SUMMARY OF CHAPTER 6 SLIDES – NUMDIFF

11/12-2015

Waves and hyperbolics

Wave equation: $u_{tt} = c^2 u_{xx}$ or $u_t + c u_x = 0$

Conservation law: $u_t + (f(u))_x = 0$ (inviscid flow)

This equation is solved by d'Alembert:

$$u(x,t) = g(x - ct)$$ d'Alembert solution

The solution is constant on the characteristics:

$$x - ct = \text{constant}$$

The advection equation

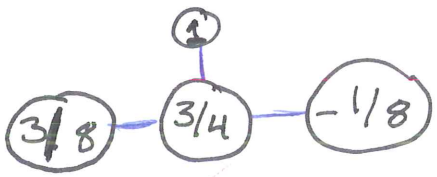
$$u_t + V u_x = 0$$

This can be solved by upwind, downwind and symmetric schemes.

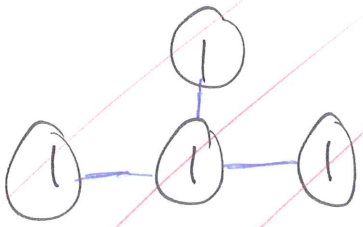
Lax-Wendroff scheme solves this,

$$u_l^{n+1} = \frac{a\mu}{2}(1 + a\mu)u_{l-1}^n + (1 - a^2\mu^2)u_l^n - \frac{a\mu}{2}(1 - a\mu)u_{l+1}^n$$

$a\mu = 1/2 \Rightarrow$



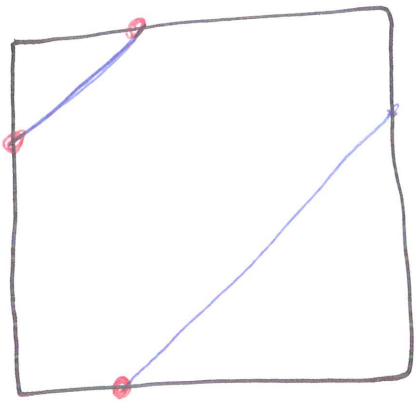
$a\mu = 1 \Rightarrow$



Characteristic!

Periodic BV conditions

$u(t,0) = u(t,1)$ for all $t \geq 0$.



$u^{n+1} = A(a\mu)u^n$, where $A(a\mu)$ is a circulant matrix given by the corner expressions.

$$A(a\mu) = \begin{pmatrix} 1-a^2\mu^2 & \frac{a\mu}{2}(a\mu-1) & & \frac{a\mu}{2}(a\mu+1) \\ \frac{a\mu}{2}(a\mu+1) & \ddots & \ddots & \\ & \ddots & \ddots & \frac{a\mu}{2}(a\mu-1) \\ \frac{a\mu}{2}(a\mu-1) & & \frac{a\mu}{2}(a\mu+1) & 1-a^2\mu^2 \end{pmatrix}$$

$$A(1) = \begin{pmatrix} 0 & 0 & & 1 \\ 1 & \ddots & \ddots & \\ & \ddots & \ddots & 0 \\ 0 & & 1 & 0 \end{pmatrix} \Rightarrow u_l^{n+1} = u_{l-1}^n$$

Th: Let C be a circular $N \times N$ - matrix,

then $\lambda_\kappa[C] = \sum_{j=0}^{N-1} x_j e^{\frac{2kj\pi i}{N}}$

A finite difference scheme with periodic BVCs is stable iff $|\lambda_\kappa[A(a\mu)]| \leq 1$

If characteristics collide, we get a discontinuity.