b) 45° eller i fase

$$\Rightarrow \arctan\left(\frac{R\omega C}{1-\omega^2 CL}\right) = \frac{\pi}{4}$$

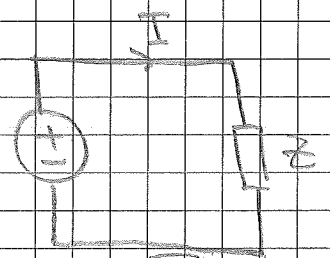
$$\Rightarrow \frac{R\omega C}{1-\omega^2 CL} = 1$$

$$R\omega C = 1-\omega^2 CL \Rightarrow$$

$$\Rightarrow R = \frac{1-\omega^2 CL}{\omega C} = \frac{1}{\omega C} - \frac{\omega^2 CL}{\omega C} =$$

$$= \frac{1}{\omega C} - \omega L$$

11.1



$V = \sqrt{2}\cdot 230\,V.$

$Z = (5-2j)\,\Omega.$

a) $I = \frac{V}{Z} = \frac{\sqrt{2}\cdot 230}{5-2j} = \frac{\sqrt{2}\cdot 230\,(5+2j)}{25+4} =$

$= \frac{1626+6504j}{29}\,A.$

b) $S = \frac{1}{2} V\cdot \tilde{I} = \frac{1}{2}\frac{(\sqrt{2}\cdot 230)^2(5-2j)}{29} = \frac{230^2}{29}(5-2j)\,VA$

c) $|S| = \frac{230^2}{29}\sqrt{25+4} = \frac{230^2}{29}\sqrt{29} = \frac{230^2}{\sqrt{29}}\,VA$

d) $\frac{230^2}{29}\cdot 5\,W$

e) $-\frac{230^2}{29}\cdot 2\,VAr.$

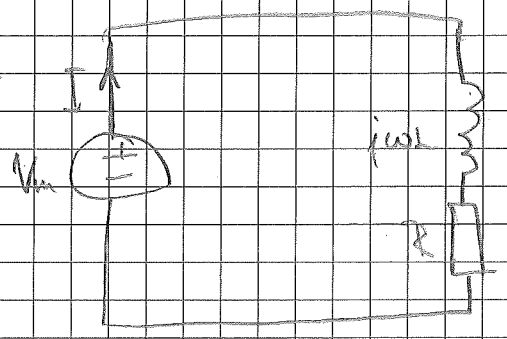
f) $P = |S|\cos\varphi \Rightarrow \cos\varphi = \frac{P}{|S|} = \frac{230^2\cdot 5}{29}\cdot\frac{\sqrt{29}}{230^2} = \frac{5\cdot\sqrt{29}}{29} = \frac{5}{\sqrt{29}}$

g) $Z = 5-2j.$

$\operatorname{Im} Z < 0$ – kapacitiv

h) $P = \frac{230^2}{29}\cdot 5\,W.$

11.2



$Z = R + j\omega L$

$$S = \frac{1}{2} V \tilde{I} = \frac{V_m}{2} \tilde{I} = \frac{V_m}{2}\left(\frac{\tilde{V_m}}{R + j\omega L}\right) = \frac{|V_m|^2}{2(R - j\omega L)} = \frac{(R + j\omega L)|V_m|^2}{2(R^2 + (\omega L)^2)}$$

$$|S| = \left|\frac{(R + j\omega L)|V_m|^2}{2(R^2 + \omega^2 L^2)}\right| = \frac{\sqrt{R^2 + \omega^2 L^2}\,|V_m|^2}{2(R^2 + \omega^2 L^2)} = \frac{|V_m|^2}{2\sqrt{R^2 + \omega^2 L^2}}$$

$$P = \frac{R|V_m|^2}{2(R^2 + \omega^2 L^2)} \text{ W}$$

$$Q = \frac{\omega L |V_m|^2}{2(R^2 + \omega^2 L^2)} \text{ VAR}$$

$$P = |S|\cos\varphi \Rightarrow \cos\varphi = \frac{P}{|S|} = \frac{R\cancel{|V_m|^2}}{\cancel{2}(R^2 + \omega^2 L^2)} \cdot \frac{\cancel{2}\sqrt{R^2 + \omega^2 L^2}}{\cancel{|V_m|^2}}$$

$$= \frac{R\sqrt{R^2 + \omega^2 L^2}}{R^2 + \omega^2 L^2} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

b) $I = \frac{V_m}{R + j\omega L} \Rightarrow |I| = \frac{|V_m|}{|R + j\omega L|} = \frac{|V_m|}{\sqrt{R^2 + \omega^2 L^2}}$

c)

