Föreläsning 4

Du förväntas kunna kapitel 5 sedan innan! (Gradienter, Gauss sats, Green-Gauss sats)
↖ Denna är ny!

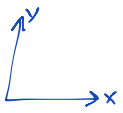
KAPITEL 6 - (3D/2D)-värmeledning

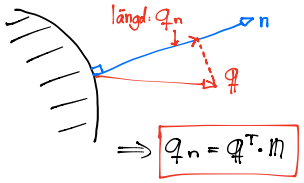
6 Strong and weak forms – two- and three-dimensional heat flow

Värmefluxvektor: $\underline{q}$

Riktn. på värmeflöde: $\frac{\underline{q}}{\|\underline{q}\|}$

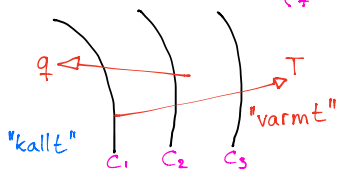
Betrakta en rand





$q_n > 0$ om värme lämnar kroppen.

Värmetransport (q behöver inte vara ortogonal)



, $c_1 < c_2 < c_3$

$\underline{q} = -K\frac{dT}{dx}$, Konstitutiv lag: $\begin{bmatrix} q_x \\ q_y \end{bmatrix} = -\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}\begin{bmatrix} T_x' \\ T_y' \end{bmatrix}$

$\underline{q}$ $\quad$ $\mathbb{D}$ $\quad$ ∇T

$(\nabla T)^T \underline{q} < 0$

$(\nabla T)^T \mathbb{D} \nabla T > 0$, ⚠ $\mathbb{D}$ är positivt definit!

Experiment säger att $\mathbb{D} = \mathbb{D}^T$ (Vi antar att detta är sant)

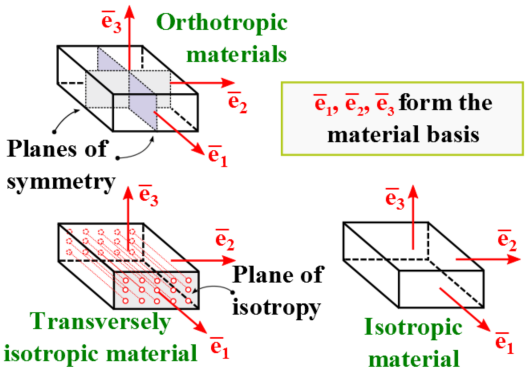
Typer av material (olika $\mathbb{D}$)

$\mathbb{D} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}$ Anisotropt material

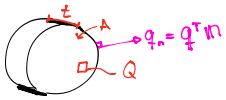
$\mathbb{D} = \begin{bmatrix} K_{xx} & 0 \\ 0 & K_{yy} \end{bmatrix}$ Ortotropt material

$\mathbb{D} = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} = K\mathbb{I}$ Isotropt material

↖ Oftast en bra approximation.



Balanslagar "In=Ut"



Global balans:

$$\int_A tQ\,dA = \int_L t\,q_n\,\delta L = -\oint_L t\,q^T n\,dL = \int_A \operatorname{div}(tq)\,dA =$$

(Gauss sats)

Stark form!

$$\int_A tQ\,dA = \int_A \operatorname{div}(tq)\,dA \Rightarrow \boxed{\int_A (\operatorname{div}(tq) - tQ)\,dA = 0}$$

Kan vi gå till svag form härifrån?

$\operatorname{div}(tq) - tQ = 0$ + RV / Stark form

1. Multiplicera med godtycklig viktfunktion $v = v(x,y)$

2. Integrera över kroppen

$$\int_A v \operatorname{div}(tq)\,dA - \int_A vtQ\,dA = 0$$

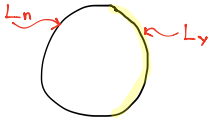
3. Generell "partialintegrering"

$$\int_A v \operatorname{div}(tq)\,dA = \int_L v t q^T n\,dL - \int_A (\nabla v)^T t q\,dA$$

Insatt

$$-\int_A (\nabla v)^T t\,q\,dA + \int_L v t q_n\,dL = \int_A vtQ\,dA$$

Randvillkor



$$\begin{cases} T = g \text{ längs } L_y \\ q_n = h \text{ längs } L_n \end{cases}$$

För att lösa systemet lägger vi till RV.

4. sätt in RV

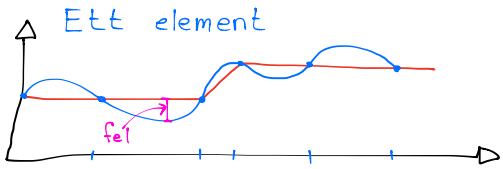
$$\begin{cases} \int_A (\nabla v)^T t\,\mathbb{D}\,\nabla T\,dA = -\int_{L_n} vth\,dL - \int_{L_y} vtq_n\,dL + \int_A vtQ\,dA \\ T = g \text{ längs } L_y \end{cases}$$

⚠ Svag term!

