17/4-2013. Galerkin.

$V = C_1\Psi_1 + C_2\Psi_2 + \dots + C_n\Psi_n$.

$$\int_a^b \frac{dv}{dx} Ak \frac{dT}{dx} = -(vqAk)_{x=a} + (vqAk)_{x=b} + \int_a^b vQ\,dx \quad (*)$$

$$\frac{dv}{dx} = \frac{d}{dx}(N\cdot C) = \frac{dN}{dx}\cdot C = B\cdot C$$

$$\left(\frac{dN}{dx}C\right)^T = (BC)^T = C^T B^T \Rightarrow$$

$$\frac{dT}{dx} = B\cdot a$$

$$V = V^T = (NC)^T = C^T\cdot N^T$$

C oberoende av x.

$\Leftrightarrow (*)$:

$$C^T\left\{\int_a^b B^T Ak\cdot B\cdot a + (N^T Aq)_{x=a} - (N^T Ak)_{x=b} - \int_a^b N^T Q\,dx\right\} = 0.$$

f_b $\quad = 0$ ty C är godtycklig. $\quad f_k$

$$\int_a^b B^T Ak\,B\,dx\;a = -\left[N^T Aq\right]_a^b + \int_a^b N^T Q\,dx$$

K $\quad a$ $\quad f_b$ $\quad f_k$

FEM-FORMULERING

$$K\cdot a = f$$

f_b - $f_{boundary}$ - randvektor.

f_k - f_{load} - lastvektor, finns över ledkoppen

Symmetriegenskaper

$$K = \int_a^b B^T Ak\,B\,dx;\quad K^T = \left(\int_a^b B^T Ak\,B\,dx\right)^T =$$

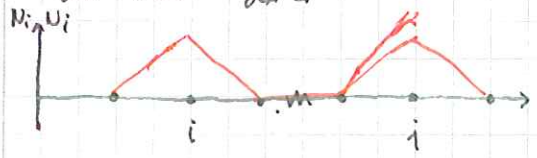
$$= \int_a^b B^T Ak^T (B^T)^T\,dx = \int_a^b B^T Ak\,B\,dx = K$$

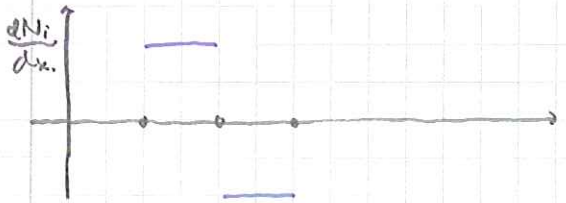
$K^T = K$ - symmetriska styvhetsmatriser.

(Matlab debug: $\Rightarrow$ spy(K))

$N = [N_1\ N_2\ \dots\ N_n]$

$B = \frac{dN}{dx} = \left[\frac{dN_1}{dx}\ \dots\ \frac{dN_n}{dx}\right]$





Lastvektorn f_k

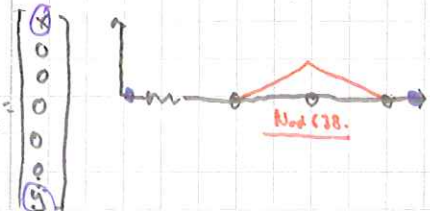
$$f_k = \int_a^b N^T Q\,dx \qquad N = [N_1\ N_2\ \dots\ N_n].$$

Randvektor f_b

$$f_b = -\left[N^T Aq\right]_a^b$$

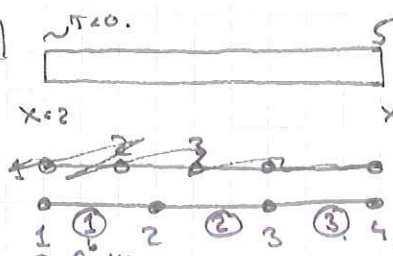
I komp:

$$\left[N_i Aq\right]_a^b = (N_i Aq)_{x=b} - (N_i Ah)_{x=a}.$$



f_b - reaktionskraft på randen. 0 förutom i ändarna.

Ex) $T=0$. $\quad Ak = 50$.



$$K_{11} = 50\int_2^8 \frac{dN_1}{dx}\frac{dN_1}{dx}\,dx = 50\int_2^4 \left(-\frac{1}{4}\right)^2 dx =$$

$$= 50\cdot\frac{1}{4}\cdot 2 = 25.$$

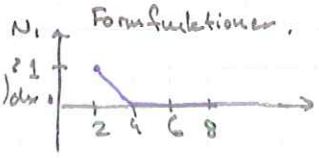
$$K_{12} = 50\int_2^8 \frac{dN_1}{dx}\frac{dN_2}{dx}\,dx = 50\int_2^4 \frac{dN_1}{dx}\frac{dN_2}{dx}\,dx =$$

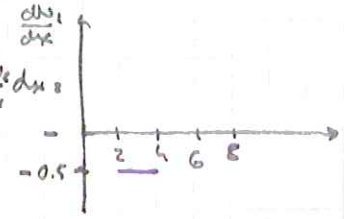
$$= -25 \Rightarrow K_{21} = -25$$

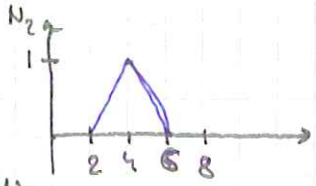
$K_{13} = 0$

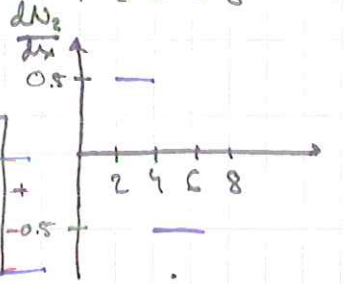
$$K = \begin{bmatrix} 25 & -25 & 0 & 0 \\ -25 & 50 & -25 & 0 \\ 0 & -25 & 50 & -25 \\ 0 & 0 & -25 & 25 \end{bmatrix}$$

Formfunktioner.









Randvillkor Väsentligt ev.

$$\begin{bmatrix} 25 & -25 & 0 & 0 \\ -25 & 50 & -25 & 0 \\ 0 & -25 & 50 & -25 \\ 0 & 0 & -25 & 25 \end{bmatrix} \begin{bmatrix} 0 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} ? \\ 0 \\ 0 \\ -100 \end{bmatrix} + \begin{bmatrix} ? \\ 0 \\ 0 \\ 100 \\ 200 \\ 200 \\ 100 \end{bmatrix}$$

K $\quad \tilde{K}$ $\quad a$ $\quad f_b$ $\quad f_k$

Alltid en obekant per rad!

$\det K = 0$

$\det \tilde{K} \neq 0$

Först temp, sen flöden.