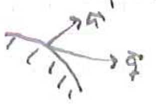
11/3-2013

2-D värmeledning.

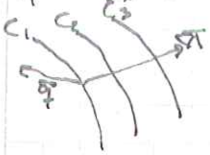
Värmeflux $\vec{q} = \begin{bmatrix} q_x \\ q_z \end{bmatrix}$ riktning på flödet $\frac{\vec{q}}{\|\vec{q}\|}$

Tillstånd på rand



$q_n = \vec{q}^T \vec{n} > 0$ när värme lämnar kroppen.

$q = -k \frac{dT}{dx}$ (1D) $(\nabla T)^T q < 0$



$$\begin{bmatrix} q_x \\ q_y \end{bmatrix} = -\begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \end{bmatrix}$$

$\vec{q} = -\vec{D}\nabla T$ (2D/3D)

Insättning av $\vec{q} = -\vec{D}\nabla T$ i $(\nabla T)^T \vec{q} < 0$

$(\nabla T)^T \vec{D} \nabla T > 0 \Rightarrow D$ pos. def. $\vec{D} = \vec{D}^T$

$$\vec{D} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \text{ anisotrop}$$

$$\vec{D} = \begin{bmatrix} k_{xx} & 0 & 0 \\ 0 & k_{yy} & 0 \\ 0 & 0 & k_z \end{bmatrix} \text{ ortotropt material}$$

$$D = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \text{ isotropt.}$$

$D = k I$

Balanslagar

$Q \left[\frac{J}{m^3 s}\right]$, dS, dV, $q_n = q^T n \left[\frac{J}{m^2 s}\right]$

"In-Ut":

$$\oint_S q_n \, dS = \int_V Q \, dV = \int_S q^T n \, dS = \int_V \mathrm{div}(q) \, dV$$

$$\int_V (Q - \mathrm{div}(q)) dV = 0 \Rightarrow \mathrm{div}(q) = Q \text{ (Stark form)}$$

Svag form.

① Mult. med godtycklig viktfunktion $v = v(x,y,z)$

$v \, \mathrm{div}(q) - vQ = 0$

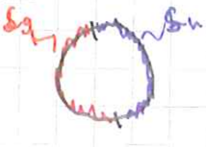
②. Integrera över kroppen.

$$\int_V v \, \mathrm{div}(q) \, dV - \int_V vQ \, dV = 0$$

③ Green-Gauss sats $\int_V v \, \mathrm{div}(q) dV = \int_S v q^T n \, dS - \int_V (\nabla v)^T q \, dV$

$$\Rightarrow \int_S v q^T n \, dS - \int_V (\nabla v)^T q \, dV - \int_V vQ \, dV = 0.$$

⑤ Randvillkor



S_g, S_h

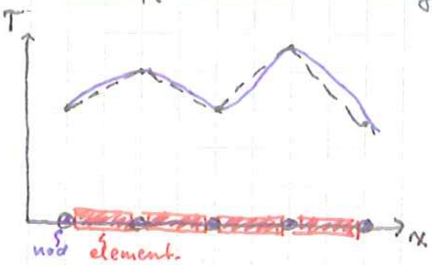
$T = g$ längs S_g

$q_n = h$ längs S_h.

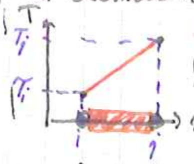
• Insättning av naturliga randvillkor

$$\boxed{\int_{S_g} v q^T n \, dS + \int_{S_h} v h \, dS - \int_V (\nabla v)^T q \, dV - \int_V vQ \, dV = 0}$$

Val av approximation Idag: 1D.



Ett element:



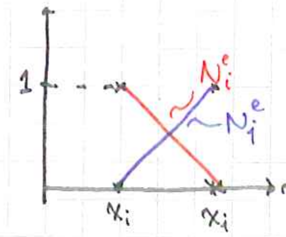
Ansats: $T = \alpha_1 + \alpha_2 x = \underbrace{[1 \quad x]}_{\bar{N}} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}}_{\alpha}$

$$\begin{cases} T(x = x_i) = T_i = \alpha_1 + \alpha_2 \cdot x_i \\ T(x = x_j) = T_j = \alpha_1 + \alpha_2 \cdot x_j \end{cases}$$

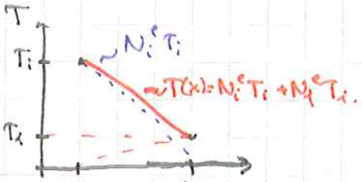
$$\Rightarrow \underbrace{\begin{bmatrix} T_i \\ T_j \end{bmatrix}}_{a^e} = \underbrace{\begin{bmatrix} 1 & x_i \\ 1 & x_j \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}}_{\alpha} \Rightarrow \alpha = C^{-1} a^e$$

$T = \bar{N}\alpha \Rightarrow T = \underbrace{\bar{N} C^{-1}}_{N^e} a^e$ / N^e: elementformfunktioner

$$C^{-1} = \frac{1}{\underbrace{x_j - x_i}_{l^e}} \begin{bmatrix} x_j & -x_i \\ -1 & 1 \end{bmatrix} \Rightarrow N^e = \frac{1}{l^e}[x_j - x, \; -x_i + x] = [N_i^e, N_j^e]$$

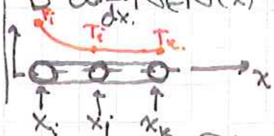


$T = N_i^e T_i + N_j^e T_j$.



$$\frac{dT}{dx} = \frac{d}{dx}(N a) = \underbrace{\frac{dN}{dx}}_{B} a + N \cancel{\frac{da}{dx}} \quad T_i$$

$= B \cdot a = \frac{dT}{dx}$. $N \in N(x)$ {innehåller x_i allt koord. beroende}.



Ansats

$$T = \alpha_1 + \alpha_2 x + \alpha_3 x^2 = \underbrace{[1 \quad x \quad x^2]}_{\bar{N}} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}}_{\alpha}$$

$T_i = \alpha_1 + \alpha_2 x_i + \alpha_3 x_i^2$

$T_j = \alpha_1 + \alpha_2 x_j + \alpha_3 x_j^2$

$T_k = \alpha_1 + \alpha_2 x_k + \alpha_3 x_k^2$.

$$\underbrace{\begin{bmatrix} T_i \\ T_j \\ T_k \end{bmatrix}}_{a^e} \underbrace{\begin{bmatrix} 1 & x_i & x_i^2 \\ 1 & x_j & x_j^2 \\ 1 & x_k & x_k^2 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}}_{\alpha} \Rightarrow \alpha = C^{-1} a^e \Rightarrow N^e = \bar{N} C^{-1}$$