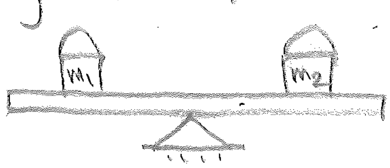
Föreläsning 8 04/02-15

Masscentrum

Tyngdpunkt: Den punkt där tyngdkraftens kraftresultant angriper.

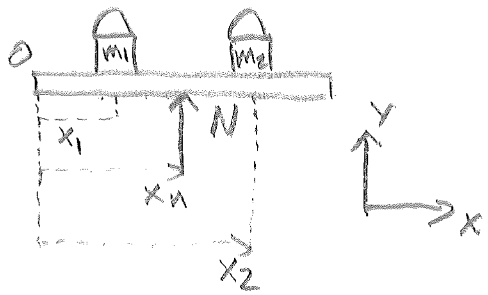
Momentprincipen

Var ska stödet placeras för att uppnå momentjämvikt för gungbrädan?
Glatta kontaktytor.



Frilägg:

Masslös gungbräda



$$\Sigma F_y = 0 \quad -m_1 g + N - m_2 g = 0 \Rightarrow N = (m_1 + m_2) g$$
$$\Sigma \overset{\curvearrowleft}{M_O} = 0 \quad -m_1 g x_1 + N x_N - m_2 g x_2 = 0$$
$$\Rightarrow x_N = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

x_N: Masscentrums placering för kroppen massa 1 + massa 2 + gungbrädan.

Tyngdpunkt tar hänsyn till variation i "g".
För ett partikelsystem med N partiklar ges masscentrum av:

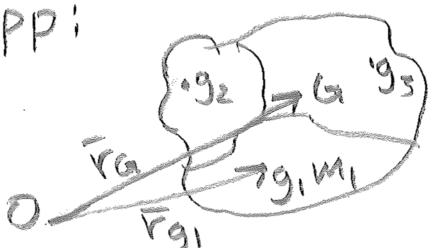
$$\bar{r}_G = \frac{\sum_k m_k \bar{r}_k}{\sum_k m_k}$$

Uttrycket kan separeras i tre oberoende riktningar:

$$z_N = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$

För en sammansatt kropp:

$$\bar{r}_G = \frac{\sum_k m_k \bar{r}_{gk}}{\sum_k m_k}$$



G: hela kroppens masscentrum
g: respektive delkropps masscentrum

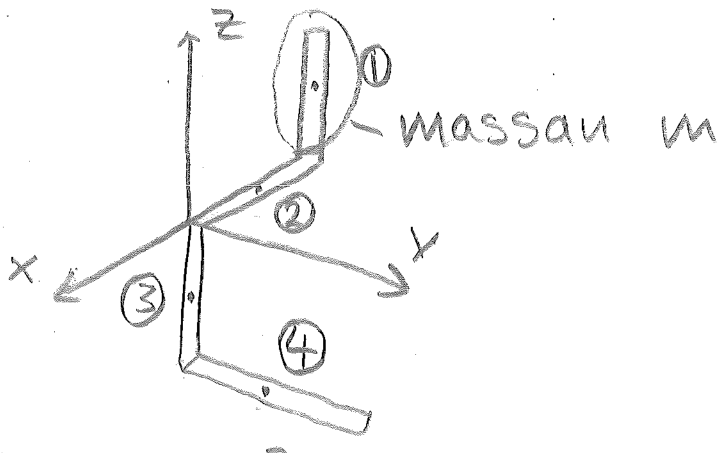
Delkropparnas storlek kan väljas som infinitesimal
Kontinuerlig massfördelning:

$$\bar{r}_G = \frac{\int \bar{r}_g \, dm}{\int dm}$$

Exempel på masscentrumsberäkningar, s. 97-101.

