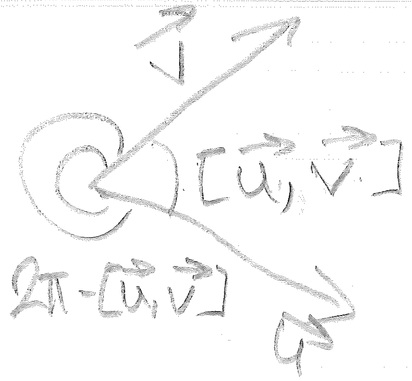
Skalärprodukt

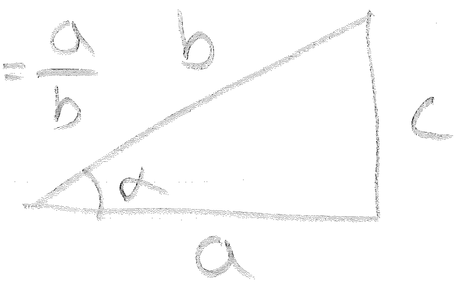
Def. Skalärprodukten av två vektorer $\vec{u}$ och $\vec{v}$ är talet givet av
$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos[\vec{u},\vec{v}]$$

Anm.
a) $\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$
b) "dot-product"
c) $\vec{u}=\vec{0}$ eller $\vec{v}=\vec{0} \Rightarrow \vec{u}\cdot\vec{v}=0$



Geom tolkn

$$\cos\alpha = \frac{\text{närligkatet}}{\text{hypote}} = \frac{a}{b}$$



$\Downarrow$

$a = b\cos\alpha$

$\Downarrow$

$|\vec{u}'| = |\vec{u}|\cos[\vec{u},\vec{v}]$

$\Downarrow$

$|\vec{v}||\vec{u}'| = |\vec{v}||\vec{u}|\cos[\vec{u},\vec{v}] = \vec{u}\cdot\vec{v}$

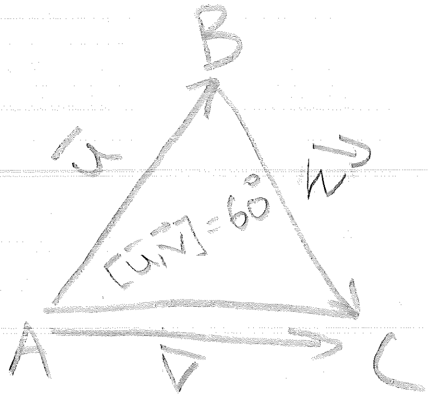
$$\vec{u}\cdot\vec{v} = \begin{cases} |\vec{u}'||\vec{v}| & \text{då } 0 \le [\vec{u},\vec{v}] < \frac{\pi}{2} \quad \text{pos.} \\ 0 & \text{då } [\vec{u},\vec{v}] = \frac{\pi}{2} \quad \text{noll} \\ -|\vec{u}'||\vec{v}| & \text{då } \frac{\pi}{2} < [\vec{u},\vec{v}] \le \pi \quad \text{neg.} \end{cases}$$

Def. Om $\vec{u}\cdot\vec{v}=0$ sägs vektorerna $\vec{u}$ och $\vec{v}$ vara ortogonala

Ex. Liksidig triangel ABC

$\vec{u}=\overrightarrow{AB}$, $\vec{v}=\overrightarrow{AC}$

$\vec{w}=\overrightarrow{BC}$



$\vec{u}\cdot\vec{v}=$

$=|\vec{u}||\vec{v}|\cos[\vec{u},\vec{v}]=$

$=1\cdot 1\cdot\cos\left(\frac{\pi}{3}\right)=$

$=\frac{1}{2}$

Sats: $\vec{u}, \vec{u_1}, \vec{u_2}, v$

$\lambda\in\mathbb{R}$

(i) $\vec{u}\cdot\vec{u}=|\vec{u}|^2$

(ii) $\vec{u}\cdot\vec{v}=\vec{v}\cdot\vec{u}$

(iii) $(\vec{u_1}+\vec{u_2})\cdot\vec{v}=\vec{u_1}\cdot\vec{v}+\vec{u_2}\cdot\vec{v}$

$(\vec{v}(\vec{u_1}+\vec{u_2})=\vec{v}\cdot\vec{u_1}+\vec{v}\cdot\vec{u_2})$

(iv) $(\lambda\vec{u})\cdot\vec{v}=\lambda(\vec{u}\cdot\vec{v})$

$\vec{u}\cdot(\lambda\vec{u})=\lambda(\vec{u}\cdot\vec{v})$