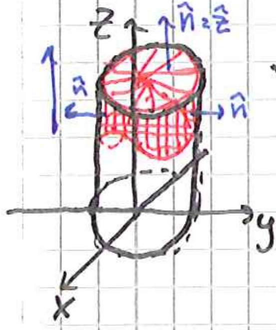
13/2-2012
(5)

2010-08-24 Ex 5

$\vec{A} = \begin{bmatrix} y+2x \\ x^2+z \\ y \end{bmatrix}$ $\quad \gamma: \vec{r}(u) = \begin{bmatrix} x(u) \\ y(u) \\ z(u) \end{bmatrix} = \begin{bmatrix} \cos u \\ \sin u \\ f(u) \end{bmatrix}$

$0 \leq u \leq 2\pi$
$f(0) = f(2\pi)$ $\quad \int_\gamma \vec{A} \cdot d\vec{r} = ?$



$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y+2x & x^2+z & y \end{vmatrix} = \begin{vmatrix} 1-1 \\ -0+0 \\ 2x-1 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2x-1 \end{bmatrix}$

$\oint_\gamma \vec{A} \cdot d\vec{r} = \iint_{\text{locket}} (\nabla \times \vec{A}) \cdot \hat{n}\, dS =$

$= \iint_{\text{locket}} (2x-1) \underbrace{\hat{z} \cdot \hat{z}}_{=1} dS =$

$= \underbrace{\iint_{\text{locket}} (2x-1)\, dS}_{=0} = -\iint_{\text{locket}} dS = -\pi$

Ersvar:

$\frac{d}{dx}(fg) = \left(\frac{\partial f}{dx}\right) g + f\left(\frac{dg}{dx}\right)$

$\int_a^b \left(\frac{df}{dx}\right) g\, dx = \left[fg\right]_a^b - \int_a^b f\left(\frac{dg}{dx}\right) dx$

Regel #5. $\underbrace{\iiint_V \nabla \cdot (f\vec{A})\, dV}_{= \oiint_{\partial V} f\vec{A} \cdot d\vec{S}} = \iiint_V f(\nabla \cdot \vec{A})\, dV + \iiint_V \vec{A} \cdot (\nabla f)\, dV$

$\Rightarrow f(\nabla \cdot \vec{A})\, dV = \oiint_S f\vec{A} \cdot d\vec{S} - \iiint_V \vec{A} \cdot (\nabla f)\, dV$

Regel #6: $\iiint (\nabla \times \vec{A}) \cdot \vec{B}\, dV = \oiint_{\partial V} (\vec{A} \times \vec{B}) \cdot d\vec{S} + \iiint_V \vec{A} \cdot (\nabla \times \vec{B})\, dV$

Regel #7: $\iint_S f(\nabla \times \vec{A}) \cdot d\vec{S} = \oint_{\partial S} f \cdot \vec{A} \cdot d\vec{r} - \iint_S (\nabla f) \times \vec{A}\, d\vec{S}$

$\vec{V} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$ $\quad \vec{V} = (V_x, V_y, V_z)$ $\quad \vec{V} = V_x\hat{x} + V_y\hat{y} + V_z\hat{z}$

$\vec{V} = V_s\hat{s} + V_\phi\hat{\phi} + V_z\hat{z}$

$\vec{V} = 2\hat{x}$ $\begin{cases} 2\hat{s} \\ -2\hat{s} \end{cases}$ Beroende på var i rummet man är.

Ex) $\vec{B} = \frac{s}{2}\hat{s} + \frac{s}{2}\hat{\phi}$