KAPITEL 7 (1D)

Ansats: $T = \alpha_1 + \alpha_2 x = \begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \bar{N}\alpha$

$$\begin{cases} T(x=x_i) = T_i = \alpha_1 + \alpha_2 x_i \\ T(x=x_j) = T_j = \alpha_1 + \alpha_2 x_j \end{cases} \Rightarrow \underbrace{\begin{bmatrix} T_i \\ T_j \end{bmatrix}}_{a^e} = \underbrace{\begin{bmatrix} 1 & x_i \\ 1 & x_j \end{bmatrix}}_{\mathcal{C}} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}}_{\alpha}$$

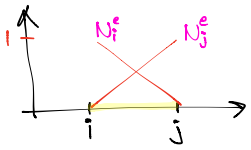


⚠ $\mathcal{C}$ är inverterbar! $\Rightarrow \alpha = \mathcal{C}^{-1} a^e$

$$T = \underbrace{\bar{N}\mathcal{C}^{-1}}_{N^e} a^e = N^e a^e \quad , \quad N^e = \text{elementformfunktioner}$$

$$\boxed{T = N^e a^e}$$

$$\mathcal{C}^{-1} = \frac{1}{\underbrace{x_j - x_i}_{L_e}} \begin{bmatrix} x_j & -x_i \\ -1 & 1 \end{bmatrix} \Rightarrow N^e = \bar{N}\mathcal{C}^{-1} = \begin{bmatrix} N_i^e & N_j^e \end{bmatrix} = \frac{1}{L_e} \begin{bmatrix} x_j - x, & -x_i + x \end{bmatrix}$$



← VAD ÄR DETTA?!

7 Choice of approximating functions for the FE method – scalar problems

4.1.3 Shape Function

The shape function is the function which interpolates the solution between the discrete values obtained at the mesh nodes. Therefore, appropriate functions have to be used and, as already mentioned, low order polynomials are typically chosen as shape functions. In this work linear shape functions are used.

For three-dimensional finite element simulations it is convenient to discretize the simulation domain using tetrahedrons, as depicted in Figure 4.1. Thus, linear shape functions must be defined for each tetrahedron of the mesh, in order to apply the Galerkin method described in Section 4.1.1.

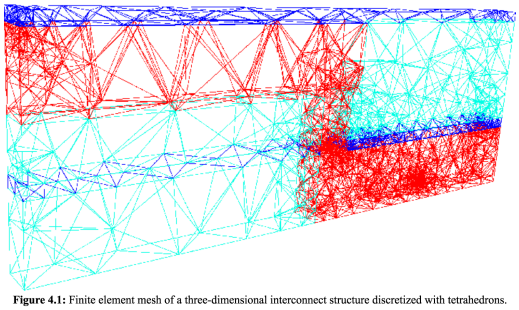


Figure 4.1: Finite element mesh of a three-dimensional interconnect structure discretized with tetrahedrons.

Chapter 18: SHAPE FUNCTION MAGIC

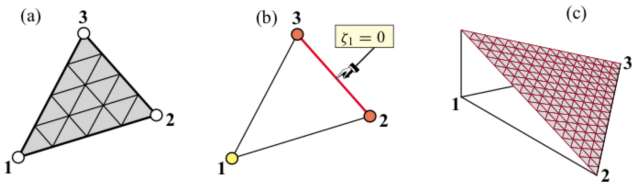


FIGURE 18.1. The three-node linear triangle: (a) element geometry; (b) equation of side opposite corner 1; (c) perspective view of the shape function $N_1 = \zeta_1$.

Globala formfunktioner

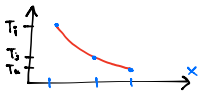
$$T = \begin{bmatrix} N_i & N_j & \dots \end{bmatrix} \begin{bmatrix} T_i \\ T_j \\ \vdots \end{bmatrix} = \mathbb{N}a$$

Storleken på $\mathbb{N}$ & a bestäms av antalet noder!

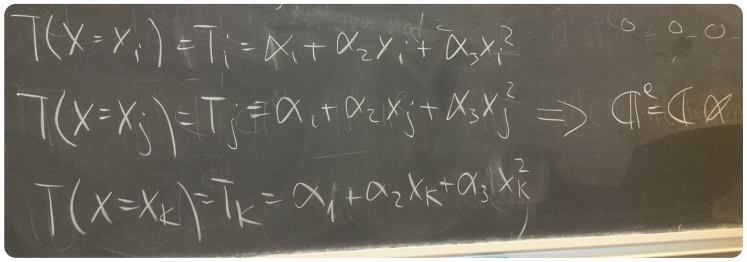
Gradienten av T.

$$\frac{\partial T}{\partial x} = \frac{\partial \mathbb{N}}{dx} a + \mathbb{N} \frac{da}{dx} = \frac{d\mathbb{N}}{dx} a = \mathbb{B}a$$

Kvadratiskt element



$$T = \alpha_1 + \alpha_2 x + \alpha_3 x^2$$

$$T = \begin{bmatrix} 1 & x & x^2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \mathbb{N}\alpha$$

$$T(x = x_i) = T_i = \alpha_1 + \alpha_2$$