Uppgift 4.1

$$\bar{r}_G = \frac{\sum_k m_k \bar{r}_{gk}}{\sum_k m_k}$$



x-led:

$$\frac{m(-a) + m(-a/2) + m \cdot 0 + m \cdot 0}{m + m + m + m} = \frac{-3}{8} a$$

y-led:

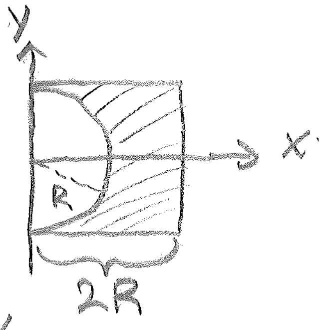
$$\frac{m \cdot 0 + m \cdot 0 + m \cdot 0 + m \cdot a/2}{m + m + m + m} = \frac{1}{8} a$$

$$\bar{r}_G = \left(-\frac{3}{8}a, \frac{1}{8}a, -\frac{1}{4}a\right)$$

z-led:

$$\frac{m \cdot a/2 + m \cdot 0 + m \cdot (-a/2) \cdot m \cdot (-a)}{m + m + m + m} = -\frac{1}{4} a$$

Exempel 9, s 101 (Jämför!)

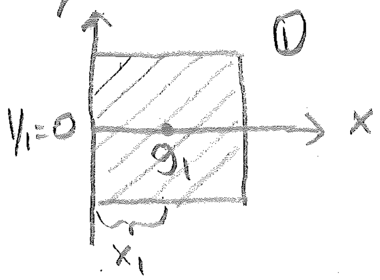


Sökt: masscentrum

Homogent material, $m = \rho A$

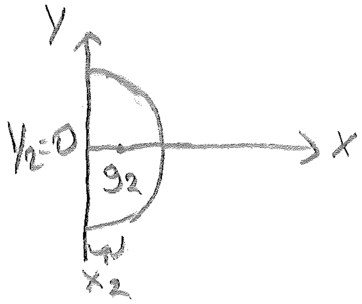
ρ: densitet [massa/areaenhet]

A: area



$$x_G = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

$$y_G = \frac{y_1 m_1 + y_2 m_2}{m_1 + m_2}$$



① $x_1 = R$

$y_1 = 0$

$m_1 = \rho A_1 = \rho 2R \cdot 2B$

② $x_2 = \frac{4R}{3\pi}$ (Tabell)

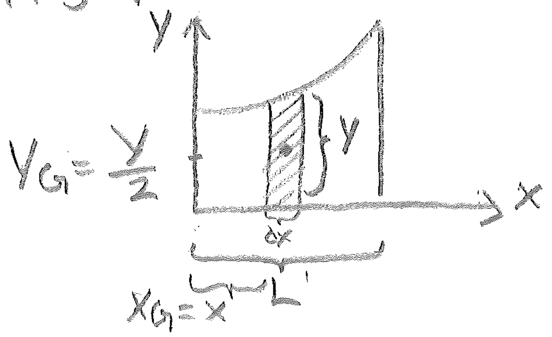
$y_2 = 0$

$m_2 = -\rho A_2 = -\rho \frac{\pi R^2}{2}$ (Tabell)

$$x_G = \frac{R(\rho \cdot 2R \cdot 2R) + \frac{4R}{3\pi}\left(-\rho \frac{\pi R^2}{2}\right)}{\rho \cdot 2R \cdot 2R - \rho \frac{\pi R^2}{2}} = \frac{20R}{24 - 3\pi}$$

$y_G = 0$

Uppgift 4.6



$y = a + bx^2$

$a = 5 \, m$

$b = 0{,}0008 \, m^{-1}$

$$x_G = \frac{\int x_g \, dm \; ②}{\int dm \; ①}$$

$$y_G = \frac{\int y_g \, dm \; ③}{\int dm \; ①}$$

① $\int_{x=0}^{L} \rho (a+bx^2) \, dx$

② $\int_{x=0}^{L} x \cdot \rho (a+bx^2) \, dx$

③ $\int_{x=0}^{L} \left(\frac{a+bx^2}{2}\right) \cdot \rho (a+bx^2) \, dx$