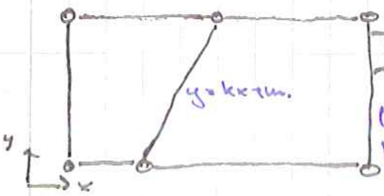
6/5-2013. Kap 19. (Isoparametriska element).



Ansats:
$T = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy$
$T = \alpha_1' + \alpha_2' y + \alpha_3' y^2$
Uppfyller ej kompatibiliteten.

$$\int_A f(x,y)\,dA = \int_E f(x(\xi,\eta), y(\xi,\eta))\,|J|\,d\xi\, d\eta$$

$$\begin{cases} x = x(\xi,\eta) \\ y = y(\xi,\eta) \end{cases}$$ 'dvs "mapping"'

Avbildning:

$$\begin{cases} x = x(\xi,\eta) \\ y = y(\xi,\eta) \end{cases} \begin{cases} dx = \frac{dx}{d\xi}d\xi + \frac{dx}{d\eta}d\eta \\ dy = \frac{dy}{d\xi}d\xi + \frac{dy}{d\eta}d\eta \end{cases} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} dx \\ dy \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{dx}{d\xi} & \frac{dx}{d\eta} \\ \frac{dy}{d\xi} & \frac{dy}{d\eta} \end{bmatrix}}_{J} \begin{bmatrix} d\xi \\ d\eta \end{bmatrix}$$

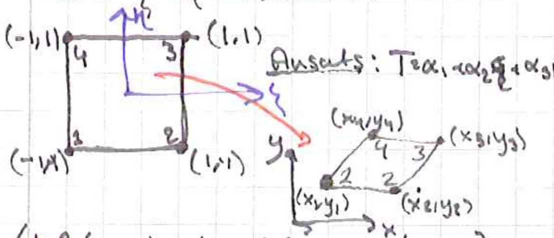
• Vi kräver $\det(J) \neq 0$ J

$$\Rightarrow \begin{bmatrix} d\xi \\ d\eta \end{bmatrix} = J^{-1} \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$dA = d\xi\, d\eta\, |\det(J)|$

Ex| 4-nodselement (Vi vet att elementränder parallella med x och y axeln uppfyller kompabiliteten).

Idé: Det. element i ξ-η domänen med elementränderna parallella med ξ-η axlarna.



Ansats: $T = \alpha_1 + \alpha_2 \xi + \alpha_3 \eta + \alpha_4 \xi\eta$

$$N_1^e(\xi,\eta) = \underbrace{(\xi-1)}_{\text{nod 2 o 3} = 0}\underbrace{(\eta-1)}_{\text{nod 4} = 0}\cdot\underbrace{\tfrac{1}{4}}_{\text{nod 1} = 1}$$

$$N_2^e(\xi,\eta) = -\tfrac{1}{4}(\xi+1)(\eta-1)$$

$$N_3^e(\xi,\eta) = \tfrac{1}{4}(\xi+1)(\eta+1)$$

$$N_4^e(\xi,\eta) = -\tfrac{1}{4}(\xi-1)(\eta+1)$$

$\Rightarrow T = N^e a^e = N^e(\xi,\eta)\, a^e$

$x = x(\xi,\eta)$ NA! $x = x(\xi,\eta) = N_1^e x_1 + N_2^e x_2 + \dots$
$y = y(\xi,\eta)$ $\quad y = y(\xi,\eta) = N_1^e y_1 + N_2^e y_2 + \dots$

Vi använder våra formfunktioner för att interpolera våra koordinater.

Hörnpunkter:

$$x(-1,-1) = \underbrace{N_1^e(-1,-1)}_{1}\cdot x_1 + \underbrace{N_2^e(-1,-1)}_{0}\cdot x_2 + \underbrace{N_3^e(-1,-1)}_{0} x_3 + \underbrace{N_4^e(-1,-1)}_{0} x_4$$

$\Rightarrow x(-1,-1) = x_1$

∴ Hörnpunkter → Hörnpunkter.

Rät linje

$x(\xi,\eta) = \alpha_1 + \alpha_2\xi + \alpha_3\eta + \alpha_4\xi\eta$
$y(\xi,\eta) = \alpha_1 + \alpha_2\xi + \alpha_3\eta + \alpha_4\xi\eta$

Välj $\xi = k$. $\Rightarrow \begin{cases} x = \alpha_1' + \alpha_2'\eta \\ y = \beta_1' + \beta_2'\eta \end{cases}$

∴ Rät linje → rät linje.