$Z_c = \frac{1}{j\omega C}$

$V = V_m$

$I_c = V_m \cdot j\omega C$

$$S = \frac{1}{2} V \cdot \tilde{I}_c = \frac{1}{2} V_m \cdot (-V_m j\omega C) = \frac{-|V_m|^2 j\omega C}{2}$$; komplexa effekten.

$$|S| = \frac{|V_m|^2}{2}|j\omega C| = \frac{|V_m|^2 \omega C}{2}$$; Skenbar effekt.

$P = \mathrm{Re}\{S\} = 0$ W ; Aktiv effekt.

$Q = \mathrm{Im}\{S\} = \frac{|V_m|^2 \omega C}{2}$ VAR ; Reaktiv effekt.

$\cos\varphi = \frac{P}{|S|} = 0$; effektfaktor.

d) $Q_c = 0 \Rightarrow$

$$\Rightarrow \frac{\omega L |V_m|^2}{2(R^2+\omega^2 L^2)} - \frac{\omega C |V_m|^2}{2} = 0$$

$$\Rightarrow \omega L |V_m|^2 - \omega C |V_m|^2 (R^2+\omega^2 L^2) = 0$$

$$\Rightarrow C = \frac{\cancel{\omega} L \cdot \cancel{|V_m|^2}}{\cancel{\omega} \cdot \cancel{|V_m|^2} \cdot (R^2+\omega^2 L^2)}$$

$$= \frac{L}{R^2+\omega^2 L^2}$$

$$S_{tot} = S + S_c \Big|_{C = \frac{L}{R^2+\omega^2 L^2}} = \frac{(R - j\omega L)|V_m|^2}{2(R^2+\omega^2 L^2)} - \frac{j\omega L |V_m|^2}{2(R^2+\omega^2 L^2)} =$$

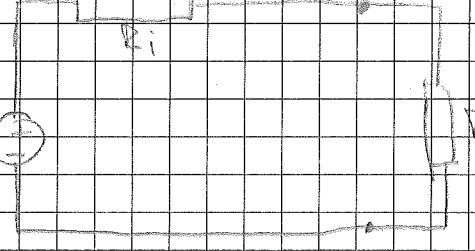
$$= \frac{|V_m|^2 (R \cancel{- j\omega L + j\omega L})}{2(R^2+\omega^2 L^2)} = \frac{R \cdot |V_m|^2}{2(R^2+\omega^2 L^2)}$$

e) $$I_1 = I + I_c = \frac{V_m}{R+j\omega L} + \frac{V_m j\omega L}{R^2+\omega^2 L^2} =$$

$$= \frac{V_m (R - j\omega L) + V_m j\omega L}{R^2+\omega^2 L^2} = \frac{V_m (R \cancel{- j\omega L + j\omega L})}{R^2+\omega^2 L^2} = \frac{R \cdot V_m}{R^2+\omega^2 L^2}$$

$$\frac{|I_1|}{|I|} = \frac{|R| \cdot \cancel{|V_m|}}{|R^2+\omega^2 L^2|} \cdot \frac{\sqrt{R^2+\omega^2 L^2}}{\cancel{|V_m|}} = \frac{R}{\sqrt{R^2+\omega^2 L^2}}$$

