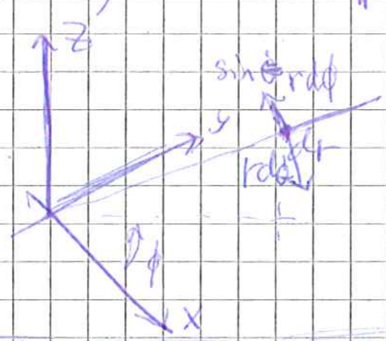
2010-12-20 ⑥

$$\int_a^b f(x)\,dx = \left[\begin{array}{l} u = u(x) \\ x = x(u) \end{array}\right] = \int f(x(u)) \left(\frac{dx}{du}\right) du = \int f(x(u)) \frac{1}{\left(\frac{du}{dx}\right)} du$$

$$\iiint_D f(\vec{r})\,dx\,dy\,dz = \iiint_{D_{ny}} f(\vec{r}(s,\phi,z)) \left(\frac{dx\,dy\,dz}{ds\,s\,d\phi\,dz}\right) ds\,d\phi\,dz$$

$$\begin{pmatrix} \frac{dx}{ds} & \frac{dx}{d\phi} & \frac{dx}{dz} \\ \frac{dy}{ds} & \frac{dy}{d\phi} & \frac{dy}{dz} \\ \frac{dz}{ds} & \frac{dz}{d\phi} & \frac{dz}{dz} \end{pmatrix}$$



Ex 6.1.58

$$\oiint_{ytan} \vec{v}\cdot d\vec{s} = \iiint (\nabla\cdot\vec{v})\,dV$$

VL: $\iint_{Sfären} \vec{v}\cdot\hat{\theta}\,dr\,d\phi\,r\sin\theta = \iint 4r^2\cos\frac{\pi}{6}\sin\frac{\pi}{6}\,d\phi\,dz$ ... $0\le r\le R$, $0\le\phi\le 2\pi$

$0\le r\le R$
$0\le\phi\le 2\pi$
$\theta = 30° = \frac{\pi}{6}$

$= \ldots + \iint_{botten} \vec{v}\cdot\hat{r}\,r\,d\theta\,d\phi\sin\theta =$ ... $0\le\theta\le\frac{\pi}{6}$, $0\le\phi\le 2\pi$ $= \iint R^2\sin\theta R^2\sin\theta\,d\theta\,d\phi =$

HL: $\nabla\cdot\vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 r^2\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, 4r^2\cos\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(r^2\tan\theta) = 4r\sin\theta + \frac{4r}{\sin\theta}(\cos^2\theta - \sin^2\theta)$

$$\iiint r^4\sin\theta\,dr\,d\theta\,d\phi =$$

$0\le r\le R$
$0\le\phi\le 2\pi$
$0\le\theta\le\frac{\pi}{6}$

2009-08-27 #4.

$\vec{A} = \nabla u$, $\frac{du}{dr} = \frac{3\cos^2(\theta)-1}{r^4}$

$\frac{1}{r}\frac{du}{d\theta} = \frac{\sin(2\theta)}{r^4} = \frac{2\sin\theta\cos\theta}{r^4}$

$\Rightarrow u = \int\left(\frac{du}{dr}\right)dr = (3\cos^2(\theta)-1)\frac{1}{(-3)r^3} + B(\theta,\phi)$

$u = \int\left(\frac{du}{d\theta}\right)d\theta = \int\frac{2\sin\theta\cos\theta}{r^3}d\theta = \frac{\sin^2\theta}{r^3} + C(r,\phi) = \frac{1-\cos^2\theta}{r^3} + C(r,\phi)$

$\Rightarrow u = \frac{-\cos^2\theta}{r^3} + \frac{1}{3r^3} + D$

$$\int_P^Q \vec{A}\cdot d\vec{r} = u(Q) - u(P) = \left(-\frac{\cos^2\frac{\pi}{2}}{3^3} + \frac{1}{3\cdot 3^3}\right) - \left(\frac{-\cos^2\frac{\pi}{4}}{1^3} + \frac{1}{3\cdot 1^3}\right) = \ldots = \frac{29}{162}$$

2011-04-28 #5

$$\oint_\Gamma \vec{K}\cdot d\vec{r} = \iint_{S\ (\text{Stoksmed } \Gamma \text{ som rand})} (\nabla\times\vec{K})\cdot\hat{n}\,dS$$

halvsfären: $r=1$, $\hat{n} = -\hat{r}$

$\begin{cases} 0\le\theta\le\pi/2 \\ 0\le\phi\le\pi/2 \end{cases}$ $dS = r\,d\theta\,r\sin\theta\,d\phi$

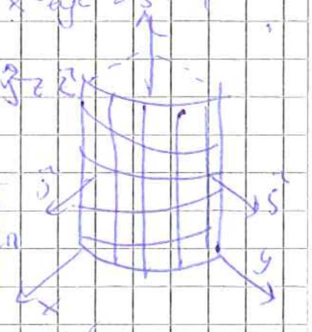
$\nabla\times\vec{K} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\phi}(-7\sin\phi)\right)\hat{r} + 0 + \frac{1}{r}\left(\frac{\partial}{\partial r}(r\cdot 7r\sin\phi)\right)\hat{\phi} = \frac{7\cos\phi}{r\sin\theta}\hat{r} + \frac{-7\sin\phi}{r}\hat{\phi}$

$$\iint_{0\le\theta\le\frac{\pi}{2},\ 0\le\phi\le\frac{\pi}{2}} \left(\frac{7\cos\phi}{r\sin\theta}\hat{r} - \frac{7\sin\phi}{r}\hat{\phi}\right)\cdot(-\hat{r})\,r^2\sin\theta\,d\theta\,d\phi = \iint -7\cos\phi\, r\,d\theta\,d\phi = -7\left(\int_0^{\pi/2} d\theta\right)\left(\int_0^{\pi/2}\cos\phi\right) = -7\frac{\pi}{2}\cdot 1$$

2010-05-11 #4

$\sqrt{x^2+y^2} = s = 4$

$\vec{A} = z\hat{x} + x\hat{y} - 3y^2 z\hat{z}$



$\hat{n}\,dS = \hat{s}\,dz\,s\,d\phi$

$\iint_{0\le z\le 5,\ 0\le\phi\le 2\pi} \vec{A}\cdot\hat{s}\,dz\,s\,d\phi =$ $\begin{cases} 0\le z\le 5 \\ 0\le\phi\le 2\pi \\ s=4 \end{cases}$

$= \iint \vec{A}\cdot(\cos\phi\,\hat{x} + \sin\phi\,\hat{y})\,s\,dz\,d\phi = \iint (z\cos\phi + x\sin\phi)\,s\,dz\,d\phi$

$\Rightarrow \iint (z\cos\phi + s\cos\phi\sin\phi)\,s\,dz\,d\phi = \ldots = 0$