Completeness

• Godtycklig konstant T
• Godtycklig konstant ∇T } (*)

(*) $T = \alpha_1 + \alpha_2 x + \alpha_3 y + \dots$

Sampla nodvärden:

$T_1 = \alpha_1 + \alpha_2 x_1 + \alpha_3 y_1$
$T_2 = \alpha_1 + \alpha_2 x_2 + \alpha_3 y_2$
$T_3 = \alpha_1 + \alpha_2 x_3 + \alpha_3 y_3$
$T_4 = \alpha_1 + \alpha_2 x_4 + \alpha_3 y_4$

$$a^e = \alpha_1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \alpha_2 x^e + \alpha_3 y^e$$

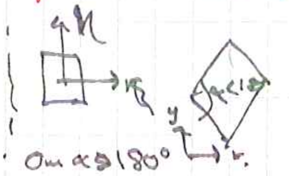
Insättning i ansats $T = N^e(\xi,\eta)\, a^e =$

$$\alpha_1 \underbrace{N^e \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}}_{\sum N_i^e = 1} + \alpha_2 \underbrace{N^e x^e}_{x} + \alpha_3 \underbrace{N^e y^e}_{y} = \alpha_1 + \alpha_2 x + \alpha_3 y$$

Completeness OK!

Styvhetsmatris

$$K^e = \int_A t B^{eT} D B^e\, dA = \int_A t \begin{bmatrix} \frac{\partial N^{eT}}{\partial x} & \frac{\partial N^{eT}}{\partial y} \end{bmatrix} D \begin{bmatrix} \frac{\partial N^e}{\partial x} \\ \frac{\partial N^e}{\partial y} \end{bmatrix} dA$$



Om $\alpha \geq 180°$ $\Rightarrow$ ej $\det \neq 0$

Hur gör vi med $\frac{\partial N^e}{\partial x}$ och $\frac{\partial N^e}{\partial y}$?

$$\frac{\partial N^e}{\partial \xi} = \frac{\partial N^e}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial N^e}{\partial y}\frac{\partial y}{\partial \xi}$$

$$\frac{\partial N^e}{\partial \eta} = \frac{\partial N^e}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial N^e}{\partial y}\frac{\partial y}{\partial \eta}$$

$$\Rightarrow \begin{bmatrix} \frac{\partial N^e}{\partial \xi} \\ \frac{\partial N^e}{\partial \eta} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}}_{J^T} \begin{bmatrix} \frac{\partial N^e}{\partial x} \\ \frac{\partial N^e}{\partial y} \end{bmatrix}$$

$$K^e = \int_A (\dots)\, dA = \int_{-1}^{1}\int_{-1}^{1} t \begin{bmatrix} \frac{\partial N^{eT}}{\partial \xi} & \frac{\partial N^{eT}}{\partial \eta} \end{bmatrix} J^{-1} D (J^T)^{-1} \begin{bmatrix} \frac{\partial N^e}{\partial \xi} \\ \frac{\partial N^e}{\partial \eta} \end{bmatrix} \det(J)\, d\xi\, d\eta$$

} → KAP 20 Numerisk integration

{ PROJEKT. }

$$\left.\begin{aligned} &\int_A B^T \sigma\, dA = f \\ &\sigma = D\cdot\varepsilon = D(\varepsilon - \varepsilon^{\Delta T}) \end{aligned}\right\} \Rightarrow \int_A B^T D B\, dA\, a = f + \int_A B^T D \varepsilon^{\Delta T} dA \;\; (*)$$

konstant.

$\varepsilon = Ba$

$Edof^{\Delta T} = [1\ 1\ 2\ 3]$ $\quad Edof^{\sigma} = [1\ 1\ 2\ 3\ 4\ 5\ 6]$

$$(*) \int_A B^T D \varepsilon^{\Delta T} dA = B^T D \int_A \alpha \Delta T \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} dA = \alpha B^T D \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \int_A \Delta T\, dA$$ — ΔT medel $\cdot A_e$.

PROJEKT.

$$S \cdot \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

$$S = [S] - \frac{(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})}{3} I$$