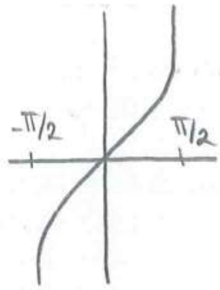
Seminarium

1 a) $\lim_{x\to\infty} \frac{\overset{\nearrow \text{begränsad}}{\arctan 2x}}{\underset{\searrow \infty}{3x}} = 0$



(0,3p)

$-\frac{\pi}{2} < \arctan y < \frac{\pi}{2}$

b) $\lim_{x\to 0} \frac{\arctan 2x}{3x} \overset{[t=\arctan 2x]}{=} \lim_{t\to 0} \frac{t}{3\frac{\tan t}{2}} = \lim_{t\to 0} \frac{t}{\frac{\sin t}{\cos t}} \cdot \frac{2}{3}$

$= \lim_{t\to 0} \frac{\overset{\nearrow 1}{\cos t}}{\underset{\searrow 1}{\frac{\sin t}{t}}} \cdot \frac{2}{3} = 1 \cdot \frac{2}{3} = \frac{2}{3}$ (0,3p)

c) $\lim_{x\to 1} \frac{x^{\overset{\text{ett tal}}{s}}-1}{\ln x} \overset{[t=x^s-1]}{=} \lim_{t\to 0} \frac{t}{\frac{\ln(t+1)}{s}} = \lim_{t\to 0} \frac{t}{\ln(t+1)} \cdot s =$

$x^s = t+1$

$s \ln x = \ln(t+1)$

$\ln x = \frac{\ln(t+1)}{s}$

$\lim_{t\to 0} \frac{1}{\frac{\ln(t+1)}{t}} s = s \cdot \frac{1}{\ln e} = s$

l'Hôpitals regel s. 236 viktig förutsättning

Sats $\lim_{x\to a} \frac{f(x)}{g(x)} \overset{\text{"}\frac{0}{0}\text{" "}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to a} \frac{f'(x)}{g'(x)}$ om gränsvärdet i HL existerar

Ex. $\lim_{x\to 0^+} \frac{2}{x} = \infty$ $\underbrace{\lim_{x\to 0^+} \frac{2}{x} \neq \lim_{x\to 0^+} \frac{(2)'}{(x)'} = \lim_{x\to 0^+} \frac{0}{1} = 0}_{\text{ej en lösning!}}$

$\boxed{(x^\alpha)' = \alpha x^{\alpha-1}}$

Ex. $\lim_{x\to 1} \frac{x^s-1}{\ln x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 1} \frac{(x^s-1)'}{(\ln x)'} = \lim_{x\to 1} \frac{s \cdot x^{s-1}}{\frac{1}{x}} = \frac{s \cdot 1}{1} = s$

Ex. $\lim_{x\to 0} \frac{1-\cos x}{x^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{(1-\cos x)'}{(x^2)'} = \lim_{x\to 0} \frac{\sin x}{2x} = \frac{1}{2}$

Ex. $\lim_{x\to 0} \frac{\sin x}{1-\sqrt{x+1}} = \lim_{x\to 0} \frac{\cos x}{0-\frac{1}{2}(x+1)^{1/2-1}\cdot 1} = \frac{1}{-\frac{1}{2}} = -2$ (0,5p)

Ex. Dela ut priser till 33 personer.

(0,5p)

Första pris: 10000 kr
Andra pris: $\frac{3}{4}$ av 1:a pris.
Tredje: $\frac{3}{4}$ av 2:a pris

⋮

Den totala prissumman $= 10000 + \frac{3}{4} \cdot 10000 + \left(\frac{3}{4}\right)^2 \cdot 10000 + \ldots + \left(\frac{3}{4}\right)^{32} \cdot 10000$

Man får en geometrisk summa

$$10000 \sum_{k=0}^{32} \left(\frac{3}{4}\right)^k = 10\,000 \, \frac{1-\left(\frac{3}{4}\right)^{32+1}}{1-\frac{3}{4}} = 40\,000 \left(1-\left(\frac{3}{4}\right)^{33}\right)$$

Ex. $\lim_{n \to \infty} \sum_{k=0}^{n} \left(\frac{4}{5}\right)^k = \sum_{k=0}^{\infty}$

8,20 Uttryck ${}^5\log x$ i ${}^3\log x$ Lösning: ${}^5\log x = \frac{{}^3\log x}{{}^3\log 5}$

8,57 ⓔ $\cos^4 x - \sin^4 x = \frac{1}{2} \iff \underbrace{(\cos^2 x + \sin^2 x)}_{1}(\cos^2 x - \sin^2 x) = \frac{1}{2}$

$$\cos^2 x - \sin^2 x = \frac{1}{2} \iff \cos 2x = \frac{1}{2} \iff 2x = \pm \frac{\pi}{3} + 2\pi n$$

$$x = \pm \frac{\pi}{6} + \pi n$$

ⓕ $\cos^4 x + \sin^4 x = \frac{1}{2}$

Lösning: $(\cos^2 x)^2 + (\sin^2 x)^2 + 2\cos^2 x \cdot \sin^2 x - 2\cos^2 x \cdot \sin^2 x = \frac{1}{2}$

$$= (\cos^2 x + \sin^2 x)^2 - 2\cos^2 x \sin^2 x = \frac{1}{2}$$

$$1 - \frac{1}{2}(2\sin x \cos x)^2 = \frac{1}{2} \qquad 1 - \frac{1}{2} = \frac{1}{2}$$

$$\sin 2x = \pm 1$$

9,40 ⓐ $\lim_{x \to \pi} \frac{\sin x}{\pi - x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to \pi} \frac{\cos x}{-1} = \frac{-1}{-1} = 1$

ⓑ $\lim_{x \to \pi} \frac{\cos x}{\pi - x} \overset{\text{"}\frac{-1}{\pm 0}\text{"}}{=} \infty$