6. Stokes sats

6.1 Beräkna rot(F)

a) $\boxed{\bar{F} = (x,y,z)}$

$\mathrm{rot}(x,y,z) = (0-0, 0-0, 0-0) = \boxed{(0,0,0)}$

b) $\boxed{\bar{F} = (\sin(xy), e^{2xy}, \cos^2 zx)}$

$\mathrm{rot}(\bar{F}) = (0-0; 0+2z\sin(zx)\cos(zx), 2ye^{2xy} - x\cos(xy)) =$

$= \boxed{(0, z\sin(2zx), 2y^{2xy} - x\cos(xy))}$

c) $\boxed{\bar{F} = (xyz, x^2y^2z^2, y^2z^2)}$

$\mathrm{rot}(\bar{F}) = \boxed{(2yz^2 - 2x^2y^2z, xy, 2xy^2z^2 - xz)}$

6.2 Bestäm rotationen för A och B.

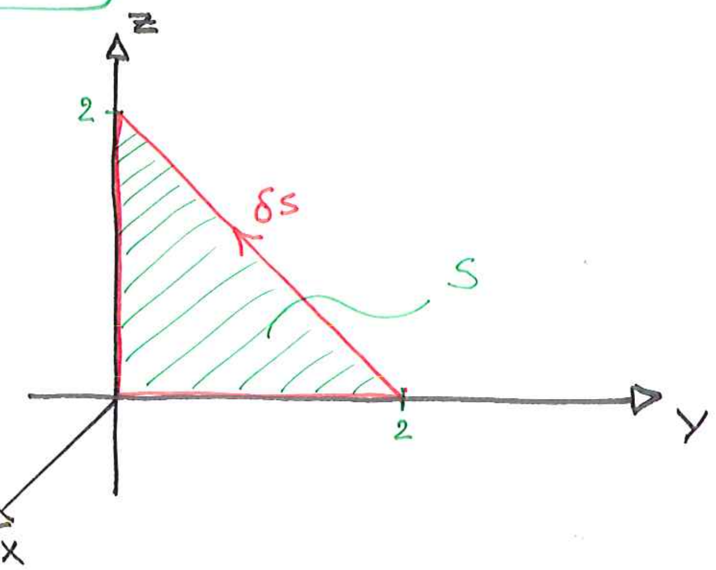
$A = (x,y,0)$, $B = (y,-x,0)$

$\mathrm{rot}(A) = (0,0,0) \leftarrow$ singularitet

$\mathrm{rot}(B) = (0,0,-2) \leftarrow$ rotation kring z-axeln i negativt led.

6.3 Kontrollera stokes sats

$$\bar{V} = (xy, 2yz, 3zx)$$



Stokes Sats

$$\iint_S \nabla \times \bar{V} \, d\bar{S} = \oint_{\delta s} \bar{V} \, d\bar{r}$$

$$\nabla \times \bar{V} = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \delta_x & \delta_y & \delta_z \\ xy & 2yz & 3zx \end{vmatrix} = (-2y, -3z, -x)$$

Vänsterledet

$d\bar{S} = dS \cdot \bar{n}$

Normalvektor

$$\iint_S \nabla \times \bar{V} \, d\bar{S} = \iint_S (-2y, -3z, -x) \, d\bar{S} = \iint_S (-2y, -3z, -x) \underbrace{(1,0,0)}_{\bar{n}} dS =$$

$$= \iint_S -2y \, dS = \left/ \begin{matrix} y: 0 \to 2 \\ z: 0 \to 2-y \\ dS = dz \cdot dy \end{matrix} \right/ = \int_0^2 \int_0^{2-y} -2y \, dz \, dy =$$

