$\boxed{11.1}$ $f(x) = \ln(1+x) =$

$$P_n(x) = f(0) + f'(0)\cdot x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n$$

a) $P_1(x) = \ln(1+0) + \frac{1}{1+0}x = \boxed{x}$

$$f'(x) = \frac{1}{1+x} = (1+x)^{-1}$$

$$f''(x) = -(1+x)^{-2}$$

$$P_2(x) = x - \frac{(1+0)^{-2}x^2}{2} = \boxed{x - \frac{1}{2}x^2}$$

$$f'''(x) = 2(1+x)^{-3}$$

$$P_3(x) = x - \frac{1}{2}x^2 + \frac{2}{1}\cdot\frac{x^3}{6} = \boxed{x - \frac{1}{2}x^2 + \frac{1}{3}x^3}$$

$$f^{(4)}(x) = -6(1+x)^{-4}$$

$$P_4(x) = P_3(x) - \frac{6}{4!}x^4 = \boxed{P_3(x) - \frac{1}{4}x^4}$$

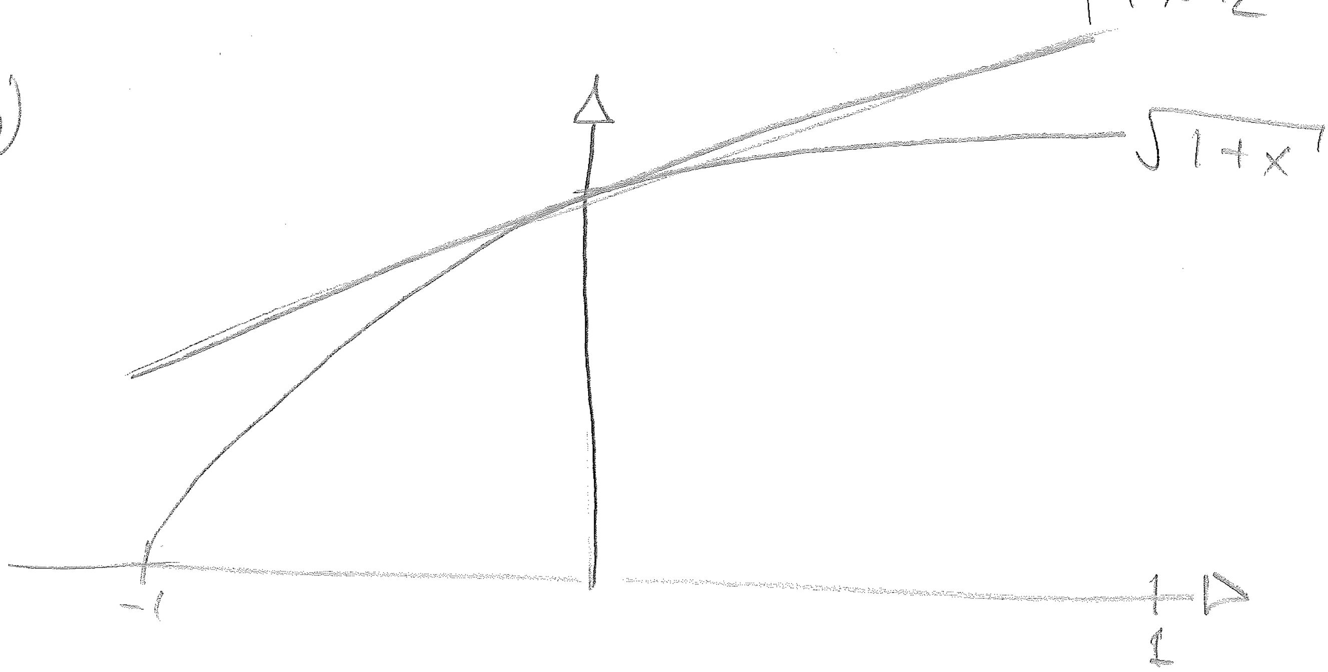
$\frac{6}{24} = \frac{3}{12} = \frac{1}{4}$

$\boxed{11.3}$

$\sqrt{1+x} \approx 1+\frac{1}{2}x$ om $x \approx 0$

a) $P_1(x) = \sqrt{1+0} + \frac{0{,}5}{\sqrt{1+0}}x = \boxed{1+\frac{1}{2}x}$

b)



c) $R_2(x) = f(x) - P_1(x) = \sqrt{1+x} - 1 + \frac{1}{2}x \approx 0$ då $x=0$

$\boxed{\text{JA}}$

d) $|R_2(x)| \leq \frac{1}{8}x^2 \quad , x > 0$

$R_2(x) + 1 + \frac{1}{2}x = \sqrt{1+x}$

$\left(R_2(x) + 1 + \frac{1}{2}x\right)^2 = 1+x$

11.4

a) $f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$P_0(x) = f(0) = \sin 0 = 0$

