

SAMMANFATTNING AV NUMDIFF (FMFF10)

Objectives

- Learn scientific computing
- Understand mathematics-numerics interaction
- Understand correspondence between math-physics
- Construct elementary MATLAB programs

Different equations

Initial value problems: $\frac{d}{dt}$ (IVP)

Boundary value problems: $\frac{d^2}{dx^2}, \frac{d}{dx}$ (BVP)

Partial differential equations: (PDE) ~~$\frac{\partial}{\partial t} \frac{\partial}{\partial x}$~~

- Parabolic: $\frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2}$



- Hyperbolic: $\frac{\partial}{\partial t} - \frac{\partial}{\partial x}$



Categories of mathematical problems:

	Algebra	Analysis
Linear	COMPUTABLE	NOT COMPUTABLE
nonlinear	NOT COMPUTABLE	NOT COMPUTABLE

Algebra: Only finite constructs

Analysis: Limits, derivates, integrals etc. ∞

DEF: COMPUTABLE = Exact solution can be obtained with finite computations.

The four principles:

- Discretization: 
- Linear algebra
- Linearization
- Iteration

Discretization (Yields computable problem)

The grid: $\Omega_N = \{x_0, x_1, \dots, x_N\}$ fr. $\Omega = [0, 1]$

Continuous function $f(x)$ is replaced by the vector $F = \{f(x_k)\}_{0}^N$.

Taylor expansion

Approximate $f(x)$ with a polynomial

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n + O(x^{n+1})$$

Derivate

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}, \text{ let } \Delta x = x_{k+1} - x_k$$

$$\Rightarrow f'(x_k) \approx \frac{f(x_k + x_{k+1} - x_k) - f(x_k)}{x_{k+1} - x_k} = \boxed{\frac{F_{k+1} - F_k}{x_{k+1} - x_k}} \boxed{f'(x_k)}$$

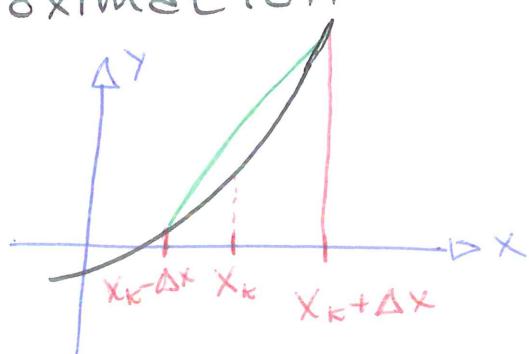
Testing the error: Plot error vs stepsize

in loglog and measure the slope.

$$\begin{aligned} r &= C \Delta x^p \\ \log(r) &= \log(C) + p \log(\Delta x) \end{aligned}$$

We can get faster convergence for $f'(x)$ if we use symmetric approximation

$$f'(x) = \boxed{\frac{F_{k+1} - F_{k-1}}{x_{k+1} - x_{k-1}}} + O(\Delta x^2).$$



Solving $\dot{Y} = q Y$

Analytical solution: $y(t) = y_0 e^{qt}$

Numerical approximation:

$$y_N = \left(1 + \frac{qT}{N}\right)^N y_0 \rightarrow e^{qT} \cdot y_0 \text{ when } N \rightarrow \infty$$

The numerical solution converges!