

6. Hyperfine structure and isotope shift

[6.1] The magnetic field in fine and hyperfine structure

Calculate B at the center of a H.

$1s^2S_{1/2}$ and $2s^2S_{1/2}$

$$\bar{B}_e = -\frac{e}{3} \mu_0 g_s \cdot \mu_B \cdot |\Psi_{ns}(0)|^2 \frac{Z^3}{a_0^3 n^3}$$

$$|\Psi_{ns}(0)|^2 = \frac{Z^3}{\pi a_0^3 n^3}$$

$$Z=1, s=1/2, n=\begin{cases} 1 \\ 2 \end{cases}, \Rightarrow B = \begin{cases} 16.7 \text{ T} \\ 2.1 \text{ T} \end{cases}$$

[6.2] Hyperfine structure in lithium.

Explain why the hyperfine splitting is of order M_e/M_p smaller than the fine structure splitting in $2p$ in Li.

6.4 Ratio of hyperfine splittings.

(spin, mag. mom)

Proton : ($\frac{1}{2}$, $2.79 \mu_N$)

Deutron : (1 , $0.857 \mu_N$)

${}^3\text{He}^+$: ($\frac{1}{2}$, $-2.13 \mu_N$)

	H	D	${}^3\text{He}^+$
I	$\frac{1}{2}$	1	$\frac{1}{2}$
F	1,0	$\frac{5}{2}, \frac{1}{2}$	1,0

a) calculate the ratio of the ground state splitting in atomic hydrogen and deuterium.

$$1s \Rightarrow J = \frac{1}{2}$$

$$\Delta \propto g_I \cdot \mu_N \cdot Z^3, g_I \propto \mu_I / I$$

$$\frac{E_H}{E_D} = \frac{A_H}{\frac{3}{2} A_D} = 4.3$$

$$b) \frac{E_H}{E_{{}^3\text{He}^+}} = \frac{A_H}{A_{{}^3\text{He}^+}} = -0.16$$

6.5 Interval for hyperfine structure.

a) Show that an interaction of the form $A \cdot \vec{I} \cdot \vec{J}$ leads to an interval rule. This means that the splitting between two sublevels is proportional to the angular momentum quantum number F of the sub-level with the larger F .

$$E_F = A \langle \vec{I} \cdot \vec{J} \rangle = \frac{A}{2} (F(F+1) - I(I+1) - J(J+1))$$

$$E_{F-1} = \frac{A}{2} ((F-1)F - I(I+1) - J(J+1))$$

$$E_F - E_{F-1} = \frac{A}{2} (F(F+1) - F(F-1)) = AF \quad \square$$

b) Is it the HF structure of $^8D_{11/2}$ in ^{153}Eu that determines the peak positions?
 What is the nuclear spin I of the isotope?
 Show that the spacing obeys the interval rule.
 Determine F for each peak.

$$I = ? , J = \frac{11}{2} , F_{\max} = 8 \Rightarrow I = F_{\max} - J = \frac{5}{2}$$

Motivering ges mha tabellen på nästa sida.

I Peak	II Position	III $\frac{E_{F-1} - E_F}{h}$	IV Ratio, x	V $\frac{x}{x-1} = F$
a	11.76	—	—	—
b	10.51	1.25	—	—
c	8.94	1.57	1.256	4.9
d	7.06	1.88	1.197	6.1
e	4.86	2.2	1.17	6.9
f	2.35	2.51	1.14	8.1

Vi ser alltså att $F_{\max} = 8$.

Hur tar vi fram J då?