$P_1(x) = f(0) + \frac{f'(0)}{1!} x = \sin 0 + \cos 0 \cdot x = \boxed{x}$

$P_2(x) = f(0) + f'(0) \cdot x + f''(0) \cdot \frac{1}{2} x^2 = 0 + x + 0 = \boxed{x}$

b) $f(x) = P_1(x) + R_2(x) = x + \boxed{\frac{f''(\theta x)}{2} x^2} = \boxed{-\frac{\sin(\theta x)}{2} x^2}$

$R_3 = \frac{f'''(\theta x)}{3!} x^3 = \boxed{-\frac{\cos \theta x}{6} x^3}$

c) $|R_2(x)| = \frac{\sin(\theta x)}{2} x^2 \leq$

$\leq \frac{1}{2} (0,1)^2 = \boxed{\frac{1}{200}}$

$|R_3(x)| = \left| -\frac{\cos \theta x}{6} x^3 \right| = \frac{\cos \theta x}{6} \cdot x^3 \leq \frac{1}{6} \cdot (0,1)^3 = \boxed{\frac{1}{6000}}$

$\boxed{11.6}$ Visa att $\left|e^x - 1 - x - \frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{24}\right| \leq \frac{|x|^5}{40}$

om $|x| \leq 1$

$f(x) = e^x = f'(x) = f^{(n)}(x)$

$P_0(x) = 1$

$P_1(x) = 1 + x$

$P_2(x) = 1 + x + \frac{1}{2}x^2$

$P_3(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$

$P_4(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$

$R_5(x) = f(x) - P_4(x) =$

$R_5(x) = \frac{e^{\theta x}}{5!}x^5 \leq \frac{3}{5 \cdot 4 \cdot 2}x^5 = \boxed{\frac{1}{40}x^5}$

$\boxed{11.7}$

$$\left|\ln(1+x) - x + \frac{x^2}{2}\right| \leq \frac{8|x|^3}{3} \quad |x| < \frac{1}{2}$$

$$f(x) = \ln(1+x) - x + \frac{x^2}{2}$$

$$f'(x) = \frac{1}{1+x} - 1 + x$$

$$f''(x) = -\frac{1}{(1+x)^2} + 1$$

$$f'''(x) = \frac{1}{(1+x)^3}$$

$$P_3(x) = x^3$$

$$R_3 = f(x) - P_3(x)$$

$$R_3 = \frac{f^3(\varepsilon)}{3!} x^3 = \frac{1}{3!\cdot(1+\varepsilon)^3} x^3 = \frac{x^3}{6(1+\theta x)^3} \leq \frac{x^3}{6}$$

$\boxed{11.10}$

$$|e^x + e^{-x} - 2 - x^2| \le \frac{1}{6}x^4$$

$$f(x) = e^x$$

$$f'(x) = f''(x) = \dots = e^x$$

$$P_2(x) = 1 + x + \frac{x^2}{2}$$

$\cancel{P_2(}$

$$|2 + 2x \cancel{+ x^2} - 2 \cancel{- x^2}| = |2x| \le \frac{1}{6}x^4$$

$$2 \le \frac{1}{6}$$

$$|e^x + e^{-x} - 2 - x^2| \le \frac{1}{6}x^4$$

$$f(x) = e^x + e^{-x}$$

$$f'(x) = e^x - e^{-x}$$

$$f''(x) = e^x + e^{-x}$$

$$f'''(x) = e^x - e^{-x}$$

$$f''''(x) = e^x + e^{-x}$$

$$\frac{e^{\beta} + e^{-\beta}}{24}x^4 = \frac{e^{\theta x} + e^{-\theta x}}{24}x^4$$

$$\le \frac{3 + \frac{1}{3}}{24}x^4 = \frac{10}{3 \cdot 24}x^4 = \boxed{\frac{5}{36}x^4 < \frac{1}{6}x^4}$$

$$f(x) = 1 + 1 + 0x + \frac{2x^2}{2} + \cancel{\frac{0x^3}{6}} + \frac{e^{\beta} + e^{-\beta}}{24}x^4$$

$\cancel{P_3(x) =}$

$$|2 + x^2 - 2 - x^2| = 0$$