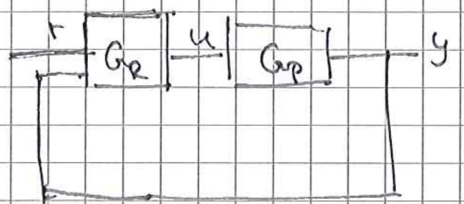
1/2 2012 (8)



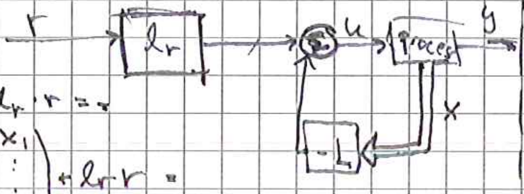
$G_r = \frac{G_R G_p}{1+G_R G_p}$

Tillståndsåterkoppling

Process: $\begin{cases}\dot{x} = Ax + Bu \\ y = Cx\end{cases}$ | $Y = C(sI-A)^{-1}BU.$

Karakteristiskt polynom: $\det(sI-A)$.

Regulator:



$u = -Lx + l_r \cdot r =$

$= -(l_1, \dots l_n)\begin{pmatrix}x_1\\ \vdots \\ x_n\end{pmatrix} + l_r r =$

$= -l_1x_1 - l_2x_2 - \dots - l_n x_n + l_r \cdot r$

Slutna systemet

$\begin{cases}\dot{x} = Ax + Bu = Ax - BLx + Bl_r \cdot r = (A-BL)x + Bl_r \cdot r \\ y = Cx\end{cases}$

$Y = C(sI-(A-BL))^{-1} B l_r R.$

Karakteristiska ekvationen:
$\det(sI-(A-BL))$

Välj L så att egenvärdena till $(A-BL)$ placeras lämpligt.

Välj l_r så att $y = r$ stationärt

Ex| (motor)

$G_p = \frac{180}{s(s+10)}$

$x_1 = \frac{100}{s+10}u \Rightarrow sX_1 + 10X_1 = 100u$

$\xrightarrow{\mathcal{L}} \dot{x}_1 + 10x_1 = 100u$

$\dot{x}_1 = -10x_1 + 100u$

$x_2 = \frac{1}{s}x_1$

$sX_2 = X_1$

$\xrightarrow{\mathcal{L}^{-1}} \dot{x}_2 = x_1$

$\therefore \begin{cases}\dot{x}_1 = -10x_1 + 100u \\ \dot{x}_2 = x_1 \\ y = x_2\end{cases}$

$\Leftrightarrow$

$\dot{x} = \underbrace{\begin{bmatrix}-10 & 0 \\ 1 & 0\end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix}100 \\ 0\end{bmatrix}}_{B} u$

$y = \underbrace{(0 \ 1)}_{C}\, x$

$u = -Lx + l_r \cdot r$

$\begin{cases}\dot{x} = (A-BL)x + Bl_r \cdot r \\ y = Cx\end{cases}$

~~det(sI-(A-B~~

$A - BL = \begin{bmatrix}-10 & 0 \\ 1 & 0\end{bmatrix} - \begin{pmatrix}100 \\ 0\end{pmatrix}(l_1 \ l_2) =$

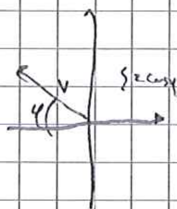
$\begin{bmatrix}-10 & 0 \\ 1 & 0\end{bmatrix} - \begin{bmatrix}100l_1 & 100l_2 \\ 0 & 0\end{bmatrix} = \begin{bmatrix}-10-100l_1 & -100l_2 \\ 1 & 0\end{bmatrix}$

$\det(sI-(A-BL)) = \begin{vmatrix}s+10+100l_1 & 100l_2 \\ -1 & s\end{vmatrix} =$

$= s^2 + (10+100l_1)s + 100l_2$

Önskat polynom: $s^2 + 2\zeta\omega s + \omega^2$

$10 + 100l_1 = 2\zeta\omega$
$100l_2 = \omega^2$
$\Rightarrow \begin{cases}l_1 = \frac{2\zeta\omega - 10}{100} \\ l_2 = \frac{\omega^2}{100}\end{cases}$



