Transponat

$A = [a_{ij}]$ $m \times n$

$A^T = [a_{ji}]$ $n \times m$

A^T kallas A:s transponat och fås genom att ställa A:s rader som kolumnerna i A^T

ex.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \; [2 \times 3] \qquad A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \; [3 \times 2]$$

ex.

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 11 \end{pmatrix} \; [3 \times 3] \qquad B^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 6 \\ 3 & 6 & 11 \end{pmatrix} \; [3 \times 3]$$

ex

$\bar{u} = (x_1, x_2, x_3)$

$\bar{v} = (y_1, y_2, y_3)$ (ON-bas)

$$u = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad v = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$\bar{v} \cdot \bar{u}$ ||

$\bar{u} \cdot \bar{v} = x_1 y_1 + x_2 y_2 + x_3 y_3 =$

$$= \underbrace{[x_1 \;\; x_2 \;\; x_3]}_{u^T} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{v} = u^T v = v^T u$$

Def. En kvadratisk matris A kallas symmetrisk om $A^T = A$

ex.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$$

Sats

$$(A^T)^T = A$$
$$(A+B)^T = A^T + B^T$$
$$(\lambda A)^T = \lambda A^T$$
$$(AB)^T = B^T A^T$$

$$A = (a_{ij}) \quad B = (b_{ij}) \quad C = AB = (c_{ij})$$

$$(C^T)_{ij} = (AB)^T_{ij} = c_{ji} = \sum_{k=1}^{p} a_{jk} b_{ki} = \sum_{k=1}^{p} (A^T)_{kj} (B^T)_{ik} =$$
$$= \sum_{k=1}^{p} (B^T)_{ik} (A^T)_{kj} = (B^T A^T)_{ij}$$

$$(AB)^T = B^T A^T$$