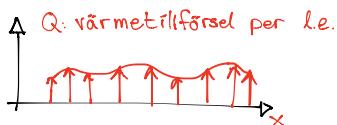
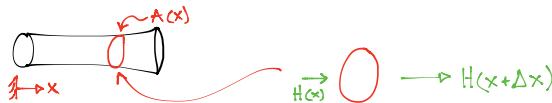


Föreläsning 3

1D-värmeledning



Lika mycket in som ut!

Stationär värmeledning

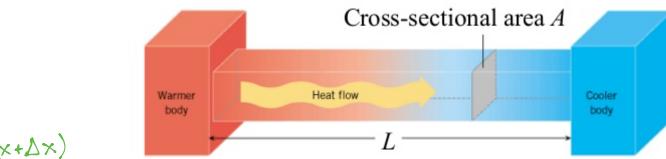
$$\Rightarrow H(x) + \int_x^{x+\Delta x} Q(\xi) d\xi = H(x+\Delta x)$$

Mha medelvärddessatsen får vi:

$$\int_x^{x+\Delta x} Q(\xi) \delta \xi = Q(\xi) \Delta x, \quad \xi \in [x, x+\Delta x]$$

$$\Rightarrow \frac{H(x+\Delta x) - H(x)}{\Delta x} - Q(\xi) = 0$$

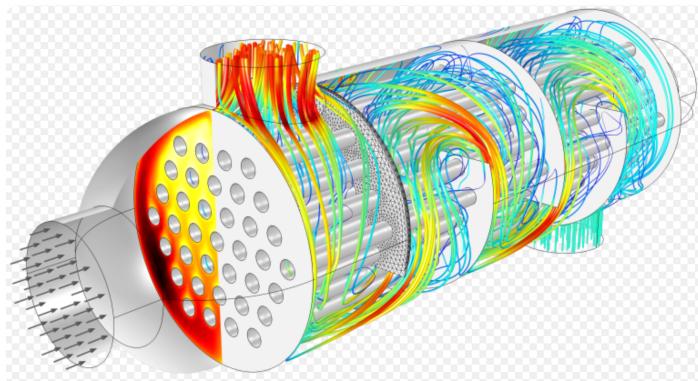
Balanslag
 $\Delta x \rightarrow 0 \Rightarrow \frac{dH}{dx} - Q = 0$



$$Q = k A \left(\frac{\Delta T}{L} \right) t$$

What is the unit of k ?

Q = heat transferred
 k = thermal conductivity
 A = cross sectional area
 ΔT = temperature difference between two ends
 L = length
 t = duration of heat transfer



Definition: Värmeflux q :

$$q = \frac{H}{A} \Rightarrow \frac{d}{dx}(Aq) - Q = 0$$

Hur tolkar vi q ? Vi gör lite rimliga antaganden om värmeledning i en stång. Typ, ju större temp差, desto större flöde. Area? Längd?

Gissning: $q = -k \frac{\Delta T}{\Delta x}$ Konstitutiv lag (Fourier)
Ej huggen i sten.

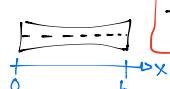
Randvillkor

Dirchlet/Neumann

$$q(x=0) = -(k \frac{dT}{dx})_0 = h$$

Neumann

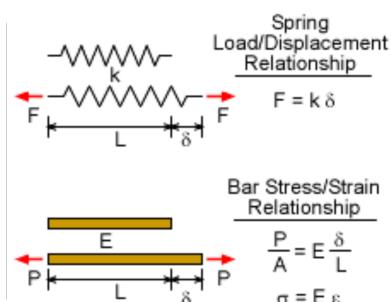
Ex.



$$T(x=L) = g$$

Dirchlet

Naturliga = Neumann
 Essentiella = Dirchlet



Värmeledning och en axiellt belastad stång beskrivs av samma diffekvationer!

Stark \Rightarrow Svag

Stark form: $\frac{d}{dx} (AK \frac{dT}{dx}) + Q = 0$

1- Multiplisera med godtycklig vikt-funktion V .

$$\Rightarrow V \frac{d}{dx} (AK \frac{dT}{dx}) + VQ = 0$$

2- Integrera över kroppen.

$$\int_0^L V \frac{d}{dx} (AK \frac{dT}{dx}) dx + \int_0^L VQ dx$$

3- Partialintegrera första termen

$$\left[VAK \frac{dT}{dx} \right]_0^L - \int_0^L \frac{dV}{dx} AK \frac{dT}{dx} + \int_0^L VQ dx = 0$$

4- Sätt in de naturliga randvillkoren

$$\left[VAK \frac{dT}{dx} \right]_0^L - \left[VAq \right]_0^L = -(VAq)_{x=L} + (VAq)_{x=0} = -(VAq)_{x=L} + (VA)h$$

Svag form: $\int_0^L \frac{dV}{dx} AK \frac{dT}{dx} dx = -(VAq)_{x=L} + (VA)h + \int_0^L VQ dx$

Kan vi gå åt andra hållet?

