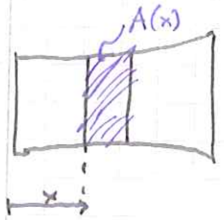
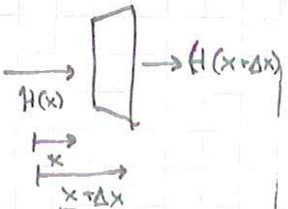
21/3-2013

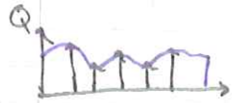
(2).

1D Värmeledning



Q: Värmetillförsel [J/m³s]





H: Totalt värmeflöde [J/s]

"Ut - in"

$$H(x+\Delta x) - H(x) - \int_x^{x+\Delta x} Q(\eta)\,d\eta = 0$$

$$\int_x^{x+\Delta x} Q(\eta)\,d\eta = Q(\xi)\Delta x \qquad \xi \in [x, x+\Delta x]$$

Analys: Medelvärdessats.

$$H(x+\Delta x) - H(x) - Q(\xi)\Delta x = 0$$

$$\frac{H(x+\Delta x) - H(x)}{\Delta x} - Q(\xi) = 0$$

$\frac{dH}{dx}$, $Q(x)$

$\Rightarrow$ Balanslag: $\boxed{\frac{dH}{dx} - Q = 0}$

Definiera värmeflux:

$$\boxed{q = \frac{H}{A}} \quad [J/m^2 s] \Rightarrow \frac{d}{dx}(qA) - Q = 0$$

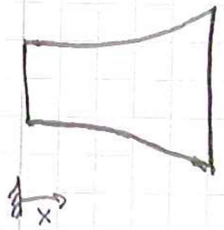
Fouriers lag (1822)

$$\begin{cases} q \propto \Delta T \\ q \propto \frac{1}{\Delta x} \end{cases} \Rightarrow q \propto \frac{\Delta T}{\Delta x} \Rightarrow q \propto \frac{dT}{dx}$$

: {linjärt antagande} $\Rightarrow q = -k\frac{dT}{dx}$

$$\Rightarrow \boxed{\frac{d}{dx}\left(Ak\frac{dT}{dx}\right) + Q = 0}$$

$T = T_0$ x=L, $q = h$ x=0 konstitutionell lag.



Randvillkor: STARK FORM (gäller punktvis)

$q(0) = 0$

$T(L) = T_0$

$q = \alpha(T - T_\infty)$.

Gör om till svag form:

1. Multiplicera med godtycklig viktsfunktion (v):

$$v \cdot 0 = v\frac{d}{dx}\left(Ak\frac{dT}{dx}\right) + vQ = 0$$

2. Integrera över kroppen

$$\int_0^L v\frac{d}{dx}\left(Ak\frac{dT}{dx}\right)dx + \int_0^L vQ\,dx = 0$$

3. Partialintegrera

$$\int_0^L v\frac{d}{dx}\left(Ak\frac{dT}{dx}\right)dx = \left[vAk\frac{dT}{dx}\right]_0^L - \int_0^L \frac{dv}{dx}Ak\frac{dT}{dx}\,dx$$

$$\Rightarrow \int_0^L \frac{dv}{dx}Ak\frac{dT}{dx} = \left[vAk\frac{dT}{dx}\right]_0^L + \int_0^L vQ\,dx$$

4. Sätt in de naturliga randvillkoren

$$\boxed{\int_0^L \frac{dv}{dx}Ak\frac{dT}{dx}\,dx = (vAq)_{x=L} + (vA)_{x=0}h + \int_0^L vQ\,dx}$$

SVAG FORM gäller över hela kroppen

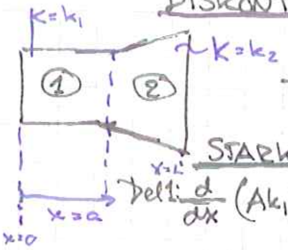
där $h = q(x=0) = -k\frac{dT}{dx}\Big|_{x=0}$

Nu vill vi visa svag form $\Rightarrow$ stark form!
Partialintegration i vänsterledet.
Välj v för att få bort oönskade termer.

"Fördelar" med svag form:

- Lägre ordning på derivatorna
- Tillåter diskontinuiteter

DISKONTINUITETER



Värmet som passerar x=a

$$-\left(Ak_1\frac{dT}{dx}\right)_{x=a} = -\left(Ak_2\frac{dT}{dx}\right)_{x=a} \quad (1)$$

STARK FORM:

Del 1: $\frac{d}{dx}\left(Ak_1\frac{dT}{dx}\right) + Q = 0 \qquad 0 < x < a \quad (2)$

$q(x=0) = h \quad (3)$

Del 2: $\frac{d}{dx}\left(Ak_2\frac{dT}{dx}\right) + Q = 0 \qquad a < x < L \quad (4)$

$T(x=L) = g \quad (5).$

SVAG FORM:

Del 1: $\int_0^a \frac{dv}{dx}Ak_1\frac{dT}{dx}\,dx = -(vAq)_{x=a} + (vAq)_{x=0} + \int_0^a vQ\,dx$

Del 2: $\int_a^L \frac{dv}{dx}Ak_2\frac{dT}{dx}\,dx = -(vAq)_{x=L} + (vAq)_{x=a} + \int_a^L vQ\,dx$

+ (Addera) $\{k_1 + k_2 = k(x)\} \Leftarrow k = \begin{cases} k_1 & 0 < x < a \\ k_2 & a < x < L \end{cases}$

$$\int_0^L \frac{dv}{dx}Ak\frac{dT}{dx}\,dx = -(vAq)_{x=L} + (vAq)_{x=0} + \int_0^L vQ\,dx$$