2. OPERATIONSKALKYL

2.1 Varför finns $\underbrace{(B\times\nabla)}_{\text{vektor}}\cdot\underbrace{A}_{\text{vektor}}$ men ej $\underbrace{B}_{\text{vektor}}\times\underbrace{(\nabla\cdot A)}_{\text{skalär}}$?

Du kan inte kryssa en vektor med en skalär.

2.2 Beräkna divergensen för funktionerna

a) $\bar{V}_a = (x^2, 3xz^2, -2xz)$

$$\nabla\cdot\bar{V}_a = 2x + 0 - 2x = \boxed{0}$$

b) $\bar{V}_b = (xy, 2yz, 3xz)$

$$\nabla\cdot\bar{V}_b = \boxed{y + 2z + 3x}$$

c) $\bar{V}_c = (y^2, 2xy + z^2, 2yz)$

$$\nabla\cdot\bar{V}_c = 0 + \boxed{2x + 2y}$$

$\boxed{2.3}$ $\boxed{\text{Beräkna rotationen för } \bar{V}_a, \bar{V}_b, \bar{V}_c.}$

$$\nabla \times \bar{V}_a = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \partial_x & \partial_y & \partial_z \\ x^2 & 3xz^2 & -2xz \end{vmatrix} = (0-6xz, 0+2z, 3z^2-0) = \boxed{(-6xz, 2z, 3z^2)}$$

$$\nabla \times \bar{V}_b = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \partial_x & \partial_y & \partial_z \\ xy & 2yz & 3zx \end{vmatrix} = \boxed{(-2y, -3z, -x)}$$

$$\nabla \times \bar{V}_c = (2z-2z, 0-0, 2y-2y) = \boxed{(0,0,0)}$$

$\boxed{2.4}$ $\boxed{A = (x, 2y, 3z), \quad B = (3y, -2x, 0)}$

a) $\boxed{\text{Kontrollera produktregeln}}$ Visa detta!

$$\boxed{\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)}$$

$$\underline{VL}: \nabla \cdot \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ x & 2y & 3z \\ 3y & -2x & 0 \end{vmatrix} = \nabla \cdot (+6xz, 9yz, -2x^2-6y^2) = +6z+9z = \boxed{15z}$$

$$\underline{HL}: B \cdot \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \partial_x & \partial_y & \partial_z \\ x & 2y & 3z \end{vmatrix} - A \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \partial_x & \partial_y & \partial_z \\ 3y & -2x & 0 \end{vmatrix} =$$

$$= B \cdot (0-0, 0-0, 0-0) - A(0-0, 0-0, -2-3) = \boxed{15z} \quad \square$$

$\boxed{2.5}$ $\boxed{\text{Visa att } rot(grad(\phi)) = 0}$

$$grad(\phi) = \nabla f = \left(\frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz}\right)$$

$$rot(grad(\phi)) = \nabla \times (\nabla \cdot f) = \left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) \times \left(\frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz}\right) =$$

$$= \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ \frac{df}{dx} & \frac{df}{dy} & \frac{df}{dz} \end{vmatrix} = \underbrace{\left(\frac{d}{dy} \cdot \frac{df}{dz} - \frac{d}{dz} \cdot \frac{df}{dy}\right)}_{=0} \bar{e}_x +$$

$$+ \underbrace{\left(\frac{d}{dz} \cdot \frac{df}{dx} - \frac{df}{dz} \cdot \frac{d}{dx}\right)}_{=0} \bar{e}_y + \underbrace{\left(\frac{d}{dx} \cdot \frac{df}{dy} - \frac{df}{dx} \cdot \frac{d}{dy}\right)}_{=0} \bar{e}_z =$$

$$= (0,0,0) = \bar{0} \quad \#$$

$\boxed{2.6}$ $\boxed{\text{Visa att } div(rot(A)) = 0}$ $\quad A = (a_1, a_2, a_3)$

$$rot(A) = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \delta_x & \delta_y & \delta_z \\ a_1 & a_2 & a_3 \end{vmatrix} = \left(\frac{\delta a_1}{dy} - \frac{\delta a_2}{\delta z}, \frac{\delta a_1}{\delta z} - \frac{\delta a_3}{\delta x}, \frac{\delta a_2}{\delta x} - \frac{\delta a_1}{\delta y}\right)$$

$$div(rot(A)) = \left(\frac{\delta a_1}{\delta y} - \frac{\delta a_2}{\delta z}\right)\frac{\delta}{\delta x} + \left(\frac{\delta a_1}{\delta z} - \frac{\delta a_3}{\delta x}\right)\frac{\delta}{\delta y} + \left(\frac{\delta a_2}{\delta x} - \frac{\delta a_1}{\delta y}\right)\frac{\delta}{\delta z} =$$

$$= 0 \quad \#$$

$\boxed{2.7}$ $\boxed{\text{Beräkna grad}(r) \text{ och div}(\bar{r}) \text{ och rot}(\bar{r})}$

$$\boxed{\bar{r} = (x, y, z) \quad , \; r = \sqrt{x^2 + y^2 + z^2}}$$

$$\text{grad}(r) = \nabla \cdot r = \left(\frac{\delta r}{\delta x}, \frac{\delta r}{\delta y}, \frac{\delta r}{\delta z} \right) = \boxed{\frac{1}{r}(x, y, z)}$$

$$\text{div}(\bar{r}) = 1 + 1 + 1 = \boxed{3}$$

$$\text{rot}(\bar{r}) = (0, 0, 0) = \boxed{\bar{0}}$$