

11.1

$$f(x) = \ln(1+x) =$$

$$P_n(x) = f(0) + f'(0) \cdot x + \frac{f''(0)}{2!} x^2 + \dots + \frac{f^{(n)}(0)}{n!} x^n$$

$$a) P_1(x) = \ln(1+0) + \frac{1}{1+0} x = \boxed{x}$$

$$\cancel{f'(x)} = \frac{1}{1+x} = (1+x)^{-1}$$

$$f''(x) = -(1+x)^{-2}$$

$$P_2(x) = x - \frac{(1+0)^{-2} x^2}{2} = \boxed{x - \frac{1}{2} x^2}$$

$$\cancel{f''(x)} = 2(1+x)^{-3}$$

$$P_3(x) = x - \frac{1}{2} x^2 + \frac{2}{1} \cdot \frac{x^3}{6} = \boxed{x - \frac{1}{2} x^2 + \frac{1}{3} x^3}$$

$$\cancel{f^{(4)}(x)} = -6(1+x)^{-4}$$

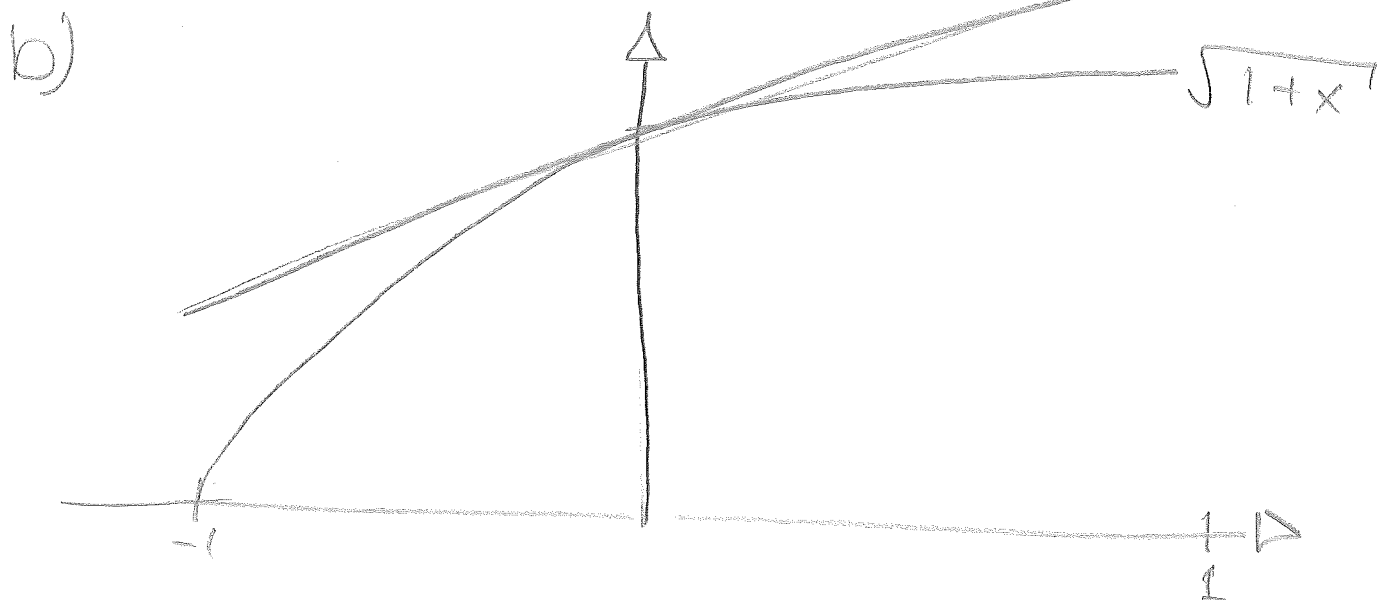
$$P_4(x) = P_3(x) - \frac{6}{4!} x^4 = \boxed{P_3(x) - \frac{1}{4} x^4}$$

$\frac{6}{24} = \frac{3}{12} = \frac{1}{4}$

11.3

$$\sqrt{1+x} \approx 1 + \frac{1}{2}x \quad \text{om } x \approx 0$$

$$a) P_1(x) = \sqrt{1+0} + \frac{0,5}{\sqrt{1+0}}x = \boxed{1 + \frac{1}{2}x}$$



$$c) R_2(x) = f(x) - P_1(x) = \sqrt{1+x} - 1 + \frac{1}{2}x \approx 0 \quad \text{da } x=0$$