$$= \boxed{-\frac{8}{3}}$$

Högerledet

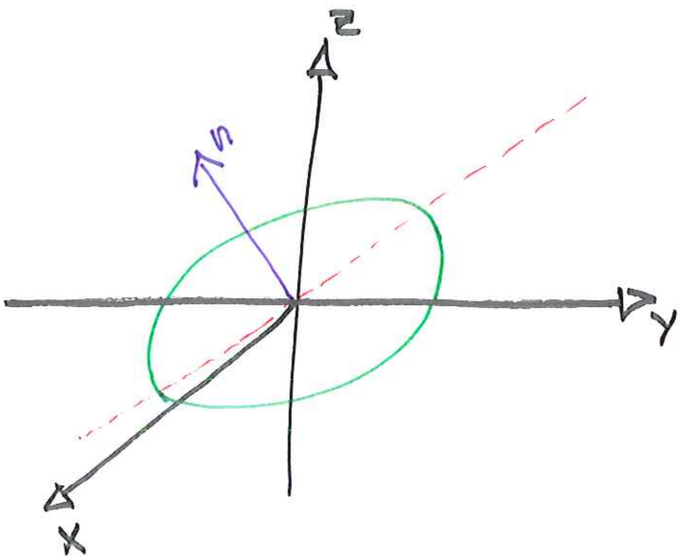
$$\oint_{\delta s} \bar{V} \, d\bar{r} =$$

$$= \underbrace{\int_0^2 \bar{V} \cdot (0,0,1) \, dy}_{=0} + \underbrace{\int_2^0 \bar{V} \cdot (0,1,0) \, dz}_{=0} + \int_2^0 \bar{V} \cdot (0,-1,1)(-dy) =$$

$$= \int_2^0 (-2yz + 3xz)(-dy) \Big|_{z=2-y} = \int_2^0 2y(2-y) \, dy = \boxed{-\frac{8}{3}}$$

#

$\boxed{6.4}$ $\boxed{\text{Beräkna } \oint_L F\,dF}$



$F = (x, x+y, x+y+z)$

$L: \begin{cases} x^2+y^2=1 \\ z = y \end{cases}$

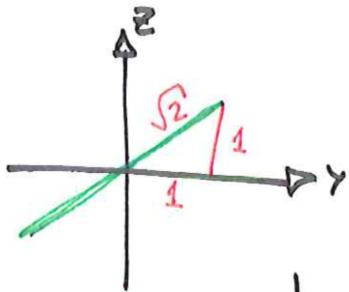
$\oint_L \bar{F} d\bar{r} = \iint_S \text{rot}(F)\cdot \bar{n}\, ds =$

$= \iint_S (1,-1,1)\cdot \bar{n}\cdot ds = \Big/ \bar{n} = \frac{1}{\sqrt{2}}(0,-1,1) \Big/ =$

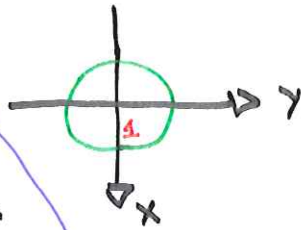
$= \iint_S (1,-1,1)(0,-1,1)\frac{1}{\sqrt{2}}\, ds = \frac{2}{\sqrt{2}} \iint_S ds =$

Arean ges av $ab\pi$.

$= \frac{2}{\sqrt{2}} \cdot \sqrt{2} \cdot 1 \cdot \pi = \boxed{2\pi}$



Här ser vi att ellipsen har halvaxlarna $a=\sqrt{2}$ och $1 = b$.