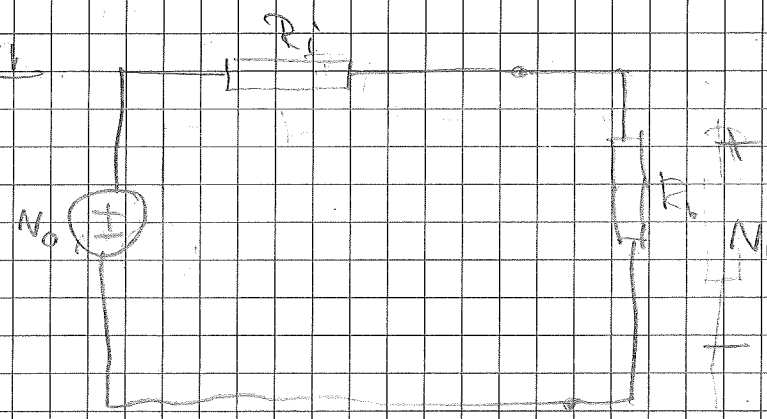
II.II



$$V_L = \frac{R_L}{R_i + R_L} V_0$$

11.12 a) falskt b) sant c) falskt

11.11



Vi har $P = \frac{1}{2} \tilde{V} \tilde{I}$

Ström och spänning i lasten:

$$I_b = \frac{N_0}{R_b + R_i} \qquad N_b = \frac{R_b}{R_b + R_i} N_0$$

Detta ger: $S = \frac{1}{2} N_b \cdot \tilde{I}_b$

$$= \frac{1}{2} \frac{N_0}{R_b + R_i} \cdot \frac{R_b}{R_b + R_i} N_0 = |N_0|^2 \frac{R_b}{|R_b + R_i|^2}$$

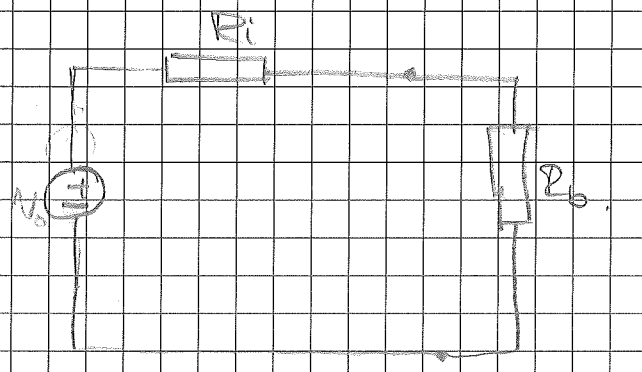
$$\frac{d}{dR_i} |N_0|^2 \frac{R_b}{(R_b + R_i)^2} = |N_0|^2 R_b \frac{d}{dR_i} (R_b + R_i)^{-2} =$$

$$= -2|N_0|^2 R_b R_i$$

Max då $R_i = 0 \Rightarrow P = |N_0|^2 \frac{1}{R_b}$

$|N_0|^2 = P R_b \Rightarrow N_0 = \sqrt{P R_b}$

11.13



Ström och spänning i detektorn:

$$I_b = \frac{V_0}{R_i + R_b} \qquad V_b = \frac{R_b}{R_i + R_b} V_0$$

$$S = \frac{1}{2} VI = |V_0|^2 \frac{R_b}{|R_i + R_b|^2} = P$$

$$\frac{dP}{dR_b} = |V_0|^2 \frac{d}{dR_b}\left(R_b \cdot (R_i + R_b)^{-2}\right) =$$

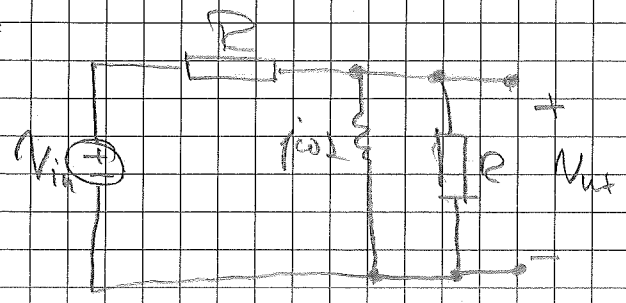
$$= |V_0|^2 \left((R_i + R_b)^{-2} - 2R_b (R_i + R_b)^{-3} \right) =$$

$$|V_0|^2 \frac{1}{(R_i + R_b)^2} - \frac{2R_b}{(R_i + R_b)^3} = |V_0|^2 \frac{R_i + R_b - 2R_b}{(R_i + R_b)^3}$$

$$\frac{dP}{dR_b} = 0 \Rightarrow R_i + R_b - 2R_b = 0$$

$$\Rightarrow R_b = R_i$$

11.14



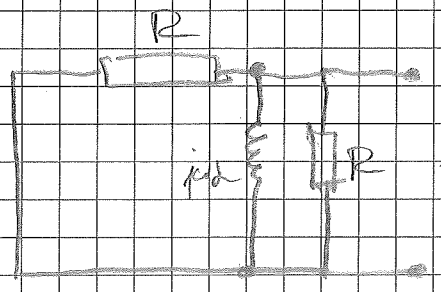
$$V_{th} = \frac{j\omega L // R}{j\omega L // R + R} V_{in} =$$