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$$d) |R_2(x)| \leq \frac{1}{8}x^2, \quad x > 0$$

$$R_2(x) + 1 + \frac{1}{2}x = \sqrt{1+x}$$

$$\left(R_2(x) + 1 + \frac{1}{2}x\right)^2 = 1+x$$

11.4

a) $f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$P_0(x) = f(0) = \sin 0 = 0$

$P_1(x) = f(0) + \frac{f'(0)}{1!} x = \sin 0 + \cos 0 \cdot x = x$

$P_2(x) = f(0) + f'(0) \cdot x + \frac{f''(0)}{2!} x^2 = 0 + x + 0 = x$

b) $f(x) = P_1(x) + R_2(x) = x + \frac{f''(\theta x)}{2} x^2$

$R_3 = \frac{f'''(\theta x)}{3!} x^3 = \frac{-\cos \theta x}{6} x^3$

$\frac{\sin(\theta x)}{2} x^2$

c) $|R_2(x)| = \frac{\sin(\theta x)}{2} x^2 \leq$

$\leq \frac{1}{2} (0,1)^2 = \frac{1}{200}$

$|R_3(x)| = \left| -\frac{\cos \theta x}{6} x^3 \right| = \frac{\cos \theta x}{6} \cdot x^3 \leq \frac{1}{6} \cdot (0,1)^3 = \frac{1}{6000}$

11.6

$$\text{Visa att } \left| e^x - 1 - x - \frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{24} \right| \leq \frac{|x|^5}{40}$$

$$\text{om } |x| \leq 1$$

$$f(x) = e^x = f'(x) = f^{(k)}(x)$$

$$P_0(x) = 1$$

$$P_1(x) = 1 + x$$

$$P_2(x) = 1 + x + \frac{1}{2}x^2$$

$$P_3(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$$

$$P_4(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$$

$$R_5(x) = f(x) - P_4(x) =$$

$$R_5(x) = \frac{e^{\theta x}}{5!} x^5 \leq \frac{3}{5 \cdot 4 \cdot 2} x^5 = \boxed{\frac{1}{40} x^5}$$

11.7

$$\left| \ln(1+x) - x + \frac{x^2}{2} \right| \leq \frac{8|x|^3}{3} \quad |x| < \frac{1}{2}$$

$$f(x) = \ln(1+x) - x + \frac{x^2}{2}$$

$$f'(x) = \frac{1}{1+x} - 1 + x$$

$$f''(x) = -\frac{1}{(1+x)^2} + 1$$

$$f'''(x) = \frac{1}{(1+x)^3}$$

$$P_3(x) = x^3$$

$$R_3 = f(x) - P_3(x)$$

$$R_3 = \frac{f'''(\xi)}{3!} x^3 = \frac{1}{3!(1+\xi)^3} x^3 = \frac{x^3}{6(1+\theta)^3} \leq \frac{x^3}{6}$$

11.10

$$|e^x + e^{-x} - 2 - x^2| \leq \frac{1}{6} x^4$$

$$f(x) = e^x$$

$$f'(x) = f''(x) = \dots = e^x$$

$$p_2(x) = \left(1 + x + \frac{x^2}{2}\right)$$

~~P2~~

$$|2 + 2x + x^2 - 2 - x^2| = |2x| \leq \frac{1}{6} x^4$$

$$2 \leq \frac{1}{6}$$

$$|e^x + e^{-x} - 2 - x^2| \leq \frac{1}{6} x^4$$

$$f(x) = e^x + e^{-x}$$

$$f'(x) = e^x - e^{-x}$$

$$f''(x) = e^x + e^{-x}$$

$$f'''(x) = e^x - e^{-x}$$

$$f^{(4)}(x) = e^x + e^{-x}$$

$$\frac{e^x + e^{-x}}{24} x^4 = \frac{e^x + e^{-x}}{24} x^4$$

$$\leq \frac{2 \cdot 3 + \frac{1}{3}}{24} x^4 = \frac{10}{3 \cdot 24} x^4 = \frac{5}{36} x^4 < \frac{1}{6} x^4$$

~~P5(x)~~

$$f(x) = 1 + 1 + 0x + \frac{2x^2}{2} + \frac{0x^3}{6} + \frac{e^x + e^{-x}}{24} x^4$$

$$|2 + x^2 - 2 - x^2| = 0$$