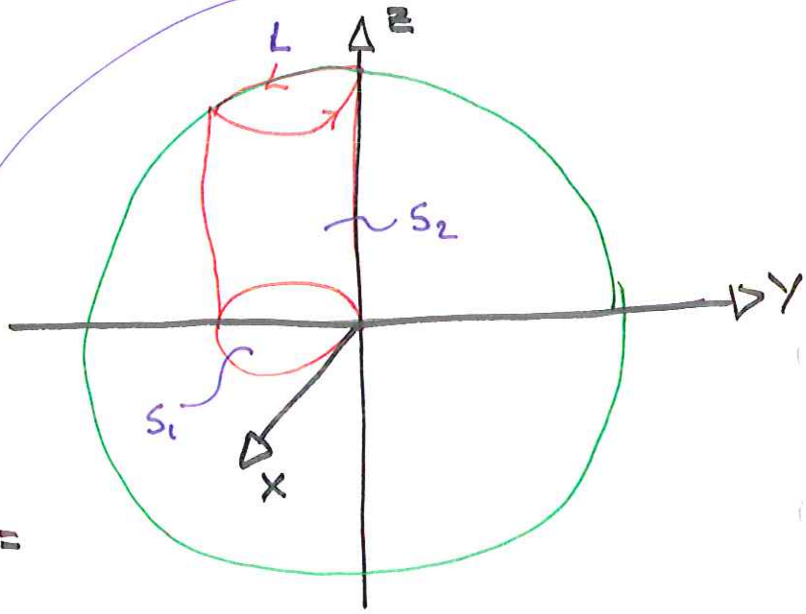


6.5 Beräkna $\oint_L A\,d\bar{r}$

$$A = \left(x^2 - a(y+z),\ y^2 - az,\ z^2 - a(x+y)\right)$$

$$C: \begin{cases} (x-a)^2 + y^2 = a^2 \\ z \geq 0 \end{cases}$$

$$S: x^2 + y^2 + z^2 = R^2$$



$$\oint_L A\,d\bar{r} = \iint_S \mathrm{rot}(A)\cdot\bar{n}\cdot ds =$$

$$= \iint_S (0,0,a)\cdot\bar{n}\cdot d\bar{s} =$$

$$= \iint_{S_1} (0,0,a)(0,0,1)\,ds + \cancel{\iint_{S_2} (0,0,a)(x,y,0)\,ds} =$$

$$= \iint_{S_1} a\,ds = a\pi a^2 = \boxed{\pi a^3}$$

Normalen har ingen komponent i z-led.
$\Rightarrow \iint_{S_2} \mathrm{rot}A\,\bar{n}\,ds = 0$

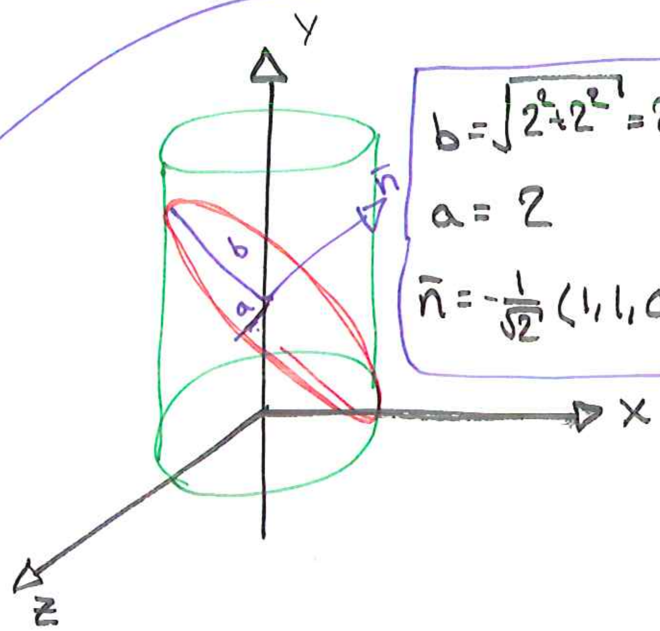
6.6 Beräkna linjeintegralen

$A = (yz + 2z,\ xy - x + z,\ xy + 5y)$

$C: x^2 + z^2 + 4$

$\pi: x + y = 2$

tangenten i $(2,0,0)$ är $(0,0,1)$



$b = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$a = 2$

$\bar{n} = -\frac{1}{\sqrt{2}}(1,1,0)$

$$\iint_S \operatorname{rot}(A)\, d\bar{S} = -\frac{1}{\sqrt{2}} \iint_S (x + 4 + 2)\, dS =$$

$$= -\frac{1}{\sqrt{2}} \iint_S (x + 6)\, dS = \Big/ \text{symmetri i x-led} \Big/ =$$

$$= -\frac{1}{\sqrt{2}} \iint_S 6\, dS = -\frac{1}{\sqrt{2}} \cdot 6 \cdot 2 \cdot 2\sqrt{2}\,\pi = \boxed{-24\pi}$$