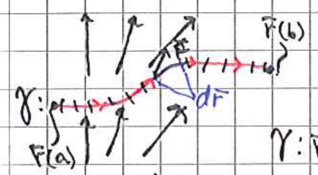
23/1 2012. (2)

$\bar{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\quad f(x,y,z) = f(\bar{r}) \quad \bar{F}(\bar{r})$

$\bar{F}$ = kraftfält.

'Arbete': $\int_\gamma \bar{F} \cdot d\bar{r}$



$\gamma: \bar{r}(t) \quad a < t < b$

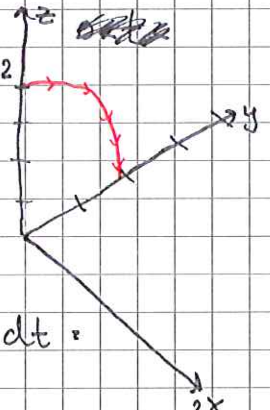
$$\int_\gamma \bar{F} \cdot d\bar{r} = \int_a^b \bar{F} \cdot \left(\frac{d\bar{r}(t)}{dt}\right) dt = \int_a^b \bar{F}(\bar{r}(t)) \cdot \left(\frac{d\bar{r}}{dt}\right) dt$$

$$\int_\gamma \bar{F} \cdot d\bar{r} = \int_\gamma \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \cdot \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \int_\gamma F_x dx + F_y dy + F_z dz$$

Ex $\bar{F} = 4z^2 \hat{x} - 3y^2 \hat{y} + 8xz \hat{z}$

$\gamma: \{z = 4 - y^2 ; x = 0 ; z, y > 0\}$

$$\bar{r}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ 4-t^2 \end{bmatrix} \quad 0 < t < 2$$



$$\frac{d\bar{r}}{dt} = \begin{bmatrix} 0 \\ 1 \\ -2t \end{bmatrix}$$

$$\int_0^2 \begin{bmatrix} 4 \cdot (4-t^2)^2 \\ -3t^2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ -2t \end{bmatrix} dt =$$

$$= \int_0^2 (0 - 3t^2 + 0)\, dt = -\int_0^2 3t^2 dt = -\Big[t^3\Big]_0^2 = -8$$

Om $\bar{F} = \nabla U(x,y,z) = \text{grad}\, U(x,y,z)$

då $\int_\gamma \bar{F} \cdot d\bar{r} = \int_\gamma (\nabla U) \cdot d\bar{r} = U(\bar{b}) - U(\bar{a})$

$\gamma: \bar{a} \to \bar{b}$

Om $\nabla \times \bar{F} \neq \bar{0}$ kan inte $\bar{F}$ ha potential.

$\bar{F} = \nabla U$

$$\nabla \times (\nabla U) = \nabla \times \begin{bmatrix} \partial U/\partial x \\ \partial U/\partial y \\ \partial U/\partial z \end{bmatrix} = \dots = 0$$

Om det ej blir 0 finns det ingen potential.

$$\text{Om } \bar{F} = \nabla U = \begin{bmatrix} \partial U/\partial x \\ \partial U/\partial y \\ \partial U/\partial z \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

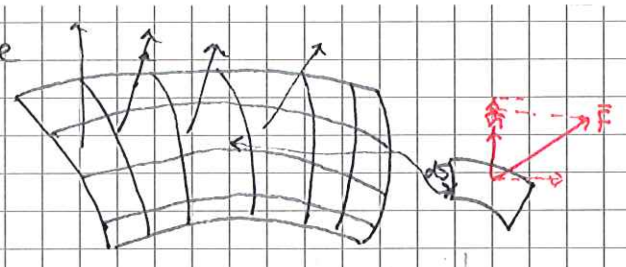
$\int F_x dx = \int 4z^2 dx = 4xz^2 + A(y,z)$

