, 1)_{(\hat{u}, \hat{v})}$ $v = (2, 3)_{(\hat{u}, \hat{v})}$

b) $\hat{u} = (3, -1)_{(u, v)}$ $\hat{v} = (-2, 1)_{(u, v)}$

2.12) Räcker visa $\overline{A_1B_1} = \overline{D_1C_1}$

$$\overline{A_1B_1} = \frac{1}{2}\left(\overline{A_1B} + \overline{A_1C}\right) = \frac{1}{2}\left(\frac{1}{2}\overline{AB} + \frac{1}{2}\overline{AB} + \overline{BC}\right) =$$

$$= \frac{1}{2}\left(\overline{AB} + \overline{BC}\right) = \frac{1}{2}\overline{AC}$$

P.S.S

$$\overline{D_1C_1} = \frac{1}{2}\overline{AC}$$

så

$$\overline{A_1B_1} = \overline{D_1C_1}$$

2.12) $A_1B_1C_1D_1$ parallellogram om

$A_1B_1 = D_1C_1$ och $B_1C_1 = A_1D_1$

$$A_1B_1 = A_1B + BB_1 = \frac{1}{2}(AB + BC) = \frac{1}{2}(AD + DC) =$$

$$= D_1D + DC_1 = D_1C_1$$

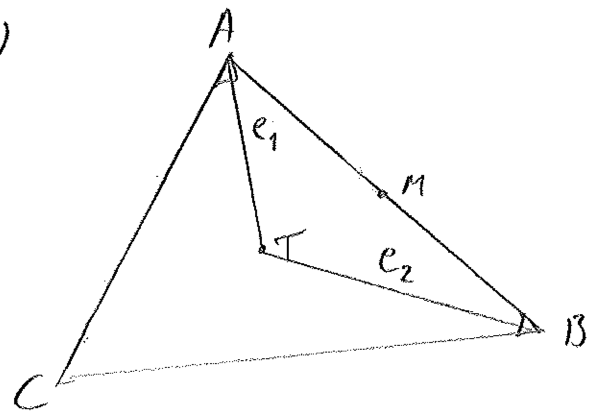
Andra likheten analogt.

2.15) Vet $\overline{OC} = \frac{3}{4}\overline{OA} + \frac{1}{4}\overline{OB}$

Med $e_1 = \overline{OA}$ och $e_2 = \overline{OB}$ fås

$$\overline{OC} = \frac{3}{4}e_1 + \frac{1}{4}e_2 = \left(\frac{3}{4}, \frac{1}{4}\right)_{(e_1,e_2)}$$

2.16)



Enl 2.8; $\overline{TA} + \overline{TB} + \overline{TC} = 0$ så $\overline{TC} = -\overline{TA} - \overline{TB} = (-1,-1)$

Enl 2.5; $\overline{TM} = \frac{1}{2}(\overline{TA} + \overline{TB}) = \left(\frac{1}{2}, \frac{1}{2}\right)$

2.17) a) $\lambda_1 \cdot (2,1,-1) + \lambda_2 \cdot (1,1,1) = (4,1,-5)$

$\Leftrightarrow$

$$\begin{cases} 2\lambda_1 + \lambda_2 = 4 \\ \lambda_1 + \lambda_2 = 1 \\ -\lambda_1 + \lambda_2 = -5 \end{cases} \Leftrightarrow \begin{matrix} \\ 2II - I \\ 2III + I \end{matrix} \begin{cases} 2\lambda_1 + \lambda_2 = 4 \\ \lambda_2 = -2 \\ 3\lambda_2 = -6 \end{cases} \Leftrightarrow \begin{cases} \lambda_1 = 3 \\ \lambda_2 = -2 \end{cases}$$

JA

b)

$$\begin{cases} 2\lambda_1 + \lambda_2 = 4 \\ \lambda_1 + \lambda_2 = 3 \\ -\lambda_1 + \lambda_2 = 2 \end{cases} \Leftrightarrow \begin{cases} 2\lambda_1 + \lambda_2 = 4 \\ \lambda_2 = 2 \\ 3\lambda_2 = 8 \end{cases}$$

Lösning saknas

NEJ

c)

$$\begin{cases} 2\lambda_1 + \lambda_2 = -9 \\ \lambda_1 + \lambda_2 = -7 \\ -\lambda_1 + \lambda_2 = -3 \end{cases} \Leftrightarrow \begin{cases} 2\lambda_1 + \lambda_2 = -9 \\ \lambda_2 = -5 \\ 3\lambda_2 = -15 \end{cases} \Leftrightarrow \begin{cases} \lambda_1 = -2 \\ \lambda_2 = -5 \end{cases}$$

JA

2.19) Fråga

2.20) a) J , lösning

b) N

c) N

d) J , $-2\cdot(1,0,2)+(-2,0,-4)=0$

e) J , ligger i samma plan $(y=0)$

f) J , ——— " ———

g) J , fyra vektorer i 3D

2.20) a) $\lambda_1 \cdot (1,1,1) + \lambda_2 \cdot (3,1,2) + \lambda_3 (0,2,1) = 0$

