30/1-2012

(7)

$$\frac{k}{1+sT}$$

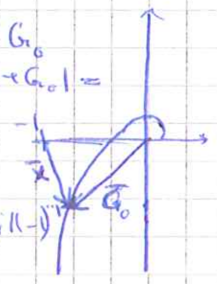
Känslighetsfunktion

$$S = \frac{1}{1+G_pG_R} = \frac{1}{1+G_0}$$

Robusthetsmarginal

$\bar{x} = 1 + G_0$

$|x| = |1+G_0| = \frac{1}{|S|}$

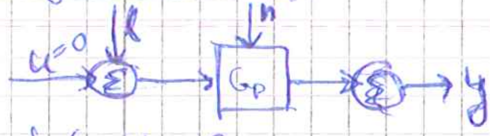


$M_s = \max |S(i\omega)| = (\text{kortaste avståndet till } -1)^{-1}$

Oftast $M_s = [1.2, 2]$

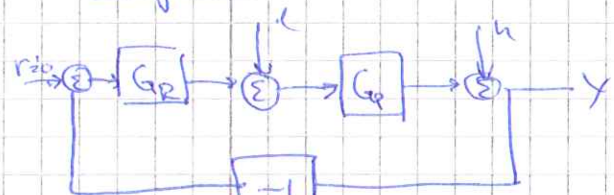
2) Störningsdämpning

Oreglerad Process:



$$Y_{ol} = N + G_p L$$

Med regulator:



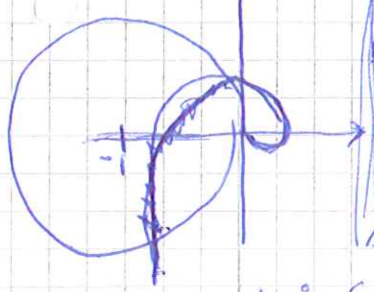
$$Y_{cl} = N + G_p(L + G_R(-Y_{cl}))$$

$$(1+G_pG_R)Y_{cl} = N + G_pL$$

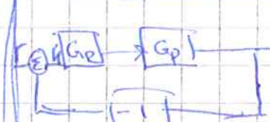
$$Y_{cl} = \frac{1}{1+G_pG_R}(N+G_pL) = S \cdot Y_{ol}$$

$\therefore \frac{Y_{cl}}{Y_{ol}} = S$ | $|S|<1$ Störningar dämpas

$|S|>1$ Störningar förstärks



Modellfel



$G_p^0 = G_p(1+\Delta G)$

G_p^0 = "verklig process"

ΔG = modellfel

$$Y^0 = \frac{G_pG_R}{1+G_pG_R}R \quad ; \; Y^0 = \text{verklig mätsignal}$$

$$Y^0 = \frac{G_p(1+\Delta G)G_R}{1+G_p(1+\Delta G)G_R}R = \frac{G_pG_R(1+\Delta G)}{1+G_pG_R(1+\Delta G)} \cdot \frac{1+G_pG_R}{1+G_pG_R}R =$$

$$= \frac{G_RG_p\left(1+G_RG_p(1+\Delta G)+\Delta G\right)}{1+G_pG_R(1+\Delta G)}R =$$

$$= \left(1+\frac{\Delta G}{1+G_pG_R(1+\Delta G)}\right)\frac{G_pG_R}{1+G_pG_R}R =$$

$$= \left(1+\frac{1}{1+G_p^0G_R}\Delta G\right)Y = (1+S^0\cdot\Delta G)Y$$

S^0 = verklig känslighetsfunktion

$\therefore Y^0 = Y + S^0\Delta G \cdot Y$

$\Rightarrow \frac{Y^0 - Y}{Y} = S^0_e\Delta G$

S = överföringsfunktion mellan relativt modellfel och relativt fel i y.

STATIONÄRA FEL

$$Y = G_p(L + G_R(R-Y))$$

$$(1+G_pG_R)Y = G_pL + G_pG_RR$$

$$Y = \frac{G_p}{1+G_pG_R}L + \frac{G_pG_R}{1+G_pG_R}R$$

Servoproblemet: $(L=0)$ (Mekaniska konstr.)

$$Y = \frac{G_pG_R}{1+G_pG_R}R = \frac{G_0}{1+G_0}R.$$

Regulatorproblemet $(R=0)$. (Processindustri)

$$Y = \frac{G_p}{1+G_pG_R}L.$$

Stationära fel - servoproblem

$Y = \frac{G_0}{1+G_0}R$. Reglerfelet: $E = R - Y =$

$$= \left(1-\frac{G_0}{1+G_0}\right)R = \frac{1}{1+G_0}R.$$

Slutvärdesteoremet: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$.

