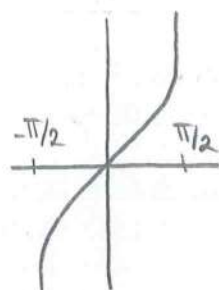


Seminarium

① $\lim_{x \rightarrow \infty} \frac{\arctan 2x}{3x} = 0$ → begränsad
 ② ∞



0,3p
 $-\frac{\pi}{2} < \arctan y < \frac{\pi}{2}$

② $\lim_{x \rightarrow 0} \frac{\arctan 2x}{3x} \quad [t = \arctan 2x] = \lim_{t \rightarrow 0} \frac{t}{3 \tan t} = \lim_{t \rightarrow 0} \frac{t}{3 \frac{\sin t}{\cos t}} = \lim_{t \rightarrow 0} \frac{t \cos t}{3 \sin t} = \frac{2}{3}$

$= \lim_{t \rightarrow 0} \frac{\cos t}{\frac{\sin t}{t}} \cdot \frac{2}{3} = 1 \cdot \frac{2}{3} = \frac{2}{3}$

0,3p

③ $\lim_{x \rightarrow 1} \frac{x^s - 1}{\ln x} \quad [t = x^s - 1] = \lim_{t \rightarrow 0} \frac{t}{\ln(t+1)} = \lim_{t \rightarrow 0} \frac{t}{\ln(t+1)} \cdot s =$

$x^s = t+1$

$\ln x = \ln(t+1)$

$\ln x = \frac{\ln(t+1)}{s}$

$\lim_{t \rightarrow 0} \frac{1}{\frac{\ln(t+1)}{t}} \cdot s = s \cdot \frac{1}{\ln e} = s$

1' Hôpital's regel s. 236 viktig förutsättning

Sats $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} \stackrel{\frac{0}{0} \text{ or } \frac{\infty}{\infty}}{=} \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ om gränsvärdet i Hz existerar

Ex. $\lim_{x \rightarrow 0^+} \frac{2}{x} = \infty \quad \lim_{x \rightarrow 0^+} \frac{2}{x} \neq \lim_{x \rightarrow 0^+} \frac{(2)'}{(x)'} = \lim_{x \rightarrow 0^+} \frac{0}{1} = 0$

ej en lösning!

$(x^\alpha)' = \alpha x^{\alpha-1}$

Ex. $\lim_{x \rightarrow 1} \frac{x^s - 1}{\ln x} \stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 1} \frac{(x^s - 1)'}{(\ln x)'} = \lim_{x \rightarrow 1} \frac{s \cdot x^{s-1}}{\frac{1}{x}} = \frac{s \cdot 1}{1} = s$

Ex. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 0} \frac{(1 - \cos x)'}{(x^2)'} = \lim_{x \rightarrow 0} \frac{\sin x}{2x} = \frac{1}{2}$

Ex. $\lim_{x \rightarrow 0} \frac{\sin x}{1 - \sqrt{x+1}} = \lim_{x \rightarrow 0} \frac{\cos x}{-\frac{1}{2}(x+1)^{-1/2} \cdot 1} = \frac{1}{-\frac{1}{2}} = -2$

0,5p

Ex. Dela ut priser till 33 personer.

0,5p

Första pris: 10000 kr

Andra pris: $\frac{3}{4}$ av 1:a pris

Tredje: $\frac{3}{4}$ av 2:a pris

⋮

Den totala prissumman = $10000 + \frac{3}{4} \cdot 10000 + \left(\frac{3}{4}\right)^2 \cdot 10000 + \dots + \left(\frac{3}{4}\right)^{32} \cdot 10000$

Man får en geometrisk summa

$$10000 \sum_{k=0}^{32} \left(\frac{3}{4}\right)^k = 10000 \frac{1 - \left(\frac{3}{4}\right)^{32+1}}{1 - \frac{3}{4}} = 40000 \left(1 - \left(\frac{3}{4}\right)^{33}\right)$$

$$\text{Ex. } \lim_{n \rightarrow \infty} \sum_{k=0}^n \left(\frac{4}{5}\right)^k = \sum_{k=0}^{\infty} \frac{1 - \frac{3}{4}}{1 - \frac{3}{4}}$$

8,20 Uttryck $^5 \log x$ i $^3 \log x$ lösning: $^5 \log x = \frac{^3 \log x}{^3 \log 5}$

$$8,57 \text{ (c)} \quad \cos^4 x - \sin^4 x = \frac{1}{2} \Leftrightarrow \underbrace{(\cos^2 x + \sin^2 x)}_1 (\cos^2 x - \sin^2 x) = \frac{1}{2}$$

$$\cos^2 x - \sin^2 x = \frac{1}{2} \Leftrightarrow \cos 2x = \frac{1}{2} \Leftrightarrow 2x = \pm \frac{\pi}{3} + 2\pi n$$

$$x = \pm \frac{\pi}{6} + \pi n$$

$$\text{(f)} \quad \cos^4 x + \sin^4 x = \frac{1}{2}$$

$$\text{lösning: } (\cos^2 x)^2 + (\sin^2 x)^2 + 2 \cos^2 x \cdot \sin^2 x - 2 \cos^2 x \cdot \sin^2 x = \frac{1}{2}$$

$$= (\cos^2 x + \sin^2 x)^2 - 2 \cos^2 x \sin^2 x = \frac{1}{2}$$

$$1 - \frac{1}{2} (2 \sin x \cos x)^2 = \frac{1}{2} \quad 1 - \frac{1}{2} = \frac{1}{2}$$

$$\sin 2x = \pm 1$$

$$9,40 \text{ (a)} \quad \lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x} \stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow \pi} \frac{\cos x}{-1} = \frac{-1}{-1} = 1$$

$$\text{(b)} \quad \lim_{x \rightarrow \pi} \frac{\cos x}{\pi - x} \stackrel{\frac{-1}{0}}{=} \infty$$