$\Leftrightarrow$

$$\begin{cases} \lambda_1 + 3\lambda_2 = 0 \\ \lambda_1 + \lambda_2 + 2\lambda_3 = 0 \\ \lambda_1 + 2\lambda_2 + \lambda_3 = 0 \end{cases} \Leftrightarrow \begin{cases} \lambda_1 + 3\lambda_2 = 0 \\ -2\lambda_2 + 2\lambda_3 = 0 \\ -\lambda_2 + \lambda_3 = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \lambda_1 + 3\lambda_2 = 0 \\ -2\lambda_2 + 2\lambda_3 = 0 \\ 2\text{III} - \text{II} \quad 0 = 0 \end{cases}$$

ej entydig lösning

så linj ber

T.ex $\lambda_1 = -3$, $\lambda_2 = 1$, $\lambda_3 = 1$ ger

$$-3(1,1,1) + 1(3,1,2) + 1 \cdot (0,2,1) = 0.$$

b)

$$\begin{cases} \lambda_2 + \lambda_3 = 0 \\ \lambda_1 + \lambda_3 = 0 \\ \lambda_1 + \lambda_2 = 0 \end{cases} \Leftrightarrow \begin{array}{l} \text{II} \\ \text{I} \\ \text{III} \end{array} \begin{cases} \lambda_1 + \lambda_3 = 0 \\ \lambda_2 + \lambda_3 = 0 \\ \lambda_1 + \lambda_2 = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \lambda_1 + \lambda_3 = 0 \\ \lambda_2 + \lambda_3 = 0 \\ \lambda_2 - \lambda_3 = 0 \end{cases} \Leftrightarrow \begin{cases} \lambda_1 + \lambda_3 = 0 \\ \lambda_2 + \lambda_3 = 0 \\ -2\lambda_3 = 0 \end{cases} \Leftrightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = 0 \\ \lambda_3 = 0 \end{cases}$$

linj ober

2.20 c) lin ober ; ses direkt.

d) $(-2, 0, -4) = -2 \cdot (1, 0, 2)$ lin ber

e) Vektorerna ligger i xz-planet ; tre stycken, alltså lin ber

f) Som i e.

g) Fyra vektorer i $\mathbb{R}^3$ alltid linjärt beroende

2.22 a) $u = 4e_1 + 3e_2 = (4,3)_e$

b) Ja, ty ej parallella

c) $2\hat{e}_1 + \hat{e}_2 = u$ så $u = (2,1)_{\hat{e}}$

d) $\begin{cases} \hat{e}_1 = e_1 + 2e_2 = (1,2)_e \\ \hat{e}_2 = 3e_1 - e_2 = (3,-1)_e \end{cases}$

e) $x_1 e_1 + x_2 e_2 = v = \hat{x}_1 \hat{e}_1 + \hat{x}_2 \hat{e}_2 = \hat{x}_1 (e_1 + 2e_2) + \hat{x}_2 (3e_1 - e_2) =$

$= (\hat{x}_1 + 3\hat{x}_2) e_1 + (2\hat{x}_1 - \hat{x}_2) e_2$

Eftersom e_1, e_2 är en bas fås

$\begin{cases} x_1 = \hat{x}_1 + 3\hat{x}_2 \\ x_2 = 2\hat{x}_1 - \hat{x}_2 \end{cases}$

Samband

$\hat{e} = Ae \iff x = A^T \hat{x}$

2.23) Vi visa att $\hat{e}_1, \hat{e}_2$ lin ober ty då är de en bas enl. bassatsen.

$\hat{x}_1 \hat{e}_1 + \hat{x}_2 e_2 = 0 \iff \hat{x}_1(-e_1 + 2e_2) + \hat{x}_2(3e_1 + 4e_2) = 0$

$\iff \begin{cases} -\hat{x}_1 + 3\hat{x}_2 = 0 \\ 2\hat{x}_1 + 4\hat{x}_2 = 0 \end{cases} \iff \underset{II+2I}{} \begin{cases} -\hat{x}_1 + 3\hat{x}_2 = 0 \\ 10\hat{x}_2 = 0 \end{cases} \iff \begin{cases} \hat{x}_1 = 0 \\ \hat{x}_2 = 0 \end{cases}$

ty e_1, e_2 bas

så lin ober.

Koord. samband ger

$\begin{cases} -\hat{x}_1 + 3\hat{x}_2 = 4 \\ 2x_1 + 4x_2 = -5 \end{cases} \iff \begin{cases} -\hat{x}_1 + 3\hat{x}_2 = 4 \\ 10\hat{x}_2 = 3 \end{cases} \iff \begin{cases} \hat{x}_1 = -\frac{31}{10} \\ \hat{x}_2 = \frac{3}{10} \end{cases}$

2.25) $\begin{cases} \hat{e}_1 = e_1 + e_2 \\ \hat{e}_2 = e_1 + e_2 - e_3 \\ \hat{e}_3 = \quad e_2 - e_3 \end{cases}$