$$= \left[\frac{1}{R'} = \frac{1}{j\omega L} + \frac{1}{R} = \frac{R + j\omega L}{j\omega L R} \Rightarrow R' = \frac{j\omega L R}{R + j\omega L} \right]$$

$$= \left(\frac{j\omega L R}{R + j\omega L} \Big/ \frac{j\omega L R}{R + j\omega L} + R \right) V_{in} = \left(\frac{j\omega L R}{R + j\omega L} \Big/ \frac{j\omega L R + R^2 + j\omega L R}{R + j\omega L} \right) V_{in}$$

$$= \frac{j\omega L \cancel{R}}{\cancel{R + j\omega L}} \cdot \frac{\cancel{R + j\omega L}}{\cancel{R}(j\omega L + R + j\omega L)} V_{in} = \left(\frac{j\omega L}{2j\omega L + R} V_{in} \right)$$

Impedansen Z_{th} → kortsluter spänningskällan:



$$\Rightarrow Z_{th} = R \,//\, j\omega L \,//\, R = \frac{j\omega L R}{R + j\omega L} \,//\, R \Rightarrow$$

$$\Rightarrow \frac{1}{Z_{th}} = \frac{R + j\omega L}{j\omega L R} + \frac{1}{R} = \frac{R + j\omega L + j\omega L}{j\omega L R}$$

$$= \frac{R + 2j\omega L}{j\omega L R} \Rightarrow Z_{th} = \boxed{\frac{j\omega L R}{R + 2j\omega L}}$$

b) $Z_b = Z_{th}^* = \dfrac{-j\omega L R}{R - 2j\omega L}$

$$P_{max} = \frac{1}{2} Re\{Z_b\} |I|^2 \qquad I = \frac{V_{th}}{Z_b + Z_{th}} \Rightarrow |I|^2 = \frac{|V_{th}|^2}{|Z_b + Z_{th}|^2} =$$

$$= \frac{\omega L}{\sqrt{4\omega L^2 + R^2}} \cdot \left| \frac{-j\omega L R}{R - 2j\omega L} + \frac{j\omega L R}{R + 2j\omega L} \right|^2$$

$$= \frac{\omega L}{\sqrt{4\omega L^2 + R^2}} \cdot \left| \frac{-j\omega L R(R + 2j\omega L) + j\omega L R(R - 2j\omega L)}{R^2 + 4\omega L^2} \right|^2 =$$

$$= \frac{\omega L}{\sqrt{4\omega L^2 + R^2}} \cdot \left| \frac{j(\cancel{\omega L R^2} - \cancel{\omega L R^2}) + 2\omega^2 L^2 R + 2\omega^2 L^2 R}{R^2 + 4\omega L^2} \right|^2 =$$

$$= \frac{\omega L}{\sqrt{4\omega L^2 + R^2}} \cdot \frac{4\omega^2 L^2 R}{R^2 + 4\omega L^2} = \frac{\cancel{\omega L}}{\sqrt{4\omega L^2 + R^2}} \cdot \frac{R^2 + 4\omega L^2}{4\omega^{\cancel{2}} L^{\cancel{2}} R} =$$

$$= \frac{\sqrt{R^2 + 4\omega L^2}}{4\omega L R}, \quad P_{max} = \frac{1}{2} Re\{Z_b\} |I|^2 = \frac{|V_{th}|^2}{8 Re\{Z_{th}\}} =$$

$$= \frac{\cancel{\omega L}}{\sqrt{4\omega L^2 + R^2}} \cdot \frac{\sqrt{R^2 + 4\omega L^2}}{16\omega^{\cancel{2}} L^{\cancel{2}} R} |V_{th}|^2 = \frac{\cancel{\sqrt{R^2 + 4\omega L^2}}}{16 R \cancel{\sqrt{4\omega L^2 + R^2}}} |V_{th}|^2 = \frac{|V_{th}|^2}{16 R}$$