Läs uppgiftstexten! Vi tittar ju på HF-strukturen i ${}^8P_{11/2} \Rightarrow J = 11/2$

$$\Rightarrow \text{--- } I = F - J = 8 - 11/2 = 5/2$$

c) What is A of ${}^8S_{7/2}$ for ${}^{153}\text{Eu}$?

$${}^{151}\text{Eu}: A({}^8S_{7/2}) = 20 \text{ MHz} \quad , \quad I = 5/2$$

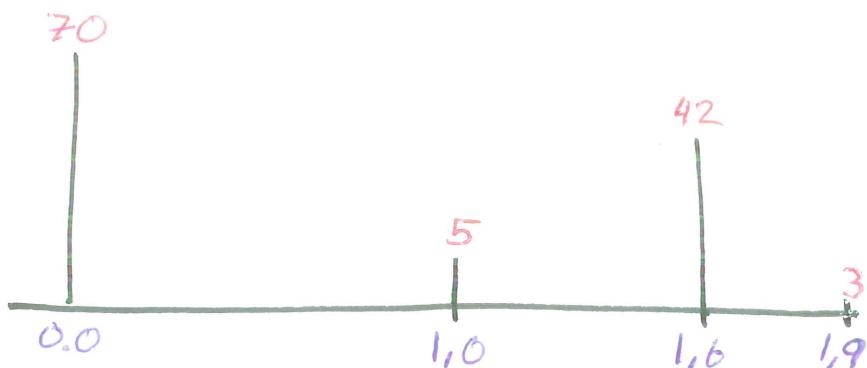
$$A({}^8S_{7/2}; 153) = 20 \cdot \frac{4.86 - 2.35}{6.412 - 0.77} = 8.9 \text{ MHz}$$

Vad gjorde jag just?

F6.7

Explain the origin of the structure and deduce nuclear spins and the ratio of the magnetic moments of the two isotopes.

Natural potassium: $^{39}\text{K} : ^{41}\text{K} = 14 : 1$



Vi undersöker förhållandet mellan intensiterna

* $\frac{70}{5} = 14$, $\frac{42}{3} = 14$ och $70/42 = \frac{5}{3}$, $\frac{5}{3} = \frac{5}{3}$.

Vi ser alltjänt att 70 och 42 förhåller sig till varandra som 5 och 3. Dessutom är förhållandet mellan 70 och 5 lika stort som mellan 42 och 3.

Utifrån detta kan slutsatsen att 70 och 42 hör till ena isotopen och 5 och 3 till den andra, dras.

Vilka hör till vilken isotop då?

Vi vet att det finns 14 ggr mer ^{39}K och df måste den hörta till 70 & 42.

Nuclear spins

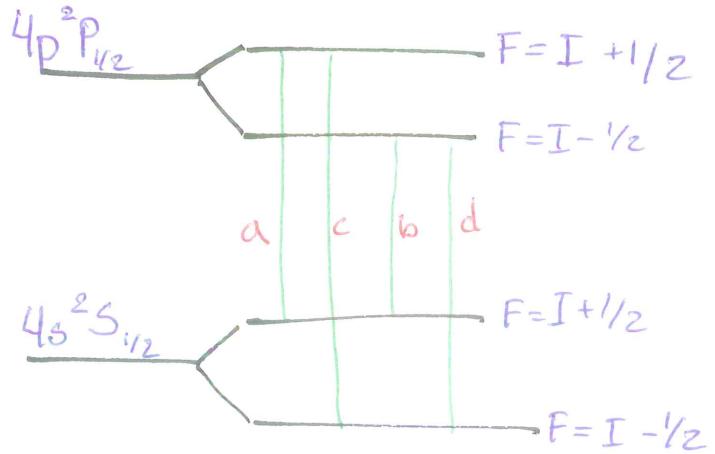
Vi använder intensity rule.

