

1. Early Atomic Physics

1.1 Beräkna skillnaden i våglängd för Balmer- α linjen i väte och deuterium.

$$\text{Balmer-}\alpha \Rightarrow n=2, n'=3$$

$$\frac{1}{\lambda} = R \left(\frac{1}{n^2} - \frac{1}{n'^2} \right)$$

För H är $R=R_H$ och för ${}^2\text{H}$ är $R=R_{{}^2\text{H}}$.

$$\lambda_H - \lambda_{{}^2\text{H}} = \frac{1}{R_H \left(\frac{1}{n^2} - \frac{1}{n'^2} \right)} - \frac{1}{R_{{}^2\text{H}} \left(\frac{1}{n^2} - \frac{1}{n'^2} \right)} =$$

$$= \underbrace{\frac{1}{\left(\frac{1}{n^2} - \frac{1}{n'^2} \right)}}_{\substack{n=2, n'=3 \\ \Rightarrow = 7.2}} \cdot \left(\frac{1}{R_H} - \frac{1}{R_{{}^2\text{H}}} \right) = 7.2 \left(\frac{1}{R_H} - \frac{1}{R_{{}^2\text{H}}} \right)$$

$$R_H = R_\infty \cdot \frac{M_p}{m_e + M_p} \approx R_\infty \left(1 - \frac{m_e}{M_p} \right)$$

$$R_{{}^2\text{H}} = R_\infty \cdot \frac{2M_p}{m_e + 2M_p} \approx R_\infty \left(1 - \frac{m_e}{2M_p} \right)$$

$$\Rightarrow \lambda_H - \lambda_{{}^2\text{H}} \approx \frac{7.2}{R_\infty} \left(\frac{1}{1 - \frac{m_e}{M_p}} - \frac{1}{1 - \frac{m_e}{2M_p}} \right) \approx 1.95 \cdot 10^{-10} \text{ m}$$

1.2

Explain the similarities and differences between the spectra for H and He⁺.

Vi tog i förra uppgiften fram $R_{\text{He}^+} = R_{\text{H}_2}$.

$$R_{\text{H}} = R_{\infty} \left(1 - \frac{m_e}{M_p}\right)$$

$$R_{\text{He}^+} = R_{\infty} \left(1 - \frac{m_e}{2M_p}\right)$$

H (nm)	He (nm)
656.28	656.01
486.13	541.16
434.05	485.93
410.17	454.16
	433.87
	419.99
	410.00

Vi kan nu bryta ut λ ur Rydbergs fantastiska formel:

$$\frac{1}{\lambda} = R \left(\frac{1}{n^2} - \frac{1}{n'^2} \right) \Leftrightarrow \lambda = R^{-1} \left(\frac{1}{n^2} - \frac{1}{n'^2} \right)^{-1}$$

$$\frac{\lambda_{\text{H}}}{\lambda_{\text{He}^+}} = \frac{R_{\text{He}^+}}{R_{\text{H}}} = \frac{R_{\infty} \left(1 - \frac{m_e}{2M_p}\right)}{R_{\infty} \left(1 - \frac{m_e}{M_p}\right)} =$$

skit
detta.

$$= \frac{2M_p - m_e}{2(M_p - m_e)} = 1.00041$$

Detta stämmer med tabellen, jämför H med varannan He.

1.4

Show that (1.21) \approx (1.20) when $z \gg \sigma_k, \sigma_L$

$$(1.21): \frac{1}{\lambda} = R_\infty \left(\frac{(z - \sigma_k)}{1} - \frac{(z - \sigma_L)^2}{2} \right)$$

$$(1.20): \sqrt{f} \propto z$$

1.21 da $z \gg \sigma_k, \sigma_L$:

$$\approx \frac{1}{\lambda} = R_\infty \left(\frac{z^2}{1} - \frac{z^2}{4} \right) = R_\infty \cdot \frac{3}{4} z^2.$$

$$c = \lambda f \Rightarrow \frac{f}{c} = R_\infty \cdot \frac{3}{4} z^2$$

$$\Leftrightarrow \sqrt{f} = \underbrace{\sqrt{c R_\infty \frac{3}{4}}}_{\text{konstant}} \cdot z \Rightarrow \boxed{\sqrt{f} \propto z}$$



F1.9

Black body radiation

$$\lambda = 600 \text{ nm} , g_1 = 1 , g_2 = 3$$

Fraction in excited state is 0.1. $\Rightarrow \begin{cases} N_2 = 0.1 \\ N_1 = 0.9 \end{cases}$

What is the temperature of the black body?

What is the energy density per unit frequency $\rho(\omega)$?

Vi har en Boltzmanfördelning:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\frac{\Delta E}{kT}} \Rightarrow e^{\frac{h\nu}{kT}} = 27$$

$$\lambda = 600 \text{ nm} \Rightarrow \nu = \frac{2\pi c}{\lambda} = 3.13 \cdot 10^{15} \text{ rad/s}$$

$$\Rightarrow T = \frac{h \cdot 3.13 \cdot 10^{15}}{k \cdot \ln(27)} = 7.23 \cdot 10^3 = 7230 \text{ K}$$

$$\rho(\nu) = \frac{h\nu^3}{\pi^2 c^3} \cdot \frac{1}{\exp(h\nu/k_B T) - 1} \approx 4.455 \cdot 10^{-16} \text{ J s/m}^3$$

stämmer ej exakt med
facit men kanske har
jag noggrannare siffror.