Ex|

$\dot{x} = \underbrace{\begin{bmatrix}-1 & 0 \\ 0 & -2\end{bmatrix}}_{A} x + \underbrace{\begin{pmatrix}1 \\ 0\end{pmatrix}}_{B} u$

$y = Cx + Du$ | $u = -Lx + l_r \cdot r$

$A - BL = \begin{bmatrix}-1 & 0 \\ 0 & -2\end{bmatrix} - \begin{pmatrix}1 \\ 0\end{pmatrix}(l_1 \dots l_2) = \begin{bmatrix}-1-l_1 & -l_2 \\ 0 & -2\end{bmatrix}$

$\det(sI-(A-BL)) = \begin{vmatrix}s+1+l_1 & l_2 \\ 0 & s+2\end{vmatrix} = (s+1+l_1)(s+2)$

$\begin{cases}\dot{x}_1 = -x_1 + u \\ \dot{x}_2 = -2x_2\end{cases}$

Styrbarhet:

Ett tillstånd x_0 är styrbart om det finns en styrsignal som för tillståndet x från origo till x_0 på ändlig tid.

Ett system är styrbart $\Leftrightarrow$ Samtliga tillstånd är styrbara $\Leftrightarrow$ Egenvärdena till $(A-BL)$ kan placeras godtyckligt.

Styrbarhetsmatrisen: $W_s = [B \ AB \ A^2B \ A^{n-1}B]$

W_s har n linjärt oberoende kolonner

$\Leftrightarrow$ $\quad \Leftrightarrow \det \neq 0$

Systemet styrbart.

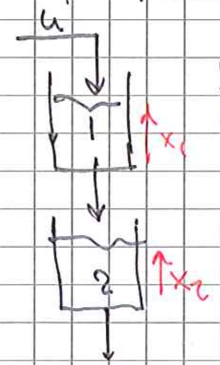
Ex 1:
Motorn: $A = \begin{pmatrix}-10 & 0 \\ 1 & 0\end{pmatrix}$, $B = \begin{pmatrix}100 \\ 0\end{pmatrix}$

$W_s = \begin{pmatrix}100 & -1000 \\ 0 & 100\end{pmatrix}$ $\det W_s = 10000 \neq 0 \Rightarrow$ Styrbart

Ex 2: $A = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$ $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$W_s = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$ $\det W_s = 0 \Leftrightarrow$ Ej styrbar.

Exempel på styrbarhet

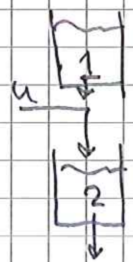
1) 

$$\begin{cases} \dot{x}_1 = u - ax_1 \\ \dot{x}_2 = ax_1 - ax_2 \end{cases}$$

$$\dot{x} = \begin{pmatrix} -a & 0 \\ a & -a \end{pmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$W_s = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & -a \\ 0 & a \end{bmatrix}.$$

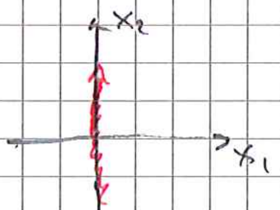
Styrbart.

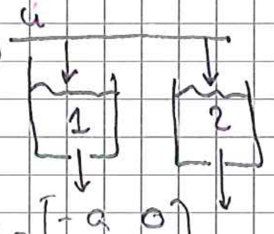
2) 

$$\begin{cases} \dot{x}_1 = -ax_1 \\ \dot{x}_2 = u - ax_1 - ax_2 \end{cases}$$

$$\dot{x} = \underbrace{\begin{bmatrix} -a & 0 \\ a & -a \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

$$W_s = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & -a \end{bmatrix}$$ Ej styrbart.



3) 

$$\begin{cases} \dot{x}_1 = \frac{1}{2}u - ax_1 \\ \dot{x}_2 = \frac{1}{2}u - ax_2 \end{cases}$$

$$\dot{x} = \begin{bmatrix} -a & 0 \\ 0 & -a \end{bmatrix} x + \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} u$$

$$W_s \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0{,}5 & -0{,}5a \\ 0{,}5 & -0{,}5a \end{bmatrix}$$ Ej styrbart.