Vi undersöker om $\hat{e}_1, \hat{e}_2, \hat{e}_3$ är lin ober, det leder till

$$\begin{cases} \lambda_1 + \lambda_2 = 0 \\ \lambda_1 + \lambda_2 + \lambda_3 = 0 \\ -\lambda_2 - \lambda_3 = 0 \end{cases} \Leftrightarrow \text{II}-\text{I} \begin{cases} \lambda_1 + \lambda_2 = 0 \\ \lambda_3 = 0 \\ -\lambda_2 - \lambda_3 = 0 \end{cases} \Leftrightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = 0 \\ \lambda_3 = 0 \end{cases}$$

så lin ober.

och därmed bas

$$\begin{cases} \hat{x}_1 + \hat{x}_2 = 2 \\ \hat{x}_1 + \hat{x}_2 + \hat{x}_3 = 3 \\ -\hat{x}_2 - \hat{x}_3 = -2 \end{cases} \Leftrightarrow \begin{cases} \hat{x}_1 + \hat{x}_2 = 2 \\ \hat{x}_3 = 1 \\ -\hat{x}_2 - \hat{x}_3 = -2 \end{cases} \Leftrightarrow \begin{cases} \hat{x}_1 = 1 \\ \hat{x}_2 = 1 \\ \hat{x}_3 = 1 \end{cases}$$

5.15) $\hat{e}_1 = \frac{1}{\sqrt{3}}(1,1,-1)$

$\hat{e}_2 \perp$ mot $\hat{e}_1$ och $\bar{n} = (1,1,1)$

$\hat{e}_2$ riktad som $\hat{e}_1 \hat{n} = \frac{1}{\sqrt{3}}(2,-2,0)$

så $\hat{e}_2 = \frac{1}{\sqrt{2}}(1,-1,0)$

$$\hat{e}_3 = \hat{e}_1 \times \hat{e}_2 = \frac{1}{\sqrt{6}}(-1,-1,-2) = \frac{-1}{\sqrt{6}}(1,1,2)$$

$$\begin{cases} x_1 = \frac{1}{\sqrt{3}}\hat{x}_1 + \frac{2}{\sqrt{3}}\hat{x}_2 - \frac{1}{\sqrt{6}}\hat{x}_3 \\ x_2 = \frac{1}{\sqrt{3}}\hat{x}_1 - \frac{2}{\sqrt{3}}\hat{x}_2 - \frac{1}{\sqrt{6}}\hat{x}_3 \\ x_3 = -\frac{1}{\sqrt{3}}\hat{x}_1 \qquad - \frac{2}{\sqrt{6}}\hat{x}_3 \end{cases}$$

så $x_1 + x_2 + x_3 = \frac{1}{\sqrt{3}}\hat{x}_1 - \frac{4}{\sqrt{6}}\hat{x}_3 = 0$

$\Leftrightarrow \hat{x}_1 - \frac{4}{\sqrt{2}}\hat{x}_3 = 0$

2.26)
$$\begin{cases} e_1' = \quad e_2 + e_3 \\ e_2' = -e_1 + 2e_2 + e_3 \\ e_3' = e_1 \quad + 2e_3 \end{cases}$$

e_1', e_2', e_3' bas om lin ober. Vi undersöker

$$0 = \lambda_1 e_1' + \lambda_2 e_2' + \lambda_3 e_3' = \lambda_1(e_2 + e_3) + \lambda_2(-e_1 + 2e_2 + e_3) +$$
$$+ \lambda_3(e_1 + 2e_3)$$

Eftersom e_1, e_2, e_3 bas så

$$\begin{matrix} \mathrm{II} \\ \mathrm{I} \\ \\ \end{matrix} \begin{cases} -\lambda_2 + \lambda_3 = 0 \\ \lambda_1 + 2\lambda_2 = 0 \\ \lambda_1 + \lambda_2 + 2\lambda_3 = 0 \end{cases} \Leftrightarrow (-1) \begin{cases} \lambda_1 + 2\lambda_2 = 0 \\ -\lambda_2 + \lambda_3 = 0 \\ \lambda_1 + \lambda_2 + 2\lambda_3 = 0 \end{cases} \Leftrightarrow (-1) \begin{cases} \lambda_1 + 2\lambda_2 = 0 \\ -\lambda_2 + \lambda_3 = 0 \\ -\lambda_2 + 2\lambda_3 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \lambda_1 + 2\lambda_2 = 0 \\ -\lambda_2 + \lambda_3 = 0 \\ \lambda_3 = 0 \end{cases} \Leftrightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = 0 \\ \lambda_3 = 0 \end{cases}$$

lin ober

Koordinatsamband

$$\begin{cases} x_1 = \quad -x_2' + x_3' \\ x_2 = x_1' + 2x_2' \\ x_3 = x_1' + x_2' + 2x_3' \end{cases}$$

Samma koord

$$\begin{cases} x_1 = \quad -x_2 + x_3 \\ x_2 = x_1 + 2x_2 \\ x_3 = x_1 + x_2 + x_3 \end{cases} \Leftrightarrow \begin{cases} x_1 + x_2 - x_3 = 0 \\ x_1 + x_2 = 0 \\ x_1 + x_2 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = t \\ x_2 = -t \\ x_3 = 0 \end{cases}$$