1.19 a) $f = u + iv$ är holomorf

ortogonala om $\nabla u \cdot \nabla v = 0$

$$\begin{cases} u'_x = v'_y \quad , \quad u''_{xx} + u''_{yy} = 0 \\ u'_y = -v'_x \quad , \quad v''_{xx} + v''_{yy} = 0 \end{cases}$$

$$\left.\begin{array}{l} \nabla u = (u'_x, u'_y) \\ \nabla v = (v'_x, v'_y) \end{array}\right\} \Rightarrow \nabla u \cdot \nabla v = u'_x v'_x + u'_y v'_y = 0$$

$$\|\nabla u\| = \sqrt{u'^2_x + u'^2_y} = \sqrt{v'^2_y + v'^2_x}$$

$$\|\nabla v\| = \sqrt{v'^2_x + v'^2_y}$$

$$f' = u'_x - iu'_y \Rightarrow |f'| = \sqrt{(u'_x - iu'_y)(u'_x + iu'_y)} = \sqrt{u'^2_x + u'^2_y}$$

b) $f'(p) \neq 0$ & $u(x,y) = \operatorname{Re} f(p)$ ; $v(x,y) = \operatorname{Im} f(p) \Rightarrow$

$f(p) = u(x,y) + iv(x,y)$

$p = x + iy$

$\nabla u \perp u$ & $\nabla v \perp v$

Eftersom $u \perp v \Rightarrow \nabla u \perp \nabla v$

1.22 $f$ holomorf på hela $\mathbb{C}$ (hel)

Visa att $g(z) = \overline{f(\bar{z})}$ också är hel

$$f(z) = f(x+iy) = u(x,y) + i\,v(x,y)$$

$$\begin{cases} u'_x = v'_y \\ u'_y = -v'_x \end{cases}$$ måste vara uppfyllt

$$g(z) = g(x+iy) = \underbrace{u(x,-y)}_{a(x,y)} + i\cdot\underbrace{-v(x,-y)}_{b(x,y)}$$

$\operatorname{Re} g = a$, $\operatorname{Im} g = b$

$a'_x = u'_x$, $a'_y = -u'_y$, $b'_x = -v'_x$, $b'_y = v'_y$

$$\left.\begin{array}{l} u'_x = v'_y \Rightarrow a'_x = b'_y \\ u'_y = -v'_x \Rightarrow -a'_y = -(-b'_x) \Rightarrow a'_y = -b'_x \end{array}\right\}$$ stämmer