29/4-2013

Töjningar

$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$

$\varepsilon_{yy} = \frac{\partial u_y}{\partial y}$

$\varepsilon_{zz} = \frac{\partial u_z}{\partial z}$

$\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$

$\gamma_{xz} = \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}$

$\gamma_{yz} = \frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}$.

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{zx} \end{bmatrix} = \underbrace{\begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & \partial/\partial z \\ \partial/\partial y & \partial/\partial x & 0 \\ \partial/\partial z & 0 & \partial/\partial x \\ 0 & \partial/\partial z & \partial/\partial y \end{bmatrix}}_{\tilde{\nabla}} \underbrace{\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}}_{u}$$

Hookes lag.

$\tilde{\nabla}^T \sigma + b = 0$

$$\underset{3\times1}{q} = -\underset{3\times3}{D}\,\underset{3\times1}{\nabla T} \qquad \underset{6\times1}{\sigma} = \underset{6\times6}{\left[\ ?\ \right]}\,\underset{6\times1}{\varepsilon^e}$$

$\boxed{\varepsilon} = \cancel{\varepsilon^{t}} + \cancel{\varepsilon^{p}} + \cancel{\varepsilon^{c}} + \boxed{\varepsilon^{\Delta T}} + \boxed{\varepsilon^{e}}$

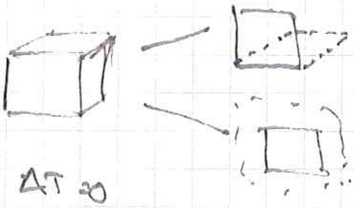
1D: $W = \frac{1}{2}E(\varepsilon^e)^2 > 0$

3D: $W = \frac{1}{2}(\varepsilon^e)^T D \varepsilon^e > 0 \Rightarrow D$ pos def.

LÄS GÄRNA: Beräkningsbaserad mat. modellering.

Initialtöjningar ε_0 ($\varepsilon^{\Delta T}$)

$\varepsilon = \varepsilon^e + \varepsilon^{\Delta T} \qquad \sigma = D\varepsilon^e = D(\varepsilon - \varepsilon^{\Delta T})$



$\sqrt{x+y} - \sqrt{x-y} =$

$\Delta T = 0$ $\qquad$ $\Delta T \neq 0$.

$$\varepsilon^{\Delta T} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta T \alpha$$

~ konstitutiv parameter

Sammanfattning

Jmv: $\tilde{\nabla}^T \sigma + b = 0$

Töjn: $\varepsilon = \tilde{\nabla} u$

Konst. samband $\sigma = D(\varepsilon - \varepsilon^T)$

$\operatorname{div}(q) + Q = 0.$ $\quad q = D\nabla T$

∇T

Vad händer när värmen tillförs stången.

Svag form 3D-elast.

$$\tilde{\nabla}^T\sigma + b = 0 \Rightarrow \begin{cases} \operatorname{div}(S_x) + b_x = 0 & (1) \\ \operatorname{div}(S_y) + b_y = 0 & (2) \\ \operatorname{div}(S_z) + b_z = 0 & (3). \end{cases}$$

x-led $\int v_x \operatorname{div}(S_x)\, dV + \int_V v_x b_x\, dV = 0.$

Green-Gauss

$\oint_S v_x S_x^T n - \int (\nabla v_x)^T S_x + \int v_x b_x = 0.$

$$+\begin{cases} \int (\nabla v_x)^T S_x\, dV = \oint v_x t_x\, dS + \int v_x b_x\, dV \\ \int_V (\nabla v_y)^T S_y\, dV = \oint_S v_y t_y\, dS + \int_V v_y b_y\, dV \\ \int_V (\nabla v_z)^T S_z\, dV = \oint_S v_z t_z\, dS + \int_V v_z b_z\, dV. \end{cases}$$

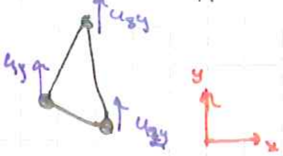
$\sum_{x,y,z} \Rightarrow \int (\tilde{\nabla} v)^T \sigma\, dV = \oint_S v^T t\, dS + \int_V v^T b\, dV$

Svag form erhållen.

(Närmeledning $T = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 z + \dots$).

$$\begin{cases} u_x = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 z + \dots \\ u_y = \beta_1 + \beta_2 x + \beta_3 y + \beta_4 z + \dots \\ u_z = \gamma_1 + \gamma_2 x + \gamma_3 y + \gamma_4 z + \dots \end{cases}$$

Approximation (2D).



$u_y = \alpha_1 + \alpha_2 x + \alpha_3 y = \bar{N}\alpha$

C-matrismetoden $u_y = N_y^e a_y^e$

~~$u = [u_x\ u_y] = [N_x^1\ 0\ N_x^2\ 0]\,[a]$~~

$$u = \begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} N_x^1 & 0 & N_x^2 & 0 & N_x^3 & 0 \\ 0 & N_y^1 & 0 & N_y^2 & 0 & N_y^3 \end{bmatrix} \begin{bmatrix} a_x^1 \\ a_y^1 \\ a_x^2 \\ a_y^2 \\ a_x^3 \\ a_y^3 \end{bmatrix} \begin{matrix} \} \text{ nod 1.} \\ \\ \} \text{ nod 2.} \\ \\ \} \text{ nod 3.} \\ \\ \end{matrix}$$

$\Rightarrow u = N^e a^e$

$\varepsilon = \tilde{\nabla} u = \tilde{\nabla} N^e a^e = B^e a^e$

FE-formulering

$$c^T\Big(\underbrace{\int_V B^T \sigma\, dV - \oint_S N^T t\, dS - \int N^T b\, dV}_{=0}\Big) = 0$$

Inga temperatureffekter $\sigma = D\varepsilon = D\tilde{\nabla} N a = DBa$

$$\boxed{\int_V B^T D B\, dV\, a = \oint_S N^T t\, dS + \int_V N^T b\, dV.}$$

Temperatureffekter $\sigma = D(\varepsilon - \varepsilon^{\Delta T})$

$$\int_V B^T D B\, dV\, a = \int_V B^T D \varepsilon^{\Delta T}\, dV + \oint_S N^T t\, dS + \int_V N^T b\, dV$$

TIPS PROJEKT: plantf.m