$\int F_y dy = \int -3y^2 dy = -y^3 + B(x,z)$

$\int F_z dz = \int 8xz\, dz = 4xz^2 + C(x,y)$

$\Rightarrow U = 4xz^2 - y^3 + D$

$\bar{F}$ = flöde

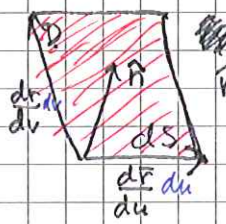


$\bar{F} \cdot \hat{n}\, dS$

dS: arean av ytelementet. $\iint_{ytan} \bar{F} \cdot \hat{n}\, dS = \iint_{ytan} \bar{F} \cdot d\bar{S}$

Totala flödet genom ytan.

ytan: $\bar{r}(u,v) \quad (u,v) \in D$

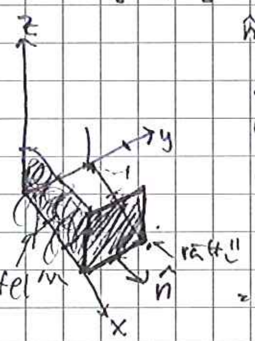


$$\hat{n} = \frac{\frac{d\bar{r}}{du} \times \frac{d\bar{r}}{dv}}{\left|\frac{d\bar{r}}{du} \times \frac{d\bar{r}}{dv}\right|} \quad ; \quad dS = \left|\frac{d\bar{r}}{du} \times \frac{d\bar{r}}{dv}\right| du\, dv$$

$$\text{Totala flödet: } \iint_{(u,v) \in D} \bar{F}(\bar{r}(u,v)) \cdot \overbrace{(\pm)\underbrace{\left(\frac{d\bar{r}}{du} \times \frac{d\bar{r}}{dv}\right)}_{\neq \hat{n}} \underbrace{du\, dv}_{\neq dS}}^{\hat{n} \cdot dS}$$

Ex $\bar{F} = \begin{bmatrix} \sqrt{y+z} \\ \cos x \\ -5 \end{bmatrix}$ ytan: $\bar{r}(u,v) = \begin{bmatrix} 3 \\ u \\ v \end{bmatrix}$ $\quad 0 < u < 2$, $\quad 0 < v < 1$

$\hat{n} = \hat{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

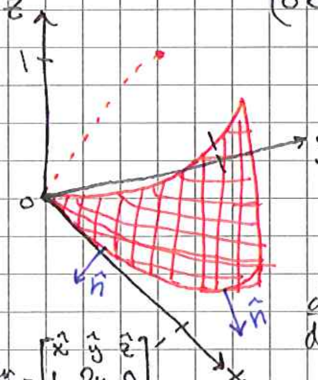


$$\int_0^2 \int_0^1 \underbrace{\begin{bmatrix} \sqrt{u+v} \\ \cos 3 \\ -5 \end{bmatrix}}_{\bar{F}} \cdot \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{\hat{n}} \underbrace{du\, dv}_{dS} =$$

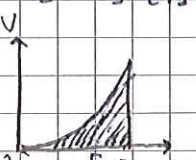
$$= \int_0^2 \int_0^1 \sqrt{u+v}\, du\, dv$$

$(\dots)$

Ex $\bar{F}$ som ovan, ytan: $\begin{cases} y = x^2 \\ z < y \\ 0 < x, y, z < 1 \end{cases}$



$$\bar{r}(u,v) = \begin{bmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{bmatrix} = \begin{bmatrix} u \\ u^2 \\ v \end{bmatrix} \quad 0 < u < 1, \quad 0 < v < u^2$$



$$\frac{d\bar{r}}{du} = \begin{bmatrix} 1 \\ 2u \\ 0 \end{bmatrix} \quad \frac{d\bar{r}}{dv} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{d\bar{r}}{du} \times \frac{d\bar{r}}{dv} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2u & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{bmatrix} 2u \\ -1 \\ 0 \end{bmatrix}$$

$$\int_0^1 \int_0^{u^2} \begin{bmatrix} \sqrt{u^2+v} \\ \cos u \\ -5 \end{bmatrix} \cdot \begin{bmatrix} 2u \\ -1 \\ 0 \end{bmatrix} du\, dv = \int_0^1 \int_0^{u^2} \left(2u\sqrt{u^2+v} - \cos u + 0\right) du\, dv$$