Gränsvärdet existerar $\Leftrightarrow$ $sE(s)$ har samtliga poler i VHP.

(VIKTIGT: VISA ALLTID ATT GRÄNSVÄRDET EXISTERAR(!!!)).

Ex) $G_R = K$, $G_p = \frac{1}{s(1+sT)}$, $G_0 = \frac{k}{s(1+sT)}$

a) $r(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$ $\quad R(s) = \frac{1}{s}$

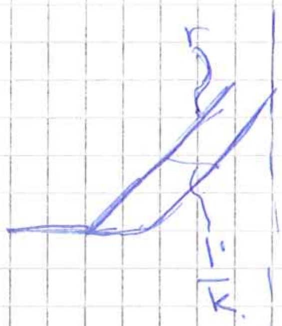
$$E(s) = \frac{1}{1+G_0(s)}R(s) = \frac{1}{1+\frac{k}{s(1+sT)}}\cdot\frac{1}{s} = \frac{s(1+sT)}{s(1+sT)+k}\cdot\frac{1}{s}.$$

SV: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) =$

$$= \lim_{s\to 0} s\cdot\frac{s(1+sT)}{s(1+sT)+k}\cdot\frac{1}{s} = \lim_{s\to 0}\frac{s(1+sT)}{s(1+sT)+k} = 0$$

$K, T > 0 \Rightarrow$ STABILT

b) $r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$



$$E(s) = \frac{s(1+sT)}{s(1+sT)+k} \cdot \frac{1}{s^2}$$

SV:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{1+sT}{s(1+sT)+k} = \underline{\underline{\frac{1}{k}}}$$

Generellt: $G_o = \frac{k(1+b_1 s + b_2 s^2 + \dots)}{s^n(1+a_1 s + a_2 s^2 + \dots)} = \frac{kB(s)}{s^n A(s)} e^{-sL}$

$A(0) = B(0) = 1$

$r(t) = \begin{cases} \frac{t^m}{m!} & t \geq 0 \\ 0 & t < 0 \end{cases}$ $\quad R(s) = \frac{1}{s^{m+1}}$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \frac{1}{1+\frac{kB}{s^n A} e^{-sL}} \cdot \frac{1}{s^{m+1}} =$$

$$= \lim_{s\to 0} s \frac{s^n \cdot A}{s^n A + kBe^{-sL}} \cdot \frac{1}{s^{m+1}} = \lim_{s\to 0} \frac{\cancel{s^n} A}{s^n A + kBe^{-sL}} \cdot s^{n-m} =$$

$$= \lim_{s\to 0} \frac{1}{s^n + k} s^{n-m}$$

1) $n > m$ $\quad e_\infty = 0$
2) $n = m = 0$ $\quad e_\infty = \frac{1}{1+k}$
3) $n = m \geq 1$ $\quad e_\infty = \frac{1}{k}$
4) $n < m$ $\quad e_\infty$ saknas

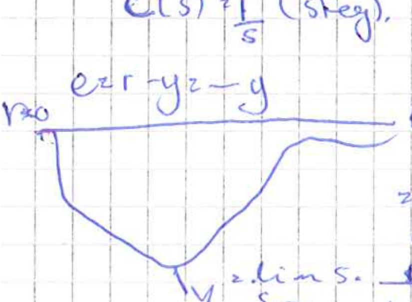
Regulatorproblemet.

$$Y = \frac{G_p}{1+G_p G_R} L$$

Ex| a) $G_R = \frac{k}{s}$, $G_p = \frac{1}{1+sT}$, $G_o = \frac{k}{s(1+sT)}$

$\mathcal{L}(s) = \frac{1}{s}$ (steg).

$e = r - y = -y$



$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) =$$

$$= \lim_{s\to 0} s \cdot \frac{G_p}{1+G_p G_R} L =$$

$$= \lim_{s\to 0} s \cdot \frac{\frac{1}{1+sT}}{1+\frac{k}{s(1+sT)}} \cdot \frac{1}{s} =$$

$$= \lim_{s\to 0} \cancel{s} \cdot \frac{s}{s(1+sT)+k} \cdot \frac{1}{\cancel{s}} = \underline{\underline{0}}$$

b) $G_R = k$, $G_p = \frac{1}{s(1+sT)}$, $G_o = \frac{k}{s(1+sT)}$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s \frac{\frac{1}{s(1+sT)}}{1+\frac{k}{s(1+sT)}} \cdot \frac{1}{s} =$$

$$= \lim_{s\to 0} \cancel{s} \frac{1}{s(1+sT)+k} \cdot \frac{1}{\cancel{s}} = \underline{\underline{\frac{1}{k}}}$$