Svag \Rightarrow Stark

1. Partialintegrera V.L.

$$\left[VA K \frac{dT}{dx} \right]_0^L - \int_0^L V \frac{d}{dx} (AK \frac{dT}{dx}) dx = -(VAq)_{x=L} + (VA)h + \int_0^L VQ dx$$

eller,

$$\left\{ \begin{array}{l} (VAq)_{x=0} - \int_0^L V \frac{d}{dx} (AK \frac{dT}{dx}) dx = (VA)h + \int_0^L VQ dx \\ T(x=L) = g \end{array} \right.$$

2. Välj ett smart V , $V(x) = \phi \left[\frac{d}{dx} (AK \frac{dT}{dx}) + Q \right]$, $\phi = \begin{cases} 0 & x=0 \\ 0 & 0 < x < L \\ 1 & x=L \end{cases}$

$$\Rightarrow 0 = \int_0^L \phi \left[\frac{d}{dx} (AK \frac{dT}{dx}) + Q \right]^2 dx \Rightarrow \frac{d}{dx} (AK \frac{dT}{dx}) + Q = 0, \text{ Nytt val av } V(x) = \begin{cases} 0 & x=0 \\ 0 & \text{för övrigt} \end{cases} \Rightarrow [Q=h]$$

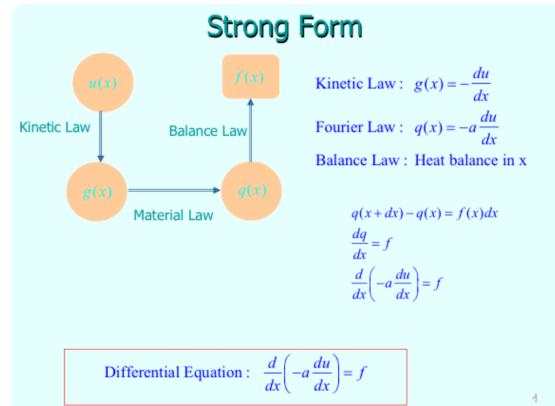
Senare i kursen kommer vi ej kunna gå åt båda hållen!

Weak formulation tillåter diskontinuiteter

Page issues

Weak formulations are an important tool for the analysis of mathematical equations that permit the transfer of concepts of **linear algebra** to solve problems in other fields such as **partial differential equations**. In a weak formulation, an equation is no longer required to hold absolutely (and this is not even well defined) and has instead **weak solutions** only with respect to certain "test vectors" or "test functions". This is equivalent to formulating the problem to require a solution in the sense of a distribution.

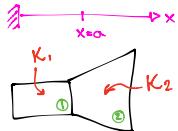
We introduce weak formulations by a few examples and present the main theorem for the solution, the **Lax-Milgram theorem**.



4

7

En ny "endimensionell" stång.

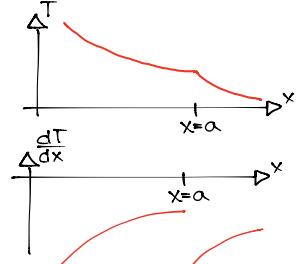


Värmet som passar $x=a$ från kropp

① till kropp ② kan betecknas:

$$-(AK_1 \frac{dT}{dx})_{x=a} = -(AK_2 \frac{dT}{dx})_{x=a} \quad (H_1 = H_2)$$

Kan bli meckigt



Del 1

$$\frac{d}{dx}(AK_1 \frac{dT}{dx}) + Q = 0$$

$$q(x=0) = h \text{ (antagande)} \quad q(x=L) = h \text{ (antagande)}$$

Del 2

$$\frac{d}{dx}(AK_2 \frac{dT}{dx}) + Q = 0$$

Vi kombinerar detta

$$\Rightarrow -(AK_1 \frac{dT}{dx})_{x=a} = -(AK_2 \frac{dT}{dx})_{x=a}$$

Swag form på del 1

$$\int_0^L \frac{d}{dx} (AK_1 \frac{dT}{dx}) dx = -(VAq)_{x=0} + (VAq)_{x=L} + \int_0^L VQ dx$$

Swag form på del 2

$$\int_a^L \frac{d}{dx} (AK_2 \frac{dT}{dx}) dx = -(VAq)_{x=L} + (VAq)_{x=a} + \int_a^L VQ dx$$

Vad händer om vi summerar dessa?

$$\int_a^L \frac{d}{dx} (AK(x) \frac{dT}{dx}) dx = (VAq)_{x=0} - (VAq)_{x=L} + \int_a^L VQ dx, \quad K = \begin{cases} K_1, & x < a \\ K_2, & x > a \end{cases}$$