a - b

$$70 = K(2(I + \frac{1}{2}) + 1) \quad (*)$$

c - d

$$42 = K(2(I - \frac{1}{2}) + 1) \quad (\#)$$



(*) ÷ (#):

$$\frac{70}{42} = \frac{2I+1+1}{2I} = \frac{I+1}{I} \quad \xrightarrow{\text{SVAR}} \frac{70}{42} I = I+1 \quad \Leftrightarrow \boxed{I = \frac{3}{2}}$$

$$\text{Ratio: } \frac{1.6}{1.9 - 1.0} = \boxed{1.8}$$

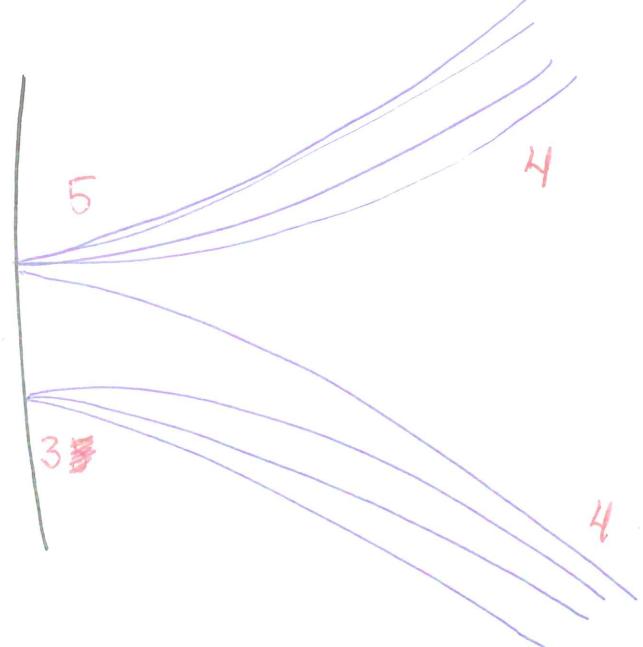
6.8

a) Nuclear spin?

$$F = I + J \quad (*)$$

Low B-field

$$\begin{aligned} 2F_1 + 1 &= 5 \\ 2F_2 + 1 &= 3 \end{aligned} \quad \left\{ \Leftrightarrow \begin{aligned} F_1 &= 2 \\ F_2 &= 1 \end{aligned} \right.$$



High B-field

$$\begin{aligned} 2J_1 + 1 &= 4 \\ 2J_2 + 1 &= 4 \end{aligned} \quad \left\{ \quad J = J_1 = J_2 = 3/2 \right.$$

$$(*) \Rightarrow I = F - J = 2 - 3/2 = \boxed{1/2}$$

b) Appropriate quantum numbers.

Hmm, which is your favourite quantum number? Mine is M_J , it's a good quantum number.

c) Show that the separation is the same in upper and lower HF levels for weak field.

We will start from $g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}$

$$\text{Low: } g_F = g_J \cdot \frac{1 \cdot 2 + \frac{1}{2} \cdot \frac{3}{2} + \frac{3}{2} \cdot \frac{5}{2}}{2 \cdot 6} = -\frac{1}{4} g_J$$

$$\text{High: } g_F = g_J \cdot \frac{2 \cdot 3 + \frac{1}{2} \cdot \frac{3}{2} - \frac{1}{2} \cdot \frac{5}{2}}{2 \cdot 6} = \frac{1}{4} g_J$$

□

d) Show that the states (in high energy) in the upper-most regime have the same separation Δ .

$$E_{ZE} = g_J \mu_B B M_J + A M_I M_J \quad (6.33)$$

Vi vet att hyperfinstrukturkonstanten A ges av:

$$A = \frac{2}{3} \mu_0 \cdot g_S \cdot \mu_B \cdot g_I \cdot \mu_N \cdot \frac{Z^3}{\pi a_0^3 \cdot n^3}$$

High energy $\Rightarrow \mu_B B \gg A$

\downarrow är samma för tillstånden.

\Rightarrow Zeemanuppsplittningen är samma, $\Delta = \tilde{\Delta}$. \square

e) Definiera vad som menas med "strong ~~energy~~ field".
Uppskatta gränsen numeriskt.

DEF: $\mu_B B \gg A$

Gräns: $\mu_B B = A = 3.4 \text{ GHz}$

$$\mu_B = 14 \Rightarrow B = \frac{3.4}{14} = 0.24 \text{ T} \quad \text{SVAR}$$

6.9 Isotope shift

Estimate the contributions to the isotope shift between $^{85}_{37}\text{Rb}$ and $^{87}_{37}\text{Rb}$ that arises from the following transitions.

a) $5s - 5p$ at a wavelength of $\sim 790\text{ nm}$

$$\lambda = 790\text{ nm}, A' = 85, A'' = 87, \delta A = 2$$

Mass shift

$$\Delta \tilde{\nu}_{\text{Mass}} = \frac{m_e}{M_p} \cdot \frac{\delta A}{A' \cdot A''} \cdot \tilde{\nu}_\infty$$

(Foot, s.106)

Vad är $\tilde{\nu}_\infty$?

$\tilde{\nu}_\infty$ är vägtalet för en teoretisk atom med oändligt tung kärna.

$$\tilde{\nu} = \frac{1}{\lambda} = \frac{1}{590\text{ nm}}$$

$$\Rightarrow \Delta \tilde{\nu}_{\text{Mass}} = \frac{9.109 \cdot 10^{-31}}{1.6726 \cdot 10^{-27}} \cdot \frac{2}{85 \cdot 87} \cdot \frac{1}{790 \cdot 10^9} = 0.186 \quad \Delta \tilde{\nu}_{\text{mass}}$$

Volume shift

$$\Delta \tilde{\nu}_{\text{vol}} = \frac{\Delta E_{\text{vol}}}{hc} = \frac{\langle r_N^2 \rangle}{a_0^2} \cdot \frac{\delta A}{A} \cdot \frac{Z^2}{(n^*)^3} \cdot R_\infty \quad (\text{Foot s.108})$$

$r_N \approx 1.2 \cdot A^{1/3} \text{ fm}$, vi antar att $A \approx 86$ & $\delta A = 2$.
 $n^* = 5$, $Z = 37$.

$$\Rightarrow \Delta \tilde{\nu}_{\text{vol}} = 0.028 \quad \Delta \tilde{\nu}_{\text{vol}} \Rightarrow \Delta V_{\text{tot}} =$$

b) $|5p - 7s \text{ at a wavelength of } \sim 730 \text{ nm}|$

Pg samma sätt som i a), vi löser denna uppg.

$$\Delta \tilde{v}_{\text{Mass}} = \frac{m_e}{M_p} \cdot \frac{\delta A}{A' \cdot A''} \cdot \tilde{v}_\infty = \frac{m_e}{M_p} \cdot \frac{2}{85 \cdot 87} \cdot \frac{1}{730 \cdot 10^{-9}} = \boxed{0.202}$$

$$\Delta \tilde{v}_{V_0} = \frac{\langle r_n^2 \rangle}{a_0^2} \cdot \frac{\delta A}{A} \cdot \frac{Z^2}{(n^*)^3} \cdot R_\infty =$$

6.11

Isotope shift

Estimate the relative atomic mass A for which the volume and mass effect give similar contribution to the isotope shift for $n^* \sim 2$ and a visible transition.

Assumptions: $\underbrace{A=2Z}_{Z=A/2}$, $n^*=2$, $\tilde{v}_\infty = 1/(500 \text{ nm})$

Lös ekvationen $\Delta \tilde{v}_{\text{Mass}} \approx \Delta \tilde{v}_{\text{Vol}}$ för A.

$$\frac{m_e}{M_p} \cdot \frac{\delta A}{A' \cdot A''} \tilde{v}_\infty = \frac{\langle r_N^2 \rangle}{a_0^2} \cdot \frac{\delta A}{A} \cdot \frac{Z^2}{(n^*)^3} \cdot R_\infty$$

$Z = A/2$

$$\Leftrightarrow \frac{A' A''}{A} \cdot \left(\frac{A}{2}\right)^2 = \frac{m_e}{M_p} \cdot \tilde{v}_\infty \cdot \frac{a_0^2}{\langle r_N^2 \rangle} \cdot \frac{(n^*)^3}{R_\infty}$$

~~Väntar att $A = A' = A''$~~

$$\Rightarrow A \approx